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Effect Of The Anisotropy On The Critical Behaviour Of The High Temperature Superconductors

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Abstract. The effect of the fluctuations of the intrinsic magnetic field on the criticality of the highly anisotropic high temperature superconductors is studied. The theory of Halperin et. al. [4] is generalized to take the anisotropy into account. The effective Hamiltonian of Ginzburg-Landau type is studied using a renormalization group approach in lowest order in $\epsilon = 4-d$. It is shown that the anisotropy preserves the first-order nature of the phase transition. Nevertheless, the appearance of strong crossover effects reduces considerably its size and should make it disappear experimentally.

1 Introduction

Since the discovery of Bednorz & Müller [1], High Temperature Superconductivity has been a very active field of research. While this extensive study has led to a good knowledge of the experimental properties of the materials exhibiting this behaviour, there have been relatively few theoretical progresses, and the mechanism which suppresses their electrical resistivity remains an unsolved puzzle. It is in particular not clear whether the BCS-theory [2], the theory which describes accurately the "ante-1986" superconductors could be somehow extended or if a completely new microscopical approach should be considered [3].

Beside these efforts toward a microscopical theory, some theoretical progresses have been done on the phenomenology of the cuprates. Most of the phenomenological theories are based on the Ginzburg-Landau theory which states that the superconducting state can be described by a macroscopic wave function - sometimes called the order parameter - the magnitude of which becomes finite below the critical temperature. This theory describes qualitatively second order phase transitions such as the paramagnetic to ferromagnetic transition in metals and the normal fluid to superfluid transition in liquid Helium where the order parameter is respectively the magnetization and the superfluid density.

Superconductors are however charged. As a consequence, their order parameter is coupled to a gauge field whose fluctuations have to be taken into account. It has been pointed out among others that this coupling could turn the transition first order [4].

Another crucial feature of the cuprates is their great anisotropy. The effective mass measured alongside the z-axis of the unit cell exceeds the effective mass in the x- or y-direction by up to three orders of magnitude. This anisotropy is so great that it even raises the question of the effective dimensionality - a central point in the modern theory of phase transitions - of the high-T_c superconductors.

In this paper, we study a Ginzburg-Landau-like model which incorporates both the coupling to a gauge field and the anisotropy of the materials. We study the following free energy functional :

$$F(\Psi, \vec{A}, T) = \int d^d x \left[\gamma_\| |(\vec{\nabla}_\| - iq_0 \vec{A}_\|(\vec{r})) \Psi(\vec{r})|^2 + \gamma_\perp |(\vec{\nabla}_\perp - iq_0 \vec{A}_\perp(\vec{r})) \Psi(\vec{r})|^2 + \right.$$
$$\left. + \alpha |\Psi(\vec{r})|^2 + \frac{\beta}{2} |\Psi(\vec{r})|^4 + \frac{1}{8\pi\mu_0} (\vec{\nabla} \times \vec{A}(\vec{r}))^2 \right] \tag{1.1}$$

where $\vec{A}(\vec{r})$ is the gauge field which represents the intrinsic magnetic field, $\Psi(\vec{r})$ the n-component superconducting order parameter, $\alpha = \alpha(T) = \alpha' \frac{(T-T_{c0})}{T_{c0}}$, where $\alpha' > 0$, changes sign at the mean-field critical temperature T_{c0}, thus enabling the phase transition, $q_0 = \frac{2e}{\hbar c} = 2\pi(\Phi_0)^{-1}$, where Φ_0 is the superconducting flux quantum, couples the order parameter to the gauge field and μ_0 is the magnetic permeability of the materials. The subscripts $\perp$ and $\|$ indicate the direction perpendicular and parallel to the CuO-planes respectively. $|\Psi(\vec{r})|^6$ and higher powers have been neglected since they are irrelevant for our study [5].

