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# FRW model with vector fields in N=1 supergravity

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*Abstract.* A FRW model obtained from N=1 supergravity with supermatter is analysed in this paper. The matter content is restricted to a vector supermultiplet. The Lorentz and supersymmetry constraints are derived. Non-trivial solutions (no-boundary and wormhole states) are then found.

## 1 Introduction

Research in supersymmetric quantum cosmology using canonical methods started about 10 years or so [1]. A review on quantum N=1,2 supergravity is found in ref. [2].

Finding and identifying physical states in minisuperspaces obtained from N=1 supergravity with supermatter constitutes an important assignment. A FRW model in the presence of a scalar supermultiplet (constituted by complex scalar fields,  $\phi, \bar{\phi}$  and their spin- $\frac{1}{2}$  partners,  $\chi_A, \bar{\chi}_{A'}$ ) and a vector supermultiplet (formed by a gauge vector field  $A_\mu^{(a)}$  and its supersymmetric partner) was analysed in ref. [3]. However, the results found there were disappointing: the only allowed physical state was  $\Psi = 0$ .

The main purpose of this paper and ref. [4] is to initiate a discussion on the paradoxical situation found in ref. [3]. We will study a FRW model where supermatter is restricted to a vector supermultiplet. In section 2 we will address the ansätze for the field variables employed in ref. [3]. In section 3 we derive the quantum constraints. In contrast with ref. [3], *non-trivial* solutions are obtained. We identify a *component* of the Hartle-Hawking (no-boundary) solution [8]. These results support our approach. Finally, our discussions and conclusions close this paper in section 4.

## 2 Ansätze for the field variables

The action for our model is obtained from the *more general* theory of N=1 supergravity with gauged supermatter [6]. We put all scalar fields and corresponding supersymmetric partners equal to zero. Our field variables will be the tetrad,  $e_{\mu}^{AA'}$ , the gravitino fields,  $\psi_{\mu}^A, \bar{\psi}_{\nu}^{A'}$ , a gauge spin-1 field,  $A_{\mu}^{(a)}$ , ((a) is a gauge group index) and the spin- $\frac{1}{2}$  partners,  $\lambda_A^{(a)}, \bar{\lambda}_{A'}^{(a)}$ . The restriction to a closed FRW model requires specific ansätze for these fields.

We choose the geometry to be that of a  $k = +1$  Friedmann model with  $S^3$  spatial sections. The ansatz for the tetrad can then be written as

$$e_{a\mu} = \text{diag} (N(\tau), a(\tau)) , \tag{2.1}$$

where  $\hat{a}$  and  $i$  run from 1 to 3.  $E_{\hat{a}i}$  is a basis of left-invariant 1-forms on the unit  $S^3$  with volume  $\sigma^2 = 2\pi^2$ . The Lagrange multipliers  $\psi^A_0$  and  $\bar{\psi}^{A'}_0$  are taken to be functions of time only. The ansatz for the gravitino field further includes

$$\psi^A_i = e^{AA'}_i \bar{\psi}_{A'} , \quad \bar{\psi}^{A'}_i = e^{AA'}_i \psi_A , \tag{2.2}$$

where we introduce the new spinors  $\psi_A$  and  $\bar{\psi}_{A'}$  which are functions of time only.

In the case of pure N=1 supergravity, ansätze (2.1), (2.2) are preserved by a combination of local coordinate, Lorentz and supersymmetry transformations. This holds provided that the generators of Lorentz, coordinate and supersymmetry transformations satisfy specific conditions. The Lorentz constraint  $J_{AB} = 0$  also has to be imposed.

Let us then consider the case of a gauge group  $\hat{G} = SU(2)$ .

