QCD corrections to the W decay width within a new dimensional regularization scheme

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QCD corrections to the W decay width within a new dimensional regularization scheme

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Abstract

A new dimensional regularization scheme for infrared and collinear singularities is worked out for the example of $O(\alpha_s)$ QCD corrections to the total W decay width. This scheme can be applied to other similar processes involving the γ_5 matrix. The QCD corrections to the W width are calculated explicitly according to this new scheme for arbitrary quark masses. Furthermore the same calculations are done in a more traditional scheme, leading to identical results.

1 Introduction

We consider first order QCD corrections to the electro-weak process

$$W^+ \to t + \bar{b} \tag{1}$$

t denotes a generic up-type quark with mass m_t and $\bar{b}$ stands for a down-type antiquark with mass m_b.
For $m_t \neq 0$ and $m_b \neq 0$ the loop graphs contain ultraviolet and infrared singularities, whereas the bremsstrahlung graphs only contain infrared singularities. In the case where $m_t = m_b = 0$ both, the loop graphs and the bremsstrahlung graphs are afflicted in addition with collinear singularities. In order to regularize all these types of singularities we work in d dimensions. For $d > 4$ we only have ultraviolet singularities whereas for $d < 4$ we only have infrared (and collinear) singularities. This implies that there is no dimension d where all types of singularities are regularized simultaneously. Nevertheless, we are allowed to regularize all types of singularities with the single parameter d, because on the one-loop level the ultraviolet singularities can be neatly separated from the infrared and collinear ones, as explicit calculations show.
However, there are problems how one precisely generalizes the matrix elements from 4 to d dimensions, in particular when γ_5 matrices are involved. We present a new method, where we give prescriptions how to treat γ_5 in our specific example. As this method explicitly includes the emission of gluons, which are pseudoscalars under the four-dimensional Lorentz group, this new scheme is referred to as the pseudoscalar gluon scheme, thereafter. It will become clear in the following description of the regularization procedure, that our method is of course not restricted to QCD corrections to the specific process in equation (1). It can be used for all gluon/photon bremsstrahlung processes, where the gluon/photon is radiated from a massive or massless external particle, and for the corresponding virtual corrections. As this scheme is technically easy to handle, we already made use of it in different applications. One such application was the calculation of inclusive lepton pair production through virtual W, Z and γ gauge bosons in proton - antiproton collisions. [See references [1], [2] and [3]]. If the lepton variables are not completely integrated out, the γ_5 problem cannot be circumvented any longer and a clear prescription has to be given. Another application, where this scheme was very useful, is the calculation of the inclusive photon energy spectrum from the rare b-quark decay $b \to s + g + \gamma$, where s, g and γ denote a s-quark, a gluon and a photon, respectively. [See ref. [4]].
There are many other papers about dimensional regularization in the literature. In reference [5] only some of them are mentioned.
Our paper is organized as follows:
In order to fix the notation, we briefly review the Feynman rules which are relevant for our process. In section 2 we give the regularization prescriptions for the pseudoscalar gluon scheme without doing the explicit calculations. In section 3 we present the calculation according to the setup in section 2 for the case where $m_t = m_b = 0$. In section 4 we speak about the results in the case where $m_t \neq 0$ and $m_b \neq 0$. [This case would be important for the physical $t\bar{b}$ decay channel of W if $m_t + m_b$ turns out to be smaller than m_W after all]. In section 5 we speak about the results one gets in a more traditional regularization scheme,

in order to convince ourself that the new scheme really leads to the correct physical results. In section 6 we illustrate the results from section 4 with some plots.

1.1 Feynman rules

We only write down the Feynman rules which are relevant for describing W^+ decay including the first order QCD corrections.

Fields

$\Psi^c_{t,a}$	quark field	c: colour index t: flavour index a: Dirac index (usually suppressed)
A^A_μ	gluon field	A: colour index μ: vector index
W_μ	W field	μ: vector index

Free propagators

Because it will turn out that only the quark masses and the quark wave functions undergo renormalization it is sufficient to work in the unitary gauge in the electro-weak sector.

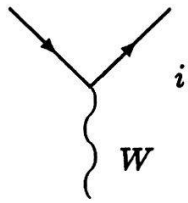
$$\begin{aligned}
\langle 0|T\, W_\mu(x)\, W^+_\nu(y)\,|0\rangle &= \frac{1}{i}\, D_{\mu\nu}(x-y) = \frac{1}{i}\int \frac{d^4\,k}{(2\pi)^4}\, e^{-i\,k\,(x-y)}\, \tilde{D}_{\mu\nu}(k) \\
\langle 0|T\, A^A_\mu(x)\, A^B_\nu(y)\,|0\rangle &= \frac{1}{i}\, G_{\mu\nu}(x-y)\, \delta^{AB} = \frac{1}{i}\int \frac{d^4\,k}{(2\pi)^4}\, e^{-i\,k\,(x-y)}\, \tilde{G}_{\mu\nu}(k)\, \delta^{AB} \\
\langle 0|T\, \Psi^c_t(x)\, \bar{\Psi}^d_{t'}(y)\,|0\rangle &= i\, S(x-y;m)\, \delta^{c\,d}\, \delta_{t\,t'} = i\int \frac{d^4\,k}{(2\pi)^4}\, e^{-i\,k\,(x-y)}\, \tilde{S}(k;m)\, \delta^{cd}\, \delta_{tt'}
\end{aligned}$$

$$\begin{aligned}
\tilde{D}_{\mu\nu}(k) &= \left\{ \frac{g_{\mu\nu} - \frac{k_\mu\, k_\nu}{m_w^2}}{k^2 - m_w^2 + i\eta} \right\} \\
\tilde{G}_{\mu\nu}(k) &= \left\{ \frac{g_{\mu\nu}}{k^2 + i\eta} + \frac{1-\lambda}{\lambda}\, \frac{k_\mu\, k_\nu}{(k^2+i\eta)^2} \right\} \\
\tilde{S}(k;m) &= \left\{ \frac{\not{k} + m}{k^2 - m^2 + i\eta} \right\}
\end{aligned}$$

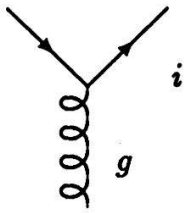
$G_{\mu\nu}(x-y)$ describes the propagation of a free massless gluon in an arbitrary covariant gauge; λ is the gauge parameter:

- $\lambda = 1$: Feyman gauge
- $\lambda = \infty$: Landau gauge

Vertices



$i\,\frac{g}{\sqrt{2}}\,\gamma_\mu\,\frac{1+\gamma_5}{2}$



$i\,h\,\gamma_\mu\,\frac{\lambda^A_{c'c}}{2}$

g and h denote the $SU(2)_L$ and the $SU(3)_C$ coupling constant, respectively. $[\alpha_s = \frac{h^2}{4\pi}]$.

2 Regularization prescriptions for the pseudoscalar gluon scheme

Before discussing the virtual correction and the bremsstahlung graphs seperately, we list some basic points which will be used for both types of graphs:

- the gluon remains massless
- the quark (antiquark) masses are denoted with m_t and m_b
- the four momenta of the external particles in each graph are four-dimensional, i.e.,

$$p^\mu = (p^0, p^1, p^2, p^3, \underbrace{0, \ldots\ldots, 0}_{\hat{d}=d-4})$$

- The polarization vector of the W boson as external particle is restricted to 4 dimensions, i.e.,

$$\epsilon^\mu_W = (\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3, 0, \ldots\ldots, 0)$$

Our calculations are done in the Landau gauge; in this gauge the Feynman integral for the vertex correction and the residue of the quark propagator are finite in the ultraviolet region in $d = 4$ on the one-loop level. For $m = 0$ the quark self-energy even vanishes in this gauge.