The uncharged anisotropic case $q_0 = 0$ and $\gamma_\perp = \gamma_\|$ has been studied a long time ago. A mean-field approximation correctly predicts a second-order phase transition with critical exponents $\nu = \frac{1}{2}$ and $\eta = 0$ [15]. Near the critical temperature, however, the fluctuations of Ψ become so important that the mean-field approximation breaks down. Ginzburg showed that the interval of temperature around the critical temperature where this approximation loses its validity - the critical region - is given by [6] :

$$\frac{|T - T_{c0}|}{T_{c0}} < \epsilon_c \equiv \frac{1}{2} \left(\frac{k_b T_{c0}}{H_{c2}(0)\xi_\perp(0)\xi_\|^2(0)} \right)^2 \tag{1.2}$$

where the temperatures are expressed in Kelvin, $\xi_\perp$ and $\xi_\|$ are the coherence length perpendicular and parallel to the CuO-planes respectively expressed in cm and $H_{c2}(0)$, the field

obtained by extrapolating the linear part of $H_{c2}(T)$, the upper critical field, near $T = T_{c0}$, expressed in Gauss. Inside this interval, the fluctuations of Ψ produce the breakdown of the mean-field approximation. For usual low-temperature superconductors, this criterion predicts at best temperature intervals $|T - T_{c0}| < 10^{-6}$ [7], i.e. the critical regime should be experimentally unaccessible. However for high-T_c superconductors the situation is reversed. Their coherence length is typically two order of magnitude smaller than in usual BCS superconductors. Together with higher critical temperatures, this increases dramatically the size of the critical region up to a few degrees around T_{c0}. Thus this criterion emphasizes the importance of fluctuations of the order parameter on the phase transition of the high-T_c superconductors.

The isotropic model ($\gamma_\perp = \gamma_\parallel$) has already been studied by Halperin et. al. [4]. They derived renormalization group (RG) recursion equations for the couplings using the method of Wilson and Fisher [8]. They noticed that the coupling to the gauge field introduces a new complex fixed point, and interpreted the non-reality of this fixed point as the occurrence of a first order phase transition. In a subsequent paper [9], the size of this transition was evaluated and shown to be too small to be experimentally observed in the superconductors available at that time. However in view of the above remarks, this question need to be revisited for the new high-T_c materials.

The paper is organised as follows :

In section 2, we define the RG transformation for our model and analyse its properties. We will see that the anisotropy introduces two new fixed points and that the RG trajectories in the parameter space are drastically modified. The complex fixed point responsible for the occurence of the first order phase transition is however still present.

The shape of the RG trajectories in the parameter space will determine the critical behaviour of our model. Due to both the structure of the recursion equations and the dimension of the parameter space, the flow is not analytically integrable, and we are led to a numerical study. We present the results of this numerical study in section 3. We see that the anisotropy sensibly reduces the size of the first order phase transition, i.e. the interval of temperature around T_c where the flow is dominantly driven by the complex fixed point.

Conclusions and final remarks are given in section 4.

2   Renormalization Group Approach

2.1   General Method

Starting from the free energy functional (1.1) we define the free energy as a function of the temperature alone by taking the trace over the configurations of the vector potential and

the order parameter:

$$\exp(-\mathcal{F}(T)) \doteq \int D\{\Psi(\vec{r}), \vec{A}(\vec{r})\} \exp(-F(\Psi, \vec{A}, T)) \tag{2.1}$$

The problem is that $F(\Psi, \vec{A}, T)$ is quartic in both Ψ and $\vec{A}$ and we don't know how to perform this integral exactly. Nevertheless the effect of these quartic terms can be evaluated perturbatively in the limit d $\to$ 4, using the Wilson-Fisher $\epsilon = 4-d$ expansion [8]. Let us remind the reader of the main steps of this method.

Because of the divergence of the correlation length, fluctuations with long wave vectors become less and less important as we approach the transition temperature. We therefore define a RG transformation which integrates out large k-modes and rescales Ψ and $\vec{A}$ by appropriate factors. Performing a Fourier transform on the free energy functional we get :