The simplest choice would be to take  $A_{\mu}^{(a)}, \lambda_A^{(a)}, \bar{\lambda}_{A'}^{(a)}$  as time-dependent only. However, this is not sufficient in ordinary quantum cosmology with Yang-Mills fields. Special ansätze are required for  $A_{\mu}^{(a)}$  [8, 5]. The ansatz described in ref. [8, 5] for  $A_{\mu}^{(a)}$  (and also employed in [3]) is the simplest one that allows vector fields to be present in a FRW geometry. The spin-1 field is taken to be

$$\mathbf{A}_{\mu}(t) \omega^{\mu} = \left( \frac{f(t)}{4} \varepsilon_{(a)i(b)} \mathcal{T}^{(a)(b)} \right) \omega^i . \tag{2.3}$$

Here  $\{\omega^{\mu}\} = \{dt, \omega^i\}$ ,  $\omega^i = \hat{E}^i_{\hat{c}} dx^{\hat{c}}$  ( $i, \hat{c} = 1, 2, 3$ ) and  $\mathcal{T}_{(a)(b)}$  are the generators of the  $SU(2)$  gauge group. We will use the more general choice for the fermionic partner of  $A_{\mu}^{(a)}$  as  $\lambda_A^{(a)} = \lambda_A^{(a)}(t)$ . It is then possible to see that these choice of field configurations are invariant for a specific and rather restrictive combination of Lorentz, gauge, supersymmetry and local coordinate transformations (see ref. [4] for more details).

### 3 Quantum constraints and solutions

We chose  $(\bar{\lambda}_{A'}^{(a)}, \psi_A, a, f)$  to be the coordinates of the configuration space and  $(\lambda_A^{(a)}, \bar{\psi}_A, \pi_a, \pi_f)$  to be the momentum operators in this representation. Hence  $\lambda_A^a \rightarrow -\frac{\partial}{\partial \bar{\lambda}^{(a)A}}$ ,  $\bar{\psi}_A \rightarrow \frac{\partial}{\partial \psi^A}$ ,  $\pi_a \rightarrow \frac{\partial}{\partial a}$ ,  $\pi_f \rightarrow -i\frac{\partial}{\partial f}$ .

The supersymmetry constraints for our FRW model have the differential operator form

$$\begin{aligned}
 S_A = & -\frac{1}{2\sqrt{6}}a\psi_A\frac{\partial}{\partial a} - \sqrt{\frac{3}{2}}\sigma^2a^2\psi_A - \frac{1}{8\sqrt{6}}\psi_B\psi^B\frac{\partial}{\partial \psi^A} \\
 & - \frac{1}{4\sqrt{6}}\psi^C\bar{\lambda}_C^{(a)}\frac{\partial}{\partial \bar{\lambda}^{(a)A}} + \frac{1}{3\sqrt{6}}\sigma^a{}_{AB'}\sigma^{bCC'}n_D{}^{B'}n_{C'}^B\bar{\lambda}^{(a)D}\psi_C\frac{\partial}{\partial \bar{\lambda}^{(b)B}} \\
 & + \frac{1}{6\sqrt{6}}\sigma^a{}_{AB'}\sigma^{bBA'}n_D{}^{B'}n_{A'}^E\bar{\lambda}^{(a)D}\bar{\lambda}_B^{(b)}\frac{\partial}{\partial \psi^E} - \frac{1}{2\sqrt{6}}\psi_A\bar{\lambda}^{(a)C}\frac{\partial}{\partial \bar{\lambda}^{(a)C}} \\
 & + \sigma^a{}_{AA'}n^{BA'}\bar{\lambda}_B^{(a)}\left(-\frac{\sqrt{2}}{3}\frac{\partial}{\partial f} + \frac{1}{8\sqrt{2}}(1-(f-1)^2)\sigma^2\right) + \frac{3}{8\sqrt{6}}\bar{\lambda}_A^a\lambda^{(a)C}\frac{\partial}{\partial \psi^C} \quad (3.1)
 \end{aligned}$$

and Hermitian conjugate.