2.1 Self-energy contributions

The full propagator for a quark with mass m has the form:

$$\begin{aligned} \langle 0|T\,\Psi^c_t(x)\,\bar{\Psi}^d_{t'}(y)\,|0\rangle &= i\,\underline{S}(x-y;m)\,\delta^{cd}\,\delta_{tt'} = i\int \frac{d^4k}{(2\pi)^4}\,e^{-ik(x-y)}\,\underline{\tilde{S}}(k;m)\,\delta^{cd}\,\delta_{tt'} \\ i\underline{\tilde{S}} &= \frac{i}{\not{p} - m - \Sigma + i\eta} \end{aligned} \tag{2}$$

The one-loop expression for the self-energy Σ reads:

$$\Sigma = \frac{4}{3}(-h^2)\int \frac{d^4k}{(2\pi)^4}\,\gamma_\alpha\,\frac{1}{\not{p}+\not{k}-m+i\eta}\,\gamma_\beta\,\tilde{G}^{\alpha\beta}(k) \tag{3}$$

We generalize this expression to d dimensions as usual:

$$\Sigma = \frac{4}{3}(-h^2)\,\mu^{4-d} \int \frac{d^d k}{(2\pi)^d}\,\gamma_\alpha \frac{1}{\not p + \not k - m + i\eta}\,\gamma_\beta\,\tilde{G}^{\alpha\beta}(k) \tag{4}$$

where μ is an arbitrary mass scale. All the gamma matrices are now matrices belonging to the Dirac algebra in d dimensions. Note, that the indices α and β run from 0 to $d-1$. The propagator can be rewritten in the form

$$i\,\underline{\tilde{S}} = \frac{i\,Z_2(m)}{(\not p - m)\,[1 + O(\not p - m)]} \tag{5}$$

where mass renormalization has been carried out tacitly. The residue $Z_2(m)$ is given by:

$$Z_2(m) = 1 + i\frac{\partial \Sigma}{\partial \not p}\Big|_{\not p = m}$$

In the Landau gauge we get:

$$Z_2(m) = 1 - \frac{\alpha_s}{3\pi}\left[4 + 3\left(\frac{1}{\epsilon} - \gamma_E + \log\frac{4\pi\mu^2}{m^2}\right)\right] \tag{6}$$

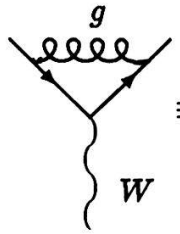
where $2\epsilon = 4 - d$ and $\gamma_E = 0.577...$ denotes the Euler constant. Note, that the $\frac{1}{\epsilon}$ -poles in $Z_2(m)$ are of infrared origin. [In the case where $m = 0$ a direct evaluation of integral (4) yields $\Sigma = 0$. Therefore $Z_2(0) = 1$ in the Landau gauge.]
The summation of the zeroth order graph and the quark self-energy correction diagrams is achieved by attaching a factor

$$\sqrt{Z_2(m_t)} \cdot \sqrt{Z_2(m_b)}$$

to the zeroth order graph.

2.2 W vertex correction

The vertex correction is only calculated on shell. In 4 dimensions the expression for the W vertex correction reads:



$$\equiv C_{ba;\mu} = (4/3)\,\tfrac{g}{\sqrt{2}}\,I_\mu\,\delta_{ab}$$

$$I_\mu = h^2 \int \frac{d^4 k}{(2\pi)^4}\,\gamma_\sigma\,\frac{\not k + \not p + m_t}{k^2 + 2pk + i\epsilon}\,\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\frac{\not k - \not p' + m_b}{k^2 - 2p'k + i\epsilon}\,\gamma_\rho\,\tilde{G}^{\rho\sigma}(k) \tag{7}$$

p' : four momentum of the outgoing antiquark $\quad (p')^2 = (m_b)^2$
p : four momentum of the outgoing quark $\quad (p)^2 = (m_t)^2$

The regularization of the formal expression is done in three steps :

STEP 1

The formal 4-dimensional expression can be rewritten in a way where the chirality structure of the outgoing (anti)quarks is manifest:

$$
\begin{aligned}
I_\mu &= \left(\frac{1-\gamma_5}{2}\right) I_\mu^{(a)} \left(\frac{1+\gamma_5}{2}\right) + \left(\frac{1+\gamma_5}{2}\right) I_\mu^{(b)} \left(\frac{1+\gamma_5}{2}\right) + \\
&\quad + \left(\frac{1+\gamma_5}{2}\right) I_\mu^{(c)} \left(\frac{1-\gamma_5}{2}\right) + \left(\frac{1-\gamma_5}{2}\right) I_\mu^{(d)} \left(\frac{1-\gamma_5}{2}\right) \\
I_\mu^{(a)} &= h^2 \int \frac{d^4 k}{(2\pi)^4} \gamma_\sigma \frac{\not k + \not p}{k^2 + 2pk + i\epsilon} \gamma_\mu \frac{\not k - \not p'}{k^2 - 2p'k + i\epsilon} \gamma_\rho \tilde{G}^{\rho\sigma}(k) \\
I_\mu^{(b)} &= h^2 \int \frac{d^4 k}{(2\pi)^4} \gamma_\sigma \frac{m_t}{k^2 + 2pk + i\epsilon} \gamma_\mu \frac{\not k - \not p'}{k^2 - 2p'k + i\epsilon} \gamma_\rho \tilde{G}^{\rho\sigma}(k) \\
I_\mu^{(c)} &= h^2 \int \frac{d^4 k}{(2\pi)^4} \gamma_\sigma \frac{m_t}{k^2 + 2pk + i\epsilon} \gamma_\mu \frac{m_b}{k^2 - 2p'k + i\epsilon} \gamma_\rho \tilde{G}^{\rho\sigma}(k) \\
I_\mu^{(d)} &= h^2 \int \frac{d^4 k}{(2\pi)^4} \gamma_\sigma \frac{\not k + \not p}{k^2 + 2pk + i\epsilon} \gamma_\mu \frac{m_b}{k^2 - 2p'k + i\epsilon} \gamma_\rho \tilde{G}^{\rho\sigma}(k)
\end{aligned}
\tag{8}
$$

Note, that the integrals do not contain a γ_5 anymore.

STEP 2

In this step we generalize the Dirac algebra and the Dirac spinors to d dimensions:

- Dirac algebra

  In d dimensions (d even) we have d $(2^{d/2} \times 2^{d/2})$ matrices $\Gamma_0, \Gamma_1, \ldots, \Gamma_{d-1}$ satisfying the algebra

$$
\begin{aligned}
\{\Gamma_\mu, \Gamma_\nu\} &= 2\, g_{\mu\nu}\, \mathbf{1} \qquad \mu, \nu = 0, 1, 2, \ldots, d-1 \\
g_{\mu\nu} &= \mathrm{diag}(1, -1, \ldots\ldots, -1)
\end{aligned}
$$

  Especially we have $\Gamma_\mu \Gamma^\mu = d\,\mathbf{1}$. For some aspects it is convenient to decompose the Dirac algebra into a direct product:

$$
\begin{aligned}
d &= 4 + \hat{d} \qquad\qquad \hat{d} : \text{extra dimensions} \\
\underbrace{\gamma_0, \gamma_1, \gamma_2, \gamma_3}_{4} &\ ; \ \underbrace{\hat{\gamma}_1, \hat{\gamma}_2, \ldots, \hat{\gamma}_{\hat{d}}}_{\hat{d}} \\
\{\gamma_\mu, \gamma_\nu\} &= 2 g_{\mu\nu}\, \mathbf{1} \qquad \mu, \nu = 0, 1, 2, 3 \quad (4 \times 4) \text{ matrices} \\
\{\hat{\gamma}_i, \hat{\gamma}_j\} &= -2\delta_{ij}\, \mathbf{1} \qquad i, j = 1, 2, \ldots, \hat{d} \quad (2^{\hat{d}/2} \times 2^{\hat{d}/2}) \text{ matrices} \\
\gamma_5 &= i \gamma_0 \gamma_1 \gamma_2 \gamma_3 \\
\hat{\gamma}_{\hat{d}+1} &= i^{\frac{3\hat{d}}{2}}\, \hat{\gamma}_1 \cdots \hat{\gamma}_{\hat{d}}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_0 &= \gamma_0 \otimes \mathbf{1} \\
\Gamma_1 &= \gamma_1 \otimes \mathbf{1} \\
\Gamma_2 &= \gamma_2 \otimes \mathbf{1} \\
\Gamma_3 &= \gamma_3 \otimes \mathbf{1} \\
\Gamma_4 &= \gamma_5 \otimes \hat{\gamma}_1 \\
\Gamma_5 &= \gamma_5 \otimes \hat{\gamma}_2 \\
&\vdots \\
\Gamma_{d-1} &= \gamma_5 \otimes \hat{\gamma}_{\hat{d}}
\end{aligned} \tag{9}
$$

We list some properties of these different gamma matrices :

$$
\begin{aligned}
\Gamma_{d+1} &= i^{\frac{3d-2}{2}}\, \Gamma_0 \cdots \Gamma_{d-1} = \gamma_5 \otimes \hat{\gamma}_{\hat{d}+1} \\
\Gamma_{d+1}^2 &= \mathbf{1} \qquad \Gamma_{d+1}^+ = \Gamma_{d+1} \\
\gamma_5^2 &= \mathbf{1} \qquad \gamma_5^+ = \gamma_5 \\
\hat{\gamma}_{\hat{d}+1}^2 &= \mathbf{1} \qquad \hat{\gamma}_{\hat{d}+1}^+ = \hat{\gamma}_{\hat{d}+1}
\end{aligned}
$$

- <u>Spinors</u>
  Because the momenta of the external particles are four-dimensional , it is also convenient to decompose the spinors into a direct product. The Dirac equation in d dimensions for a particle with mass m reads:

$$\not{p} U(p) = p_\mu \Gamma^\mu U(p) = m U(p)$$

Because p_μ is four-dimensional the solution to this equation can be written in the form:

$$U(p) = u(p) \otimes \chi$$

where we used the notation :

U(p)	Dirac spinor in d dimensions
u(p)	Dirac spinor in 4 dimensions
χ	constant spinor in $\hat{d}$ dimensions: $2^{\hat{d}/2}$ degrees of freedom