$$F(\Psi, \vec{A}, T) = \int d^d k \left[(\gamma_\parallel \vec{k}_\parallel^2 + \gamma_\perp \vec{k}_\perp^2) \Psi_k \Psi_{-k} + \alpha \Psi_k \Psi_{-k} + \frac{1}{8\pi\mu_0} \vec{k}^2 \vec{A}_k^2 \right.$$
$$-2q_0 \frac{1}{(2\pi)^{\frac{d}{2}}} \int d^d k' \Psi_k \Psi_{-k-k'} (\vec{k}_\parallel \vec{A}_{k'}^\parallel + \vec{k}_\perp \vec{A}_{k'}^\perp) + \frac{\beta}{2(2\pi)^d} \int d^d k' \int d^d k'' \Psi_k \Psi_{k'} \Psi_{k''} \Psi_{-k-k'-k''}$$
$$\left. + q_0^2 \frac{1}{(2\pi)^d} \int d^d k' \int d^d k'' \Psi_k \Psi_{k'} (\vec{A}_{k''}^\parallel \vec{A}_{-k-k'-k''}^\parallel + \vec{A}_{k''}^\perp \vec{A}_{-k-k'-k''}^\perp) \right] \tag{2.2}$$

where we used the Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$. Now we set :

$$F_0(\Psi, \vec{A}, T) = \int d^d k \left[(\gamma_\parallel \vec{k}_\parallel^2 + \gamma_\perp \vec{k}_\perp^2 + \alpha) \Psi_k \Psi_{-k} + \frac{1}{8\pi\mu_0} \vec{k}^2 \vec{A}^2 \right] \tag{2.3}$$

$$F_1(\Psi, \vec{A}, T) = \int d^d k \left[-2q_0 \frac{1}{(2\pi)^{\frac{d}{2}}} \int d^d k' \Psi_k \Psi_{-k-k'} (\vec{k}_\parallel \vec{A}_{k'}^\parallel + \vec{k}_\perp \vec{A}_{k'}^\perp) \right.$$
$$+ \frac{\beta}{2(2\pi)^d} \int d^d k' \int d^d k'' \Psi_k \Psi_{k'} \Psi_{k''} \Psi_{-k-k'-k''}$$
$$\left. + q_0^2 \frac{1}{(2\pi)^d} \int d^d k' \int d^d k'' \Psi_k \Psi_{k'} (\vec{A}_{k''}^\parallel \vec{A}_{-k-k'-k''}^\parallel + \vec{A}_{k''}^\perp \vec{A}_{-k-k'-k''}^\perp) \right] \tag{2.4}$$

If we now take only F_0 into account, the RG transformation can be performed exactly. The behaviour of the various couplings in parameter space near the fixed-point of this transformation leads us to the mean-field results $\nu = \frac{1}{2}$ and $\eta = 0$. The crucial point is that for d $\geq$ 4, these results are exact. There is therefore hope that not too far below the critical dimension d = 4, we can approximate the integral in equation (2.1) by a perturbation expansion around the gaussian model (F=F_0) [10], that is we expand $\exp(-F_1)$ in a power serie, where the small parameter of this expansion is ϵ=4-d. In analogy with field-theory, the gaussian model defines the Feynman rules, while the expansion determines the graphs to be computed. Our perturbation theory is therefore constructed with the following gaussian propagators (see figure 1, from left to right) :

$$G_0^\psi = (\alpha + \gamma_\perp k_\perp^2 + \gamma_\parallel k_\parallel^2)^{-1} \tag{2.5}$$
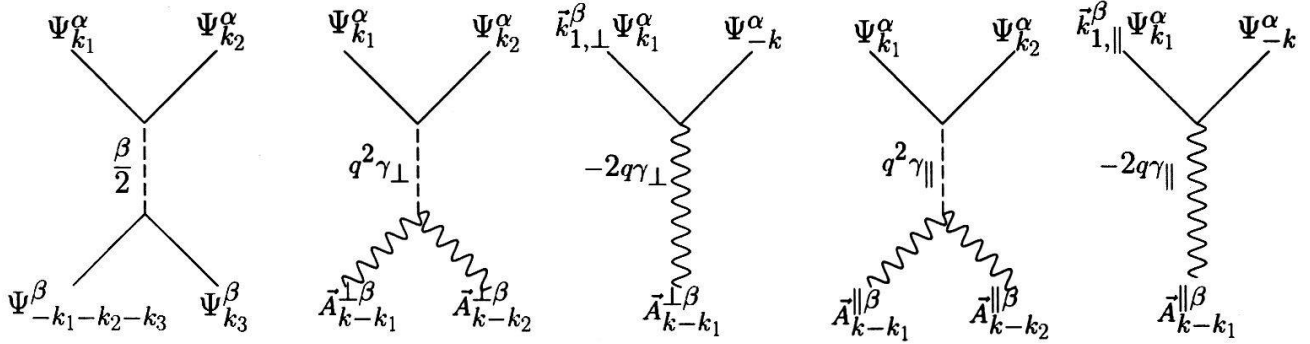
$$A_0^\parallel = 8\pi\mu_0 k_\parallel^{-2} \tag{2.6}$$