When matter fields are taken into account we have  $J_{AB} = \psi_{(A}\bar{\psi}^{B'}n_{B)B'} - \lambda_{(A}^{(a)}\bar{\lambda}^{(a)B'}n_{B)B'} = 0$ . The Lorentz constraint  $J_{AB}$  implies that a physical wave function should be a Lorentz scalar:

$$\begin{aligned}
 \Psi = & A + B\psi^C\psi_C + d_a\lambda^{(a)C}\psi_C + c_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}_C^{(b)} + e_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}_C^{(b)}\psi^D\psi_D \\
 & + c_{abc}\bar{\lambda}^{(a)C}\bar{\lambda}_C^{(b)}\bar{\lambda}^{(c)D}\psi_D + c_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}_C^{(b)}\bar{\lambda}^{(c)D}\bar{\lambda}_D^{(d)} + d_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}_C^{(b)}\bar{\lambda}^{(c)D}\bar{\lambda}_D^{(d)}\psi^E\psi_E \\
 & + \mu_1\bar{\lambda}^{(2)C}\bar{\lambda}_C^{(2)}\bar{\lambda}^{(3)D}\bar{\lambda}_D^{(3)}\bar{\lambda}^{(1)E}\psi_E \\
 & + \mu_2\bar{\lambda}^{(1)C}\bar{\lambda}_C^{(1)}\bar{\lambda}^{(3)D}\bar{\lambda}_D^{(3)}\bar{\lambda}^{(2)E}\psi_E + \mu_3\bar{\lambda}^{(1)C}\bar{\lambda}_C^{(1)}\bar{\lambda}^{(2)D}\bar{\lambda}_D^{(2)}\bar{\lambda}^{(3)E}\psi_E \\
 & + F\bar{\lambda}^{(1)C}\bar{\lambda}_C^{(1)}\bar{\lambda}^{(2)D}\bar{\lambda}_D^{(2)}\bar{\lambda}^{(3)E}\bar{\lambda}_E^{(3)} + G\bar{\lambda}^{(1)C}\bar{\lambda}_C^{(1)}\bar{\lambda}^{(2)D}\bar{\lambda}_D^{(2)}\bar{\lambda}^{(3)E}\bar{\lambda}_E^{(3)}\psi^F\psi_F. \quad (3.2)
 \end{aligned}$$

where  $A, B, \dots, G$  are functions of  $a, f$  only.

From  $S_A\Psi = 0, \bar{S}_A\Psi = 0$  we obtain

$$A = e^{-3\sigma^2a^2}e^{\frac{3}{16}\sigma^2}\left(-\frac{f^3}{3}+f^2\right), \quad G = e^{3\sigma^2a^2}e^{\frac{3}{16}\sigma^2}\left(\frac{f^3}{3}-f^2\right). \quad (3.3)$$

The last solution in (3.3) is present in the Hartle-Hawking (no-boundary) solution of ref. [8]. However, the wave function (3.3) represents only one of the components of the wave function in ref. [8]. The first solution in (3.3) has wormhole features. The Dirac bracket of the supersymmetry constraints induces an expression whose bosonic sector corresponds to the (*decoupled*) gravitational and vector field components of the Hamiltonian constraint in ref. [8].

## 4 Discussions and Conclusions

Summarizing our work, we considered the canonical formulation of the more general theory of  $N = 1$  supergravity with supermatter [6] subject to a  $k = +1$  FRW geometry. Ansätze for the the gravitational and gravitino fields, the gauge vector field  $A_\mu^a$  and fermionic partners were introduced. The scalar fields and their partners were set equal to zero.

Concerning the ansätze employed here (and also in ref. [3]), the form of the tetrad and gravitinos were consistent with the FRW geometry. Supersymmetry invariance was achieved for  $A_\mu^{(a)}$  and  $\lambda_A^{(a)}$  if further conditions were imposed.

Interesting physical features were derived in section 3. After a dimensional reduction, we obtained the supersymmetric constraints. We found a non-trivial solution that can be interpreted as a (Hartle-Hawking) no-boundary solution. This result were quite supportive. Namely, the Hartle-Hawking solution found here corresponded to a component of a solution found from a Wheeler-DeWitt equation in ordinary quantum cosmology [8].

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