A similar decomposition also holds for antiparticle spinors V(p) satisfying

$$\not{p} V(p) = p_\mu \Gamma^\mu V(p) = -m V(p)$$

<u>STEP 3</u>

Now we are ready to write down the regularized version for I_μ . We do the following replacements in equations (8) :

- $\frac{1\pm\gamma_5}{2} \longrightarrow \frac{1\pm\gamma_5}{2} \otimes \mathbf{1}$

- $\gamma_\alpha \to \Gamma_\alpha$ for all gamma matrices, e.g. ,
  $\not{p} = p_\alpha \gamma^\alpha \to p_\alpha \Gamma^\alpha$
- $\frac{d^4 k}{(2\pi)^4} \to \frac{d^d k}{(2\pi)^d}$
- $h^2 \to h^2 \cdot \mu^{4-d}$

Note, that the indices ρ and σ vary from 0 to $d-1$, whereas the index μ, which couples to the external W polarization vector only take the values 0,...,3. Now the expression for I_μ is well-defined and can be evaluated. Note, that I_μ is understood to stand between spinors in d-dimensional space. Therefore, doing the algebraic manipulations the Dirac equation is used, e.g. ,

$$\not{p}(u(p) \otimes \chi) = m(u(p) \otimes \chi)$$

i.e., we only calculate the vertex correction I_μ on shell.
We do not give the detailed results here, but we mention its structure:

$$I_\mu = \tilde{I}_\mu \otimes \mathbf{1} \tag{10}$$

where $\tilde{I}_\mu$ is

$$\begin{aligned} \tilde{I}_\mu \;=\; & \frac{-ih^2}{16\pi^2} \left\{ \gamma_\mu \left(\frac{1+\gamma_5}{2}\right) \bar{\mathbf{A}} + \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) \bar{\mathbf{B}} + \right. \\ & -i\sigma_{\mu\nu} p'^\nu \left(\frac{1+\gamma_5}{2}\right) \bar{\mathbf{C}} - i\sigma_{\mu\nu} p'^\nu \left(\frac{1-\gamma_5}{2}\right) \bar{\mathbf{D}} + \\ & \left. +i\,\sigma_{\mu\nu} p^\nu \left(\frac{1+\gamma_5}{2}\right) \bar{\mathbf{E}} + i\sigma_{\mu\nu} p^\nu \left(\frac{1-\gamma_5}{2}\right) \bar{\mathbf{F}} \right\} \end{aligned} \tag{11}$$

The form factors $\bar{\mathbf{A}}, \bar{\mathbf{B}}, ..., \bar{\mathbf{F}}$ depend only on the scalar product $2pp'$, the quark masses m_t , m_b , and on the mass scale μ introduced through dimensional regularization. We should point out that all singularities are contained in the form factor $\bar{\mathbf{A}}$.

2.3 Matrix element and phase space for W decay [Zeroth order + loop corrections]

The matrix element M including zeroth order and all one-loop contributions to the W decay width reads (in d dimensions):

$$\begin{aligned} M \;=\; & \left\{ i\,\frac{g}{\sqrt{2}} \sqrt{Z_2(m_t) Z_2(m_b)}\; \bar{U}(p)\Gamma_\mu \left(\frac{1+\gamma_5}{2} \otimes \mathbf{1}\right) V(p')\delta_{ab} + \right. \\ & \left. + (4/3)\,\frac{g}{\sqrt{2}}\, \bar{U}(p) I_\mu V(p')\delta_{ab} \right\} \cdot \epsilon_W^{\star\mu} \end{aligned} \tag{12}$$

Because of the direct product structure of the different quantities in M it can be written as

$$M = \tilde{M}\;(\chi^+ \chi) \tag{13}$$

where

$$\tilde{M} = \left\{ i \frac{g}{\sqrt{2}} \sqrt{Z_2(m_t) Z_2(m_b)}\, \bar{u}(p) \gamma_\mu \left(\frac{1+\gamma_5}{2} \right) v(p')\, \delta_{ab} + \right.$$
$$\left. + (4/3) \frac{g}{\sqrt{2}} \bar{u}(p) \tilde{I}_\mu v(p')\, \delta_{ab} \right\} \cdot \epsilon_W^{\star\mu} \tag{14}$$

Taking $|M|^2$ and summing over the extra spinorial degrees of freedom we get

$$\sum_{\text{extra spins}} |M|^2 = 2^{\hat{d}/2}\, |\tilde{M}|^2$$

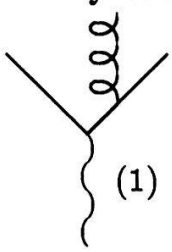
On this level, the ultraviolet divergences have cancelled and we are left with infrared singularities . They are regularized by the dimension parameter d, i.e., we have to stay in d dimensions. However we are allowed to omit the factor $2^{\hat{d}/2}$ due to its universality: The factor $2^{\hat{d}/2}$ tends to 1 as $\hat{d} \to 0$. It will also be present in the bremsstrahlung contributions. Therefore we leave it out, i.e., what we really calculate is $|\tilde{M}|^2$.
The phase space integrals will be worked out in d dimensions.

2.4 Summary concerning the loop-corrections

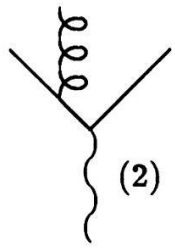
The matrix element M involving the zeroth order graph and the loop-corrections has been regularized in such a way that the four-dimensional chirality structure is maintained. In particular, for massless (anti)quarks this means that only quarks with helicity -1 and antiquarks with helicity +1 appear in the final state.

2.5 Gluon bremsstrahlung

For W^+ decay there are two bremsstrahlung diagrams:



(1)



(2)

Again all four momenta of the external particles are taken to be four-dimensional; this is no loss of generality as long as the number of particles involved is ≤ 5. The polarization vector of the W^+ boson also lies in four dimensions. However, we are not allowed to consider only four-dimensional gluon polarizations, i.e., we also have to include the additional $\hat{d} = d - 4$ gluon polarization vectors:

$$\begin{aligned} \hat{\epsilon}_\mu^{(1)} &= (0,0,0,0;1,\ldots\ldots,0) \\ &\vdots \\ \hat{\epsilon}_\mu^{(\hat{d})} &= (0,0,0,0;0,\ldots\ldots,1) \end{aligned}$$

Under the four-dimensional Poincaré group these vectors transform as pseudoscalars. Therefore we denotes these extra degrees of freedom as pseudoscalar gluons.
The reason, why we have to include these pseudoscalars, can be seen in the discussion of the virtual corrections, because the regularization we did there includes the propagation of pseudoscalar degrees of freedom: This is most easily seen in the gluon propagator:

$$\tilde{G}_{\rho\sigma}(k) = \frac{g_{\rho\sigma}}{k^2 + i\epsilon} + \text{other terms}$$

Of course ρ and σ are not restricted to 4 dimensions, because otherwise we would have been forced to put

$$\Gamma_\rho \Gamma_\sigma g^{\rho\sigma} = 4$$

instead of

$$\Gamma_\rho \Gamma_\sigma g^{\rho\sigma} = d$$

as we did.
Next we want to give the regularized version of sum of the two matrix elements (1) and (2) : In a first step we write down the matrix elements in four dimensions. In a second step we do exactly the same replacements in the four dimensinal expression as we did in STEP 3 when discussing the vertex correction. These two steps lead to:

$$\begin{aligned} M &= M_1 + M_2 \\ M &= \frac{g\,h\,\mu^{4-d}}{\sqrt{2}} \frac{\lambda^A_{ab}}{2} \left\{ \bar{U}_a(p) \left[\not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \otimes 1 \right) \frac{-\not k - \not p' + m_b}{2\,p'k} \not\epsilon^*_g + \right.\right. \\ &\qquad \left.\left. + \not\epsilon^*_g \frac{\not k + \not p + m_t}{2\,pk} \not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \otimes 1 \right) \right] V_b(p') \right\} \end{aligned} \tag{15}$$

Keeping in mind that the W^+ polarizations are restricted to $d = 4$, M can be split into two terms:

$$\begin{aligned} M_{vect.} &= \frac{g\,h\,\mu^{4-d}}{\sqrt{2}} \frac{\lambda^A_{ab}}{2} \left\{ \bar{u}_a(p) \left[\not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \right) \frac{-\not k - \not p' + m_b}{2\,p'k} \not\epsilon^*_g + \right.\right. \\ &\qquad \left.\left. + \not\epsilon^*_g \frac{\not k + \not p + m_t}{2\,pk} \not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \right) \right] v_b(p') \right\} (\chi^+ \chi) \end{aligned} \tag{16}$$

$$\begin{aligned} M_{scal.} &= \frac{g\,h\,\mu^{4-d}}{\sqrt{2}} \frac{\lambda^A_{ab}}{2} \left\{ \bar{u}_a(p) \left[\not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \right) \frac{-\not k - \not p' + m_b}{2\,p'k} \gamma_5 + \right.\right. \\ &\qquad \left.\left. + \gamma_5 \frac{\not k + \not p + m_t}{2\,pk} \not\epsilon^*_W \left(\frac{1+\gamma_5}{2} \right) \right] v_b(p') \right\} (\chi^+ \hat{\gamma}_i \chi) \end{aligned} \tag{17}$$

where $M_{vect.}$ stands for the emission of a vector gluons with polarization vector $\epsilon^*_\mu(k)$ and $M_{scal.}$ stands for the emission of a pseudoscalar gluon.