$$A_0^\perp = 8\pi\mu_0 k_\perp^{-2} \tag{2.7}$$

$$K_0^\psi = \vec{k}(\alpha + \gamma_\perp k_\perp^2 + \gamma_\parallel k_\parallel^2)^{-1} \tag{2.8}$$
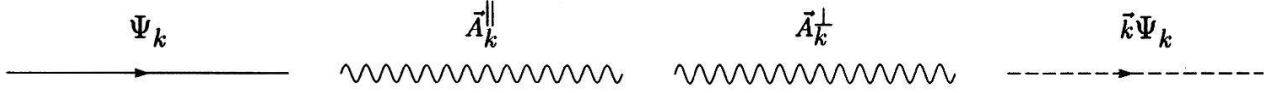
Figure 1 : **Gaussian Propagators**

The interaction vertices are shown in figure 2.

Figure 2 : **Interaction Vertices**

The transformation is the following [11] :

$$
\begin{aligned}
\exp\left(-F' - C\right) &= \left[\int D\{\Psi_k, \vec{A}_k\}_{\frac{\Delta}{s} \leq |\vec{k}| \leq \Lambda} \exp\left(-F_0 - F_1\right)\right]_T \\
&= \left[\int D\{\Psi_k, \vec{A}_k\}_{\frac{\Delta}{s} \leq |\vec{k}| \leq \Lambda} \exp\left(-F_0\right) \sum_{m=0}^{\infty} \frac{F_1^m}{m!}\right]_T
\end{aligned} \tag{2.9}
$$

The value m_c at which we will cut this expansion is then determined by the self-consistency of the transformation [12]. C is a constant without influence on the critical behaviour of the model that arises from the integration of the large k modes of H_0, while s is the rescaling factor satisfying $s > 1$. The subscript T indicates that after integration we must perform the following rescaling :

$$
\vec{k} \to s\vec{k} \qquad\qquad \Psi_k \to s^{1-\frac{\eta}{2}}\Psi_{sk} \qquad\qquad \vec{A}_k \to s^{1-\frac{\eta_A}{2}}\vec{A}_{sk} \tag{2.10}
$$

η and η_A are the critical exponents characterizing the behaviour at criticality of the two-point correlation function of the order parameter and the vector potential respectively

$$
< \Psi_k \Psi_{-k} > \propto k^{-2+\eta} \quad \text{for} \quad T \to T_c \tag{2.11}
$$

$$
< \vec{A}_k \vec{A}_{-k} > \propto k^{-2+\eta_A} \quad \text{for} \quad T \to T_c \tag{2.12}
$$

2.2   Recursion Equations

Equation (2.9) defines the recursion equations for the various couplings of the free energy functional (1.1) and (2.2). These recursion equations determine the flow of the vector in the parameter space $\{(\gamma_\parallel, \gamma_\perp, \alpha, \beta, \mu_0, q_0)\}$ representing the free energy functional, i.e. the evolution of the parameters under repeated iterations of the RG transformation. The critical behaviour of the model is then determined by the linearization of these equations around the various fixed points, i.e. points of the parameter space that go onto themselves under the effect of the RG transformation. For a coupling X^i these equations have the general structure :

$$X^i_{l+1} = s^{y_{x^i}} \left[X^i_l + \text{graphs} \right] \tag{2.13}$$

where y_{x^i} is a critical exponent associated with the coupling X^i while "graphs" stand for the corrections due to the power expansion of $\exp(-F_1)$. For $y_{x^i} = O(\epsilon)$ we have :

$$s^{y_{x^i}} = 1 + y_{x^i} \ln s + O(\epsilon^2) \tag{2.14}$$

and the fixed-point equation reads :

$$X^{i*} = s^{y_{x^i}} \left[X^{i*} + \text{graphs} \right] = \left[1 + y_{x^i} \ln s \right] \left[X^{i*} + \text{graphs} \right] \tag{2.15}$$

where the subscript "*" means the fixed-point value. Equation (2.15) implies

$$X^{i*} y_{x^i} \ln s = -\text{graphs} \tag{2.16}$$

The consistency of this method requires then that

$$\text{graphs} = W(\{X^i\}) \ln s \tag{2.17}$$

for a function W of the couplings, since the fixed-point value of the couplings must be independent of the rescaling factor.

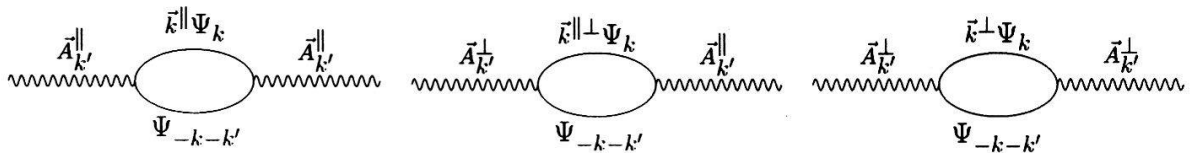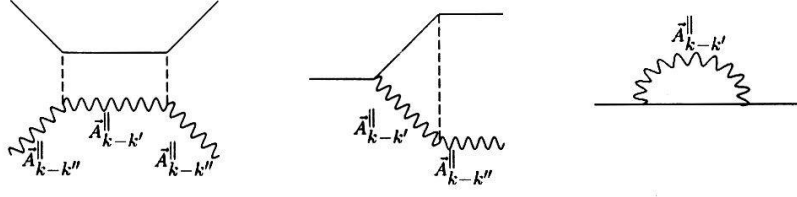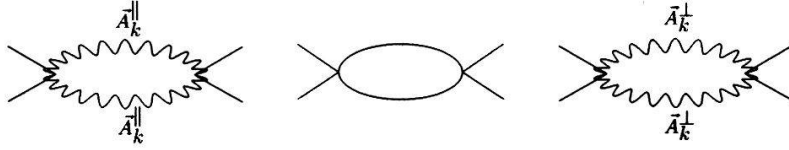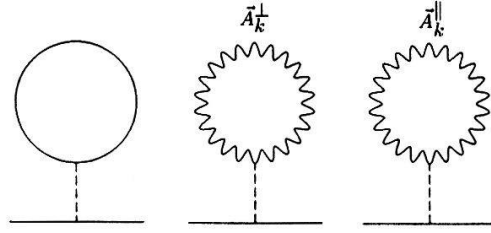
Figure 3 : **Radiative corrections for the renormalization of μ_0**

Figure 3, 4, 5 and 6 show the graphs giving corrections to the various couplings up to first order in ϵ. Two points deserve discussion :

-The three graphs in figure 3 give contributions to the renormalization of the magnetic permeability. The fact that, unlike the isotropic case, these three graphs give nonequal

Figure 4 : **Radiative corrections for the renormalization of $\gamma_\parallel$ and $\gamma_\perp$**

Figure 5 : **Radiative corrections for the renormalization of β**

Figure 6 : **Radiative corrections for the renormalization of α**

contributions forces us to introduce three magnetic permeabilities increasing thereby the dimension of our parameter space [13]. We must thus perform the following transformation on the free energy functional :

$$\frac{\vec{k}^2 \vec{A}_k^2}{8\pi\mu} \longrightarrow \frac{\vec{k}_\perp^2 \vec{A}_k^{\perp 2}}{8\pi\mu_\perp} + \frac{\vec{k}_\parallel^2 \vec{A}_k^{\parallel 2}}{8\pi\mu_\parallel} + \frac{\vec{k}_\perp^2 \vec{A}_k^{\parallel 2} + \vec{k}_\parallel^2 \vec{A}_k^{\perp 2}}{8\pi\mu_{tr}} \tag{2.18}$$

-None of these graphs result in a renormalization of the charge q_0. This is consistant with the fact that q_0 can be scaled out of the free energy functional by the transformation :

$$\vec{A}_k \longrightarrow \frac{\vec{A}_k}{q_0} \qquad\qquad \mu \longrightarrow \frac{\mu}{q_0^2} \tag{2.19}$$

As a consequence, the permeability will appear in the recursion equations only in the product $q_l^2 \mu_l$. Computing the contributions from each of these graphs is equivalent to solving equation (2.9) with $m_c = 2$.