In a next step one has to take the square of M. We sum over the pseudoscalar gluons and also over the additional spinorial (χ) degrees of freedom.

$$\sum_{\text{spins}} |\chi^+ \chi|^2 = 2^{\hat{d}/2}$$

$$\sum_{\text{spins,scal.}} |\chi^+ \hat{\gamma}_i \chi|^2 = 2^{\hat{d}/2}\, \hat{d}$$

The factor $2^{\hat{d}/2}$ tends to 1 as $\hat{d} \to 0$. Furthermore it is universal, i.e., it appears also in the vertex correction graph and in the 0th order graph. Therefore we omit this factor.
We now introduce new equivalent matrix elements $\tilde{M}_{vect.}$ and $\tilde{M}_{scal.}$, in which only four-dimensional objects appear:

$$\begin{aligned}\tilde{M}_{vect.} \;=\; \frac{g\,h\,\mu^{4-d}}{\sqrt{2}}\,\frac{\lambda^A_{ab}}{2}\Bigg\{\bar{u}_a(p)\Bigg[\not{\epsilon}^*_W\left(\frac{1+\gamma_5}{2}\right)\frac{-\not{k}-\not{p}'+m_b}{2\,p'k}\not{\epsilon}^*_g+ \\ +\not{\epsilon}^*_g\frac{\not{k}+\not{p}+m_t}{2\,pk}\not{\epsilon}^*_W\left(\frac{1+\gamma_5}{2}\right)\Bigg]\,v_b(p')\Bigg\}\end{aligned} \tag{18}$$

$$\begin{aligned}\tilde{M}_{scal.} \;=\; \sqrt{d-4}\,\frac{g\,h\,\mu^{4-d}}{\sqrt{2}}\,\frac{\lambda^A_{ab}}{2}\Bigg\{\bar{u}_a(p)\Bigg[\not{\epsilon}^*_W\left(\frac{1+\gamma_5}{2}\right)\frac{-\not{k}-\not{p}'+m_b}{2\,p'k}i\gamma_5+ \\ +i\gamma_5\frac{\not{k}+\not{p}+m_t}{2\,pk}\not{\epsilon}^*_W\left(\frac{1+\gamma_5}{2}\right)\Bigg]\,v_b(p')\Bigg\}\end{aligned} \tag{19}$$

$\tilde{M}_{vect.}$ and $\tilde{M}_{scal.}$ are equivalent to $M_{vect.}$ and to $M_{scal.}$ in the following sense :

$$|\tilde{M}_{vect.}|^2 \cong \sum_{\text{extra spins}} |M_{vect.}|^2$$

$$|\tilde{M}_{scal.}|^2 \cong \sum_{\text{extra spins,scal. gluons}} |M_{scal.}|^2$$

where $\cong$ means equal up to the factor $2^{\hat{d}/2}$. We mention that $\tilde{M}_{scal.}$ can be interpreted to be generated through the Lagrangian

$$L_{\text{pseudoscalar gluon}} = h\sqrt{d-4}\,\bar{\Psi}_a(x)\,i\gamma_5\,\lambda^A_{ab}\,\Psi_b(x)\;\Phi^A(x) \tag{20}$$

where Φ^A denotes <u>one real</u> pseudoscalar gluon field with colour A.

2.6 Summary concering the bremsstrahlung graphs

The regularization has been done in such a way that the (anti)quarks have the same chirality structure as in four dimensions when a vector gluon is emitted. However, if a pseudoscalar gluon is emitted by a (anti)quark its chirality is flipped. (We should point

out, that because of the extra factor $\sqrt{d-4}$ in equation (19) a pseudoscalar gluon does not contribute if it is not emitted collinearly). In particular, for massless (anti)quarks this means that in the final state a quark with the 'wrong' helicity +1 together with a collinear pseudoscalar gluon can appear (or, equivalently an antiquark with the helicity -1 together with a collinear pseudoscalar gluon). However, these states cannot be distinguished experimentally from the corresponding (anti)quark states with the 'right' helicity. It may be conjectured, that in the case where a massive quark emits a collinear pseudoscalar gluon, the latter could be separated from the quark because of the different speed of the two particles. However, in this case the contribution of $\tilde{M}_{scal.}$ vanishes, which proves the intrinsic consistency of the scheme.

3 W decay into a massless quark and antiquark

We present the explicit calculation for the QCD corrections to the total decay width of the W^+ boson into a massless quark and antiquark according to the pseudoscalar gluon scheme described in section 2. The decay of the W^+ boson is considered in its rest frame. In the following discussion q,p,p' and k stand for the momenta of the W^+ boson, the quark, the antiquark and the gluon, respectively.

3.1 Zeroth order and loop corrections

As the self-energy vanishes in the Landau gauge, we have $Z_2(0) = 1$. For the regularized version of the vertex correction I_μ (see equation (7)) we get:

$$\begin{aligned} I_\mu &= \left(\frac{1-\gamma_5}{2}\otimes 1\right) I_\mu^{(a)} \left(\frac{1+\gamma_5}{2}\otimes 1\right) \qquad \text{with} \\ I_\mu^{(a)} &= h^2(\mu^2)^{4-d}\int \frac{d^d k}{(2\pi)^d}\,\Gamma_\sigma\,\frac{\not k+\not p}{k^2+2pk+i\epsilon}\,\Gamma_\mu\,\frac{\not k-\not p'}{k^2-2p'k+i\epsilon}\,\Gamma_\rho\,\tilde{G}^{\rho\sigma}(k) \end{aligned} \tag{21}$$

This integral yields:

$$\begin{aligned} I_\mu^{(a)} &= \frac{-i}{16\pi^2}\,\Gamma_\mu\,\bar{\mathbf{A}} \\ \bar{\mathbf{A}} &= h^2\frac{\left(\frac{2pp'}{4\pi\mu^2}\right)^{-\epsilon}}{\Gamma(1-\epsilon)}\left[\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+8\right]e^{i\pi\epsilon} \end{aligned} \tag{22}$$

As mentioned above I_μ is of the form

$$I_\mu = \tilde{I}_\mu \otimes 1$$

with

$$\tilde{I}_\mu = \frac{-i}{16\pi^2}\,\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\bar{\mathbf{A}} \tag{23}$$

The matrix element $\tilde{M}$ in equation (14) can then be written as:

$$\tilde{M} = \epsilon_W^{\star\mu}\left[i\frac{g}{\sqrt{2}}\,\bar{u}(p)\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)v(p')\,\delta_{ab}\right]\cdot\left[1-\frac{1}{16\pi^2}\frac{4}{3}\bar{\mathbf{A}}\right] \tag{24}$$

From $\tilde{M}$ we calculate Γ^{virt}, which contains the zeroth order contribution and the loop-corrections to the total decay width for W^+ decaying at rest:

$$d\Gamma^{virt} = \frac{(2\pi)^d}{2m_W}\,\delta^d(p+p'-q)\,\frac{1}{3}\Sigma|\tilde{M}|^2\,d\mu(p)d\mu(p') \tag{25}$$

$$d\mu(p) \doteq \frac{d^{d-1}p}{(2\pi)^{d-1}\,2p^0} \qquad p^0 = +\sqrt{\vec{p}^2} \tag{26}$$

The sum runs over the spins of the quarks and their colour and over the polarizations of the W^+ boson. The factor (1/3) stems from averaging over the W^+ polarizations. The calculation of $\Gamma^{virt} = \int d\Gamma^{virt}$ is now straightforward. We get:

$$\Gamma^{virt} = \frac{g^2 m_W}{16\pi} + \frac{g^2 h^2 m_W}{96\pi^3}\,\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\left\{-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\pi^2\right\} \tag{27}$$

3.2 Gluon bremsstrahlung contributions

We start from the matrix elements $\tilde{M}_{vect.}$ and $\tilde{M}_{scal.}$ in equations (18) and (19), respectively. In a first step we calculate the square of these matrix elements where we immediately sum over spins and colours of the quarks and gluons and average over the W^+ polarizations:

$$\frac{1}{3}\sum_{\text{all}}|\tilde{M}_{vect.}|^2 = \frac{2\,g^2\,h^2}{3\,m_W^2}\,(\mu^2)^{4-d}\,f_{vect.}$$

$$\frac{1}{3}\sum_{\text{all}}|\tilde{M}_{scal.}|^2 = \frac{2\,g^2\,h^2}{3m_W^2}\,(d-4)\,(\mu^2)^{4-d}\,f_{scal.}$$