We thus get a new free energy functional of the same form as (1.1) but with renormalized couplings :

$$q_{l+1} = s^{\frac{1}{2}(\epsilon - \eta_A)} q_l \tag{2.20}$$

$$\mu_{\parallel,l+1}^{-1} = s^{-\eta_A} \mu_{\parallel,l}^{-1} \left(1 + q_l^2 \mu_{\parallel,l} \frac{n \ln s}{12\pi} \rho_l^{-\frac{1}{2}}\right) \tag{2.21}$$

$$\mu_{tr,l+1}^{-1} = s^{-\eta_A} \mu_{tr,l}^{-1} \left(1 + q_l^2 \mu_{tr,l} \frac{n \ln s}{12\pi} \rho_l^{\frac{1}{2}}\right) \tag{2.22}$$

$$\mu_{\perp,l+1}^{-1} = s^{-\eta_A} \mu_{\perp,l}^{-1} \left(1 + q_l^2 \mu_{\perp,l} \frac{n \ln s}{12\pi} (\rho_l^{\frac{3}{2}} - 4\rho_l^{-\frac{1}{2}}(\rho_l - 1)^2)\right) \tag{2.23}$$

$$\gamma_{\perp,l+1} = s^{-\eta} \gamma_{\perp,l} \left[1 - \frac{3}{2\pi} q^2 \gamma_{\perp,l} \ln s \left(\frac{\mu_{tr,l}^{\frac{3}{2}} \mu_{\perp,l}^{\frac{1}{2}} - \mu_{tr,l} \mu_{\perp,l} \rho_l^{\frac{1}{2}}}{\mu_{tr,l} \gamma_{\parallel,l} - \mu_{\perp,l} \gamma_{\perp,l}}\right)\right] \tag{2.24}$$

$$\gamma_{\parallel,l+1} = s^{-\eta} \gamma_{\parallel,l} \left[1 - \frac{3}{2\pi} q^2 \gamma_{\parallel,l} \ln s \left(\frac{\mu_{\parallel,l}^{\frac{3}{2}} \mu_{tr,l}^{\frac{1}{2}} - \mu_{tr,l} \mu_{\parallel,l} \rho_l^{\frac{1}{2}}}{\mu_{\parallel,l} \gamma_{\parallel,l} - \mu_{tr,l} \gamma_{\perp,l}}\right)\right] \tag{2.25}$$

$$\beta_{l+1} = s^{\epsilon - 2\eta}[\beta_l - \frac{n+8}{16\pi^2} \frac{\beta_l^2}{\gamma_{\parallel,l}^2} \sqrt{\rho_l^{-1}} \ln s$$
$$- 4q_l^4(\gamma_{\parallel,l}^2 \sqrt{\mu_{tr,l} \mu_{\parallel,l}^3} + \gamma_{\perp,l}^2 \sqrt{\mu_{\perp,l} \mu_{tr,l}^3}) \ln s] \tag{2.26}$$

$$\alpha_{l+1} = s^{2-\eta} \left[\alpha_l + \frac{\beta_l(n+2)}{16\pi^2 \gamma_{\parallel,l}} (\Lambda^2 \frac{(1 - s^{-2})}{(1 + \sqrt{\rho_l})} - \alpha_l \frac{\ln s}{\sqrt{\gamma_{\parallel,l} \gamma_{\perp,l}}}) + \right.$$
$$\left. + q_l^2 \frac{\Lambda^2(1 - s^{-2})}{2\pi} \left[(d^A - 1)\gamma_{\parallel,l} \mu_{\parallel,l} \frac{1}{1 + \sqrt{\frac{\mu_{\parallel,l}}{\mu_{tr,l}}}} + \gamma_{\perp,l} \mu_{tr,l} \frac{1}{1 + \sqrt{\frac{\mu_{tr,l}}{\mu_{\perp,l}}}}\right]\right] \tag{2.27}$$