Using energy momentum conservation

$$q = p + p' + k$$

$f_{vect.}$ and $f_{scal.}$ can be written in the form

$$\begin{aligned} f_{vect.} = {} & \frac{8(pq)(p'q)-4(p'q)^2-4(p'q)(kq)+4(kq)(pq)+2m_W^2(kq)}{p'k} + \\ & + \frac{8(pq)(p'q)-4(pq)^2-4(pq)(kq)+4(kq)(p'q)+2m_W^2(kq)}{pk} + \\ & + \frac{4(pq)(p'q)\,(2m_W^2-(kq))}{(pk)(p'k)} - 8m_W^2 \end{aligned} \tag{28}$$

$$f_{scal.} = -2m_W^2 + (kq)\left\{\frac{2(p'q)+m_W^2}{(pk)} + \frac{2(pq)+m_W^2}{(p'k)}\right\} \tag{29}$$

In a second step we write down the partial decay width $d\Gamma^{brems}_{vect.}$ and $d\Gamma^{brems}_{scal.}$ for a W^+ decaying at rest:

$$d\Gamma^{brems}_{vect.} = \frac{(2\pi)^d}{2m_W}\,\delta^d(p+p'+k-q)\,\frac{2\,g^2\,h^2\,(\mu^2)^{4-d}}{3\,m_W^2}\,f_{vect.}\,d\mu(p)\,d\mu(p')\,d\mu(k) \tag{30}$$

and the corresponding expression for $d\Gamma^{brems}_{scal.}$. The measures $d\mu(p), d\mu(p')$ and $d\mu(k)$ are defined in the same way as in the virtual contributions. Working out the phase space integrals, we get:

$$\Gamma^{brems}_{vect.} = \frac{g^2h^2\,m_W}{96\pi^3}\,\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\left\{\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{17}{2}-\pi^2\right\}$$

$$\Gamma^{brems}_{scal.} = \frac{g^2h^2\,m_W}{96\pi^3}\,\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\,\{1\} \tag{31}$$

and

$$\Gamma^{brems} \doteq \Gamma^{brems}_{vect.} + \Gamma^{brems}_{scal.} = \frac{g^2h^2\,m_W}{96\pi^3}\,\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\left\{\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}-\pi^2\right\} \tag{32}$$

3.3 Total decay width of W^+

The total decay width Γ of a W^+ boson decaying into a massless quark and antiquark including first order QCD corrections yields:

$$\Gamma = \Gamma^{virt} + \Gamma^{brems}$$

Using $\alpha_s = \frac{h^2}{4\pi}$ and $G_F = \frac{g^2}{4\sqrt{2}m_W^2}$ we get:

$$\Gamma = \frac{2\sqrt{2}\,G_F m_W^3}{8\pi}\left[1+\frac{\alpha_s}{\pi}\right] \tag{33}$$

4 W decay width into a massive quark and antiquark

We consider QCD corrections to the process

$$W^+ \to t\bar{b}$$

where the masses m_t and m_b are arbitrary. The calculations are done according to the pseudoscalar gluon scheme. As the principle steps in this calculation are the same as in the massless case, we immediately give the final results. As in section 3 we give the results for Γ^{virt} and Γ^{brems} seperately. Furthermore the result for Γ^{virt} is presented as a function of the individual form factors present in equation (11). Their explicit form is given in the appendix. As far as we know, these form factors are not given explicitly in the literature for two arbitrary,different masses m_t and m_b; however, the final result including both, the gluon bremsstrahlung and the virtual corrections , can also be found in reference [6]].

4.1 Zeroth order and loop-corrections

As before, the zeroth order contribution and the one-loop corrections are contained in Γ^{virt}. For Γ^{virt} we get:

$$\Gamma^{virt} = \frac{\rho f}{16\pi m_W^3} \frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)} \left[1 + \epsilon \log \frac{m_W^6}{\rho^2 m_t m_b} + O(\epsilon^2)\right] \tag{34}$$

with

$$\rho = \sqrt{(2pp')^2 - 4m_t^2 m_b^2} \quad .$$

Using

$$\frac{g^2}{m_W^2} = 4G_F\sqrt{2} \qquad \text{and} \qquad \kappa = \frac{h^2}{16\pi^2} \quad ,$$

f can be written as

$$\begin{aligned} f \;=\; & 2\sqrt{2}G_F V_1 + \\ & -\frac{16\sqrt{2}G_F\kappa}{3}\left[V_1\left(\Re\bar{\mathbf{A}} + 4 + \frac{3}{\epsilon}\right) + V_2\,\Re\bar{\mathbf{B}} + \right. \\ & -\frac{1}{2}m_t V_3\,\Re\bar{\mathbf{C}} - \frac{1}{2}m_b V_4\,\Re\bar{\mathbf{D}} + \\ & \left. +\frac{1}{2}m_t V_5\,\Re\bar{\mathbf{E}} + \frac{1}{2}m_b V_6\,\Re\bar{\mathbf{F}}\right] \end{aligned} \tag{35}$$

with

$$\begin{aligned} V_1 &= 2m_W^4 - m_b^4 - m_t^4 + 2m_t^2 m_b^2 - m_W^2 m_b^2 - m_W^2 m_t^2 \\ V_2 &= 6m_t m_b m_W^2 \\ V_3 &= m_W^4 + m_b^4 + m_t^4 - 2m_t^2 m_b^2 + 10m_W^2 m_b^2 - 2m_W^2 m_t^2 \\ V_4 &= 5m_W^4 - m_t^4 - m_b^4 - 4m_t^2 m_W^2 + 2m_t^2 m_b^2 - 4m_b^2 m_W^2 \\ V_5 &= V_4 \\ V_6 &= m_W^4 + m_b^4 + m_t^4 - 2m_b^2 m_W^2 + 10m_t^2 m_W^2 - 2m_t^2 m_b^2 \end{aligned} \tag{36}$$

In equation (35) the symbol $\Re\bar{\mathbf{A}}$ denotes the real part of the form factor $\bar{\mathbf{A}}$, present in equation (11). The explicit result for the form factors is given in the appendix.
In order to see the cancellation of the infrared singularities when adding the bremsstrahlung contributions we split Γ^{virt} into an infrared finite and an infrared singular part.

$$\begin{aligned} \Gamma^{virt}_{sing} &= \frac{2\sqrt{2}\,G_F\kappa}{3\pi m_W^3} V_1 \frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon} \frac{1}{\Gamma(2-2\epsilon)} \times \\ & \times\left\{(m_W^2 - m_t^2 - m_b^2)\log\frac{(m_W^2 - m_t^2 - m_b^2) + \rho}{2m_t m_b} - \rho\right\} \\ \Gamma^{virt}_{finite} &= \Gamma^{virt} - \Gamma^{virt}_{sing} \end{aligned} \tag{37}$$

4.2 Gluon bremsstrahlung contributions

We give the final result for Γ^{brems}. As in section 3.2 Γ^{brems} consists of two parts:

$$\Gamma^{brems} = \Gamma^{brems}_{vect.} + \Gamma^{brems}_{scal.}$$

In the massive case the contribution $\Gamma^{brems}_{scal.}$ vanishes, i.e, the emission of pseudoscalar gluons does not contribute to the total decay width.
For Γ^{brems} we get:

$$\begin{aligned}
\Gamma^{brems} &= \frac{2\sqrt{2}\,G_F\kappa}{3\pi m_W}\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\times \\
&\times\Big\{-4m_t^2\,V_1\,L_1+4(m_W^2-m_t^2-m_b^2)\,V_1\,L_2-4m_b^2\,V_1\,L_3+ \\
&\quad+\left[-10m_W^4+6m_W^2m_b^2-2m_W^2m_t^2-4m_b^2m_t^2+4m_b^4\right]L_4+ \\
&\quad+\left[8m_W^3+4m_Wm_t^2+4m_Wm_b^2\right]L_5+ \\
&\quad+\left[4m_W^3+8m_Wm_t^2\right]L_6+ \\
&\quad+\left[4m_b^4-2m_W^4-2m_W^2m_b^2+6m_W^2m_t^2-4m_t^2m_b^2\right]L_7+ \\
&\quad+\left[8m_Wm_t^2-12m_W^3\right]L_8+ \\
&\quad+\left[12m_Wm_t^2-4m_W^3+4m_Wm_b^2\right]L_9\Big\}
\end{aligned} \tag{38}$$

$$V_1 = 2m_W^4-m_b^4-m_t^4+2m_t^2m_b^2-m_W^2m_b^2-m_W^2m_t^2$$

The functions L_1, L_2 and L_3 contain the infrared singularities. We have worked out them explicitly:

$$\begin{aligned}
L_1 &= \left(\frac{1}{2m_W^2}-\frac{\hat{E}_t}{2m_Wm_t^2}\right)\log\frac{(\hat{E}_t+\hat{p})\,(m_W-\hat{E}_t+\hat{p})}{m_tm_b}-\frac{\hat{E}_t}{2m_Wm_t^2}\log\frac{\hat{E}_t+\hat{p}}{m_t}+ \\
&\quad-\frac{\hat{p}}{4m_Wm_t^2}\left[\frac{1}{\epsilon}+2-2\log\frac{8(\hat{p})^3}{m_Wm_tm_b}\right]
\end{aligned}$$

$$\begin{aligned}
L_2 &= \frac{1}{4m_W^2}\Big\{3\log^2\frac{\hat{E}_t+\hat{p}}{m_t}-4\log\frac{\hat{E}_t+\hat{p}}{m_t}\log\frac{m_W}{m_t}-4\log\frac{m_W-\hat{E}_t+\hat{p}}{m_b}\log\frac{m_W}{m_b}+ \\
&\quad+3\log^2\frac{m_W-\hat{E}_t+\hat{p}}{m_b}+\mathrm{Li}\left(\frac{\hat{E}_t+\hat{p}}{m_W}\right)-\mathrm{Li}\left(\frac{\hat{E}_t-\hat{p}}{m_W}\right)+ \\
&\quad+\mathrm{Li}\left(\frac{m_W-\hat{E}_t+\hat{p}}{m_W}\right)-\mathrm{Li}\left(\frac{m_W-\hat{E}_t-\hat{p}}{m_W}\right)+ \\
&\quad+3\mathrm{Li}\left(\frac{\hat{E}_t-\hat{p}}{\hat{E}_t+\hat{p}}\right)+3\mathrm{Li}\left(\frac{m_W-\hat{E}_t-\hat{p}}{m_W-\hat{E}_t+\hat{p}}\right)-\pi^2+ \\
&\quad-\frac{1}{\epsilon}\log\frac{(\hat{E}_t+\hat{p})\,(m_W-\hat{E}_t+\hat{p})}{m_tm_b}\Big\}
\end{aligned}$$

$$
\begin{aligned}
L_3 &= \left(\frac{1}{2m_W^2} - \frac{m_W - \hat{E}_t}{2m_W m_b^2}\right) \log \frac{(\hat{E}_t + \hat{p})(m_W - \hat{E}_t + \hat{p})}{m_t m_b} + \\
&\quad - \frac{m_W - \hat{E}_t}{2m_W m_b^2} \log \frac{m_W - \hat{E}_t + \hat{p}}{m_b} - \frac{\hat{p}}{4m_W m_b^2} \left[\frac{1}{\epsilon} + 2 - 2\log \frac{8(\hat{p})^3}{m_W m_t m_b}\right]
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
\hat{E}_t &= \frac{m_W^2 + m_t^2 - m_b^2}{2m_W} \\
\hat{p} &= \sqrt{\hat{E}_t^2 - m_t^2} = \frac{\sqrt{(m_W^2 - m_t^2 - m_b^2)^2 - 4m_t^2 m_b^2}}{2m_W}
\end{aligned}
\tag{40}
$$

Li(x) denotes the Spence function

$$
\mathrm{Li}(x) = -\int_0^x \frac{dt}{t} \log(1-t) \quad .
$$

Many useful properties of this function are given in [7].
The functions $L_4, ..., L_9$ are given in terms of one-dimensional finite integrals:

$$
\begin{aligned}
L_4 &= \int dE_t \frac{\bar{p}}{m_W^2 - 2m_W E_t + m_t^2} \\
L_5 &= \frac{1}{2} \int dE_t \frac{(M^2 - 2m_W E_t)\,\bar{p}\,(m_W - E_t)}{(m_W^2 - 2m_W E_t + m_t^2)^2} \\
L_6 &= \int dE_t \frac{E_t\,\bar{p}}{m_W^2 - 2m_W E_t + m_t^2} \\
L_7 &= \frac{1}{2m_W} \int dE_t \left\{2\log\frac{E_t + \bar{p}}{m_t} + \log\frac{m_W - E_t + \bar{p}}{m_W - E_t - \bar{p}}\right\} \\
L_8 &= \frac{1}{2m_W} \int dE_t\, E_t \left\{2\log\frac{E_t + \bar{p}}{m_t} + \log\frac{m_W - E_t + \bar{p}}{m_W - E_t - \bar{p}}\right\} \\
L_9 &= \frac{1}{4m_W^2} \int dE_t \left\{\frac{2m_W \bar{p}\,(M^2 - 2m_W E_t)}{m_W^2 - 2m_W E_t + m_t^2} + \right. \\
&\qquad \left. + (M^2 - 2m_W E_t) \left[2\log\frac{E_t + \bar{p}}{m_t} + \log\frac{m_W - E_t + \bar{p}}{m_W - E_t - \bar{p}}\right]\right\}
\end{aligned}
\tag{41}
$$

where

$$
\bar{p} = \sqrt{E_t - m_t^2} \qquad \text{and} \qquad M^2 = m_W^2 + m_t^2 - m_b^2 \quad .
$$

The integration variable E_t is restricted to the interval

$$
E_t \in [m_t, \hat{E}_t] \quad .
$$

Again,

$$
\Gamma^{brems} = \Gamma^{brems}_{sing} + \Gamma^{brems}_{finite}
$$

The singular part Γ^{brems}_{sing} reads:

$$\Gamma^{brems}_{sing} = -\frac{2\sqrt{2}\,G_F\,\kappa}{3\pi m_W^3}\,\frac{\left(\frac{4\pi\mu^2}{m_W^2}\right)^{2\epsilon}}{\Gamma(2-2\epsilon)}\,\frac{1}{\epsilon}\,V_1 \times$$
$$\times\left\{(m_W^2 - m_t^2 - m_b^2)\,\log\frac{(m_W - \hat{E}_t + \hat{p})(\hat{E}_t + \hat{p})}{m_t m_b} - 2m_W\hat{p}\right\} \tag{42}$$

4.3 Total decay width

We perform the sum

$$\Gamma = \Gamma^{virt} + \Gamma^{brems} \quad .$$

When doing this sum the infrared singular contributions Γ^{virt}_{sing} and Γ^{brems}_{sing} just cancel. Our result for Γ coincides with the one given in reference [6].

5 On the results in a more traditional scheme

We consider again the QCD corrections to the total decay width of a W^+ boson into a massless quark and antiquark.
One way to regularize the collinear singularities is to give (small) masses m_t and m_b to the quark and the antiquark, respectively. In order to regularize the infrared singularities one gives a small mass to the gluon. The bremsstrahlung corrections can then be worked out in 4 dimensions from the beginning to the end. In the loop graphs we have to regularize the ultraviolet divergences in addition. This we did dimensionally. When summing all the loop graphs the ultraviolet singularities vanish. At this point the limit $d \to 4$ can be done. The subsequent phase space integrals can then be worked out in d=4. All these calculations are done in detail in my doctoral thesis [8].
When adding the virtual - and bremsstrahlung contributions, the infrared singularities cancel, i.e., the gluon mass can be sent to zero. After this step the results turns out to be identical with the one we got in section 4.
In a last step we can do the limits

$$m_t \to 0 \qquad \text{and} \qquad m_b \to 0 \quad .$$

This step precisely reproduces the final result in section 3.

6 Numerical results

In this section we illustrate the results of section 4 with two plots. In the following discussion $\Gamma^{(0)}$ denotes the W^+ decay width for the $t\bar{b}$ channel without QCD corrections, whereas Γ includes first order QCD corrections.

- **Figure 1**
  $\Gamma^{(0)}$ and Γ are plotted as a function of m_t. The other parameters are:$m_W = 82\,\mathrm{GeV}$,$m_b = 5\,\mathrm{GeV}$ and $\alpha_s = 0.1$.

- **Figure 2**
  We can write Γ as
  $$\Gamma = \Gamma^{(0)} \left\{ 1 + \frac{\alpha_s}{\pi} \cdot \text{factor} \right\} .$$
  This 'factor', which only depends on the masses m_W, m_t and m_b, is plotted as a function of m_t ; two curves are plotted:
  - $m_W = 82\,\mathrm{GeV}$, $m_b = 5\,\mathrm{GeV}$
  - $m_W = 82\,\mathrm{GeV}$, $m_b = 0.1\,\mathrm{GeV}$.

  Note, that in the massless case $[m_t = m_b = 0]$ we have:
  $$\Gamma^{tot} = \Gamma^{(0)} \left\{ 1 + \frac{\alpha_s}{\pi} \right\} ,$$
  i.e., the 'factor' defined above is 1. This is represented by the dashed line in figure 2.

Acknowledgement
It is a pleasure to thank P. Minkowski for many interesting discussions and helpful suggestions.

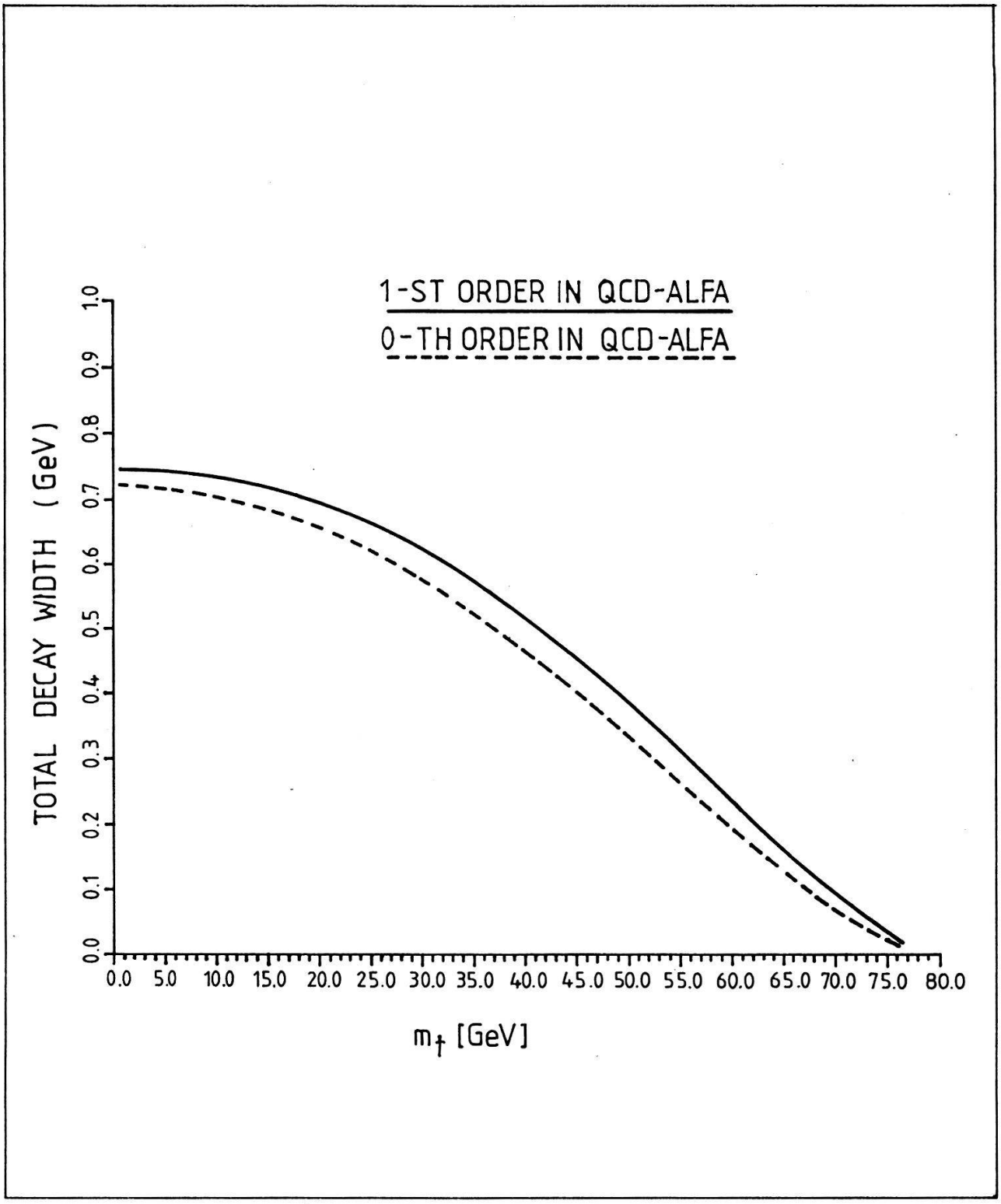


Figure 1: Total decay width of W^+ as a function of m_t ; $m_W = 82\,\mathrm{GeV}$, $m_b = 5\,\mathrm{GeV}$, $\alpha_s = 0.1$

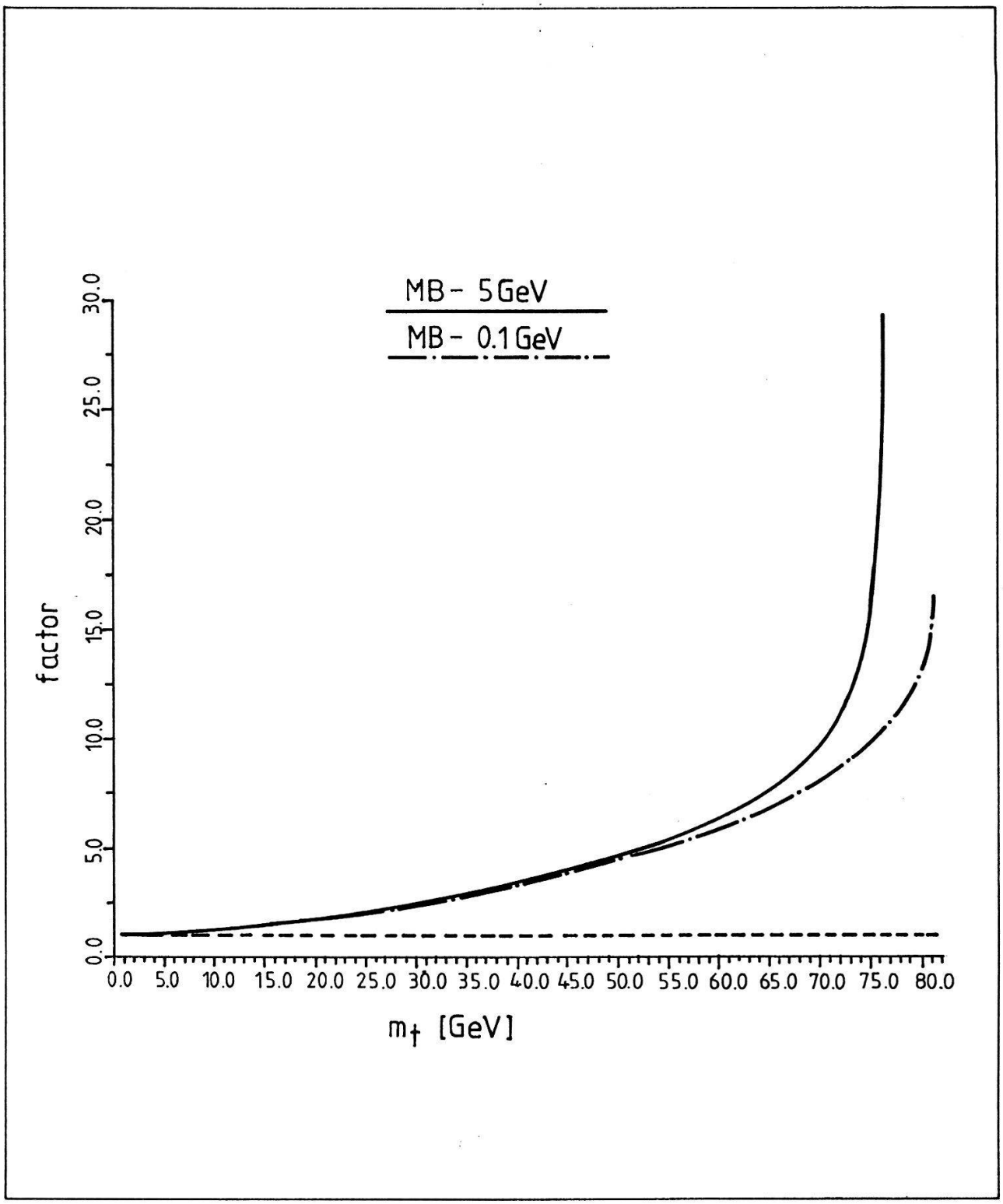


Figure 2: $\Gamma = \Gamma^{(0)} \left\{1 + \frac{\alpha_s}{\pi} \text{factor}\right\}$

Appendix: The form factors $\bar{\mathbf{A}}$,...,$\bar{\mathbf{F}}$

We give the explicit results for the form factors appearing in equation (35) :

$$
\begin{aligned}
\bar{\mathbf{A}} &= 1 + \frac{1}{2}\left[3(2pp')\bar{I}_2 + \rho^2\,\bar{H}_2\right]\frac{1}{\epsilon} + \\
&\quad + \frac{2pp'}{2}\left[-3\bar{I}_1 + 6\bar{I}_2 - 2\bar{I}_5\right] - m_t^2\,\bar{I}_6 - m_b^2\,\bar{I}_7 + \\
&\quad - \frac{\rho^2}{2}\left(\bar{K}_2 - 3\bar{H}_2\right) \\
\bar{\mathbf{B}} &= 2m_t\,m_b\left(2\bar{I}_3 + 2\bar{I}_4 - \bar{I}_2 - 2\bar{I}_5\right) \\
\bar{\mathbf{C}} &= 4m_t\,\bar{I}_3 - 2m_t\bar{I}_5 \\
\bar{\mathbf{D}} &= -2m_b\,\bar{I}_7 \\
\bar{\mathbf{E}} &= 2m_t\,\bar{I}_6 \\
\bar{\mathbf{F}} &= -4m_b\,\bar{I}_4 + 2m_b\bar{I}_5
\end{aligned}
\tag{43}
$$

where

$$\rho = \sqrt{(2pp')^2 - 4m_t^2 m_b^2} \quad .$$

Note that the scalar product $2pp'$, on which these functions depend, is

$$2pp' = m_W^2 - m_t^2 - m_b^2 \quad .$$

The functions $\bar{I}_1, ..., \bar{I}_7, \bar{H}_2, \bar{K}_2$ read:

$$
\begin{aligned}
\bar{I}_2 &= \int dx\,\frac{1}{\alpha(x)} \\
\bar{I}_3 &= \int dx\,\frac{x}{\alpha(x)} \\
\bar{I}_4 &= \int dx\,\frac{1-x}{\alpha(x)} \\
\bar{I}_5 &= \int dx\,\frac{x(1-x)}{\alpha(x)} \\
\bar{I}_6 &= \int dx\,\frac{x^2}{\alpha(x)} \\
\bar{I}_7 &= \int dx\,\frac{(1-x)^2}{\alpha(x)} \\
\bar{I}_1 &= \int dx\,\frac{1}{\alpha(x)}\log\frac{\alpha(x)}{m_t\,m_b} \\
\bar{H}_2 &= \int dx\,\frac{x(1-x)}{\alpha(x)^2}
\end{aligned}
$$

$$\begin{aligned}
\bar{K}_2 &= \int dx\, \frac{x(1-x)}{\alpha(x)^2} \log \frac{\alpha(x)}{m_t\, m_b} \\
\alpha(x) &= m_t^2 x^2 + m_b^2 (1-x)^2 - 2\, pp'\, x(1-x)
\end{aligned} \tag{44}$$

We list the real parts and the imaginary parts of these functions :

$$\begin{aligned}
\Re \bar{I}_2 &= -\frac{2}{\rho} \log \frac{2pp' + \rho}{2m_t m_b} \\
\Re \bar{I}_3 &= \frac{-2m_b^2 - 2pp'}{Q^2\, \rho} \log \frac{2pp' + \rho}{2m_t m_b} + \frac{1}{2Q^2} \log \frac{m_t^2}{m_b^2} \\
\Re \bar{I}_4 &= \frac{-2m_t^2 - 2pp'}{Q^2\, \rho} \log \frac{2pp' + \rho}{2m_t m_b} - \frac{1}{2Q^2} \log \frac{m_t^2}{m_b^2} \\
\Re \bar{I}_5 &= -\frac{1}{Q^2} - \frac{4m_b^2 m_t^2 + 2pp'\,(m_t^2 + m_b^2)}{\rho\,(Q^2)^2} \log \frac{2pp' + \rho}{2m_t m_b} + \\
&\quad + \frac{m_t^2 - m_b^2}{2(Q^2)^2} \log \frac{m_t^2}{m_b^2} \\
\Re \bar{I}_6 &= \frac{1}{Q^2} - \frac{2m_b^4 + 2m_b^2(2pp') + (2pp')^2 - 2m_t^2 m_b^2}{\rho\,(Q^2)^2} \log \frac{2pp' + \rho}{2m_t m_b} + \\
&\quad + \frac{2m_b^2 + 2pp'}{2(Q^2)^2} \log \frac{m_t^2}{m_b^2} \\
\Re \bar{I}_7 &= \frac{1}{Q^2} - \frac{2m_t^4 + 2m_t^2(2pp') + (2pp')^2 - 2m_t^2 m_b^2}{\rho\,(Q^2)^2} \log \frac{2pp' + \rho}{2m_t m_b} + \\
&\quad - \frac{2m_t^2 + 2pp'}{2(Q^2)^2} \log \frac{m_t^2}{m_b^2} \\
\Re \bar{I}_1 &= -\frac{1}{\rho} \Bigg\{ 2 \log \frac{\rho}{Q^2} \log \frac{(2pp' + 2m_t^2 + \rho)\,(2pp' + 2m_b^2 + \rho)}{4Q^2 m_b^2} + \\
&\quad + \log \frac{2pp' + 2m_t^2 + \rho}{2m_t^2} \log \frac{2pp' + 2m_t^2 + \rho}{2m_b^2} + \\
&\quad - \log \frac{2Q^2}{2pp' + 2m_b^2 + \rho} \log \frac{2m_b^2 Q^2}{(2pp' + 2m_b^2 + \rho) m_t^2} + \\
&\quad + 2 \log \frac{2pp' + 2m_t^2 + \rho}{2\rho} \log \frac{2m_t^2 Q^2}{(2pp' + 2m_t^2 + \rho)\,\rho} + \\
&\quad - \frac{4\pi^2}{3} + 2\mathrm{Li}\left(\frac{\rho - 2\mathrm{m}_\mathrm{b}^2 - 2\mathrm{pp}'}{2\rho}\right) + 2\mathrm{Li}\left(\frac{\rho - 2\mathrm{m}_\mathrm{t}^2 - 2\mathrm{pp}'}{2\rho}\right) \Bigg\} \\
\Re \bar{H}_2 &= \frac{2}{\rho^2} \left\{ \frac{2pp'}{\rho} \log \frac{2pp' + \rho}{2m_t m_b} - 1 \right\} \\
\Re \bar{K}_2 &= \Re \bar{H}_2 - \frac{2pp'}{\rho^2} \Re \bar{I}_1 + \\
&\quad + \frac{1}{\rho} \Bigg\{ \frac{8m_t^2 m_b^2 + 2m_b^2(2pp') + 2m_t^2(2pp')}{Q^2 \rho^2} \times
\end{aligned}$$

$$
\begin{aligned}
&\times \log \frac{4Q^2 m_b^2}{(2pp' + 2m_b^2 + \rho)\,(2pp' + 2m_t^2 + \rho)} + \\
&-2\log \frac{\rho}{Q^2} \left[\frac{\rho + 2pp'}{(2pp' + 2m_b^2 + \rho)\rho} + \frac{2pp' - \rho}{(2pp' + 2m_t^2 - \rho)\,\rho} \right] + \\
&+2\log \frac{2m_b Q^2}{(2pp' + 2m_b^2 + \rho)m_t} \left[\frac{1}{Q^2} - \frac{2pp' + \rho}{(2pp' + 2m_b^2 + \rho)\rho} \right] + \\
&-2\log \frac{2pp' + 2m_t^2 + \rho}{2m_t m_b} \left[\frac{1}{Q^2} - \frac{\rho - 2pp'}{(2pp' + 2m_t^2 - \rho)\rho} \right] + \\
&-\frac{4m_b^2(2pp') + 8m_t^2 m_b^2}{\rho^2\,(2pp' + 2m_b^2 - \rho)} \log \frac{2pp' + \rho + 2m_b^2}{2\rho} + \\
&+\frac{4m_t^2(2pp') + 8m_t^2 m_b^2}{\rho^2\,(2pp' + 2m_t^2 + \rho)} \log \frac{2Q^2 m_t^2}{(2pp' + 2m_t^2 + \rho)\,\rho} \Bigg\}
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
\Im \bar{I}_2 &= \frac{2\pi}{\rho} \\
\Im \bar{I}_3 &= \left(\frac{2m_b^2 + 2pp'}{Q^2\,\rho} \right) \pi \\
\Im \bar{I}_4 &= \left(\frac{2m_t^2 + 2pp'}{Q^2\,\rho} \right) \pi \\
\Im \bar{I}_5 &= \left(\frac{4m_b^2 m_t^2 + 2pp'(m_t^2 + m_b^2)}{\rho\,(Q^2)^2} \right) \pi \\
\Im \bar{I}_6 &= \left(\frac{2m_b^4 + 2m_b^2(2pp') + (2pp')^2 - 2m_t^2 m_b^2}{\rho\,(Q^2)^2} \right) \pi \\
\Im \bar{I}_7 &= \left(\frac{2m_t^4 + 2m_t^2(2pp') + (2pp')^2 - 2m_t^2 m_b^2}{\rho\,(Q^2)^2} \right) \pi \\
\Im \bar{I}_1 &= \frac{2\pi}{\rho} \log \frac{\rho^2}{Q^2 m_t m_b} \\
\Im \bar{H}_2 &= -\frac{2\pi}{\rho^2} \frac{2pp'}{\rho} \\
\Im \bar{K}_2 &= \pi \Bigg\{ -\frac{2(2pp')}{\rho^3} - \frac{2(2pp')}{\rho^3} \log \frac{\rho^2}{Q^2 m_t m_b} + \\
&\qquad + \frac{8m_t^2 m_b^2 + 2m_t^2(2pp') + 2m_b^2(2pp')}{\rho^3\,Q^2} + \\
&\qquad + \frac{2}{\rho} \left[\frac{1}{Q^2} - \frac{\rho - 2pp'}{(2pp' + 2m_t^2 - \rho)\rho} \right] + \\
&\qquad + \frac{4m_t^2(2pp') + 8m_t^2 m_b^2}{\rho^3(2pp' + 2m_t^2 + \rho} \Bigg\}
\end{aligned}
\tag{46}
$$

where

$$Q^2 = m_t^2 + m_b^2 + 2pp'$$

Li(x) denotes the Spence function:

$$\mathrm{Li}(x) = -\int_0^x \frac{dt}{t} \log(1-t)$$

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