where in the last equation, $d^A = d$ is the dimension of the gauge field. We have set $\rho_l \doteq \frac{\gamma_{\perp,l}}{\gamma_{\parallel,l}}$. Some details on the derivation of these recursion equations are given in Appendix A. Since

$$q_l^2 \gamma_{\parallel,l} \mu_{\parallel,l} \frac{1}{1 + \sqrt{\frac{\mu_{\parallel,l}}{\mu_{tr,l}}}} \sim \epsilon \tag{2.28}$$

we must set $d^A = 4$. The fact that $\gamma_{\parallel}$ and $\gamma_{\perp}$ have different recursion equations leads us to introduce two different effective critical exponents $\eta_{\parallel}$ and $\eta_{\perp}$. Their evolution is given by :

$$\eta_{\perp,l} = -\frac{3}{2\pi} q_l^2 \left(\frac{\mu_{tr,l}^{\frac{3}{2}} \mu_{\perp,l}^{\frac{1}{2}} - \mu_{tr,l} \mu_{\perp,l} \rho_l^{\frac{1}{2}}}{\mu_{tr,l} \rho_l - \mu_{\perp,l}}\right) \tag{2.29}$$

$$\eta_{\parallel,l} = -\frac{3}{2\pi} q_l^2 \left(\frac{\mu_{\parallel,l}^{\frac{3}{2}} \mu_{tr,l}^{\frac{1}{2}} - \mu_{tr,l} \mu_{\parallel,l} \rho_l^{\frac{1}{2}}}{\mu_{\parallel,l} - \mu_{tr,l} \rho_l^{-1}}\right) \tag{2.30}$$

We next turn our attention to the coupling β. In the isotropic case ($\rho = 1$, $\gamma_{\parallel} = \gamma_{\perp} = \gamma$ and $\mu_{\parallel} = \mu_{\perp} = \mu_{tr} = \mu$) after inserting the fixed-point value $q^2 \mu = \frac{12\pi\epsilon}{n}$ obtained from the

recursion equations (2.21) to (2.23), we get a fixed-point value :

$$\frac{\beta}{\gamma^2} = \frac{8\pi^2}{n+8}\left[(\frac{36}{n}+1)\epsilon \pm \sqrt{(\frac{36}{n}+1)^2\epsilon^2 - 432\frac{n+8}{n^2}\epsilon^2}\right] \qquad (2.31)$$

In particular $\frac{\beta}{\gamma^2} \in \mathbf{C}$ for n<n_c=365.9, well above any physically interesting value. As already mentioned, this can be interpreted as the signature of a first order phase transition. It is the main purpose of the present paper to determine whether the situation is modified by the anisotropy. We therefore study the anisotropic equations in more details.

2.3   Fixed Points

The critical behaviour is determined by the nature of the RG flow in the neighborhood of the fixed points of the recursion equations. Beside increasing the dimension of the phase space, the couplings introduced by the anisotropy, as well as the anisotropy itself, will possibly modify the stability of the fixed points and the flow in their neighborhood. As a consequence the flow will be possibly driven away from the unperturbed fixed point to a new one, thereby modifying the critical behaviour of the model, an occurence which is called a crossover. As a consequence, two points are of particular interest : the determination of new fixed points and their topological properties. Let us precise the second point : One says that g is a linear scaling field if around a fixed point, the evolution of g under the RG transformation is given by the linearized recursion equation

$$g_{l+1} = s^{\lambda_g} g_l \qquad (2.32)$$

By definition, the crossover exponent associated with g is $y_g \doteq \nu_0 \lambda_g$ where

$$\nu_0 = \frac{1}{2} + \frac{\epsilon(n+2)}{4(n+8)} + O(\epsilon^2) \qquad (2.33)$$

is the critical exponent associated with the divergence of the correlation length in the uncharged anisotropic model. If $y_g > 0$, g increases under iteration of the RG transformation and is called relevant. It modifies dramatically the flow around the considered fixed point. On the other hand it goes exponentially fast to zero if $y_g < 0$ and is therefore called irrelevant. For $y_g = 0$, the field is called marginal and the situation must be studied in more details [14]. In our case there are two new scaling fields of interest : the anisotropy and the charge. Before determining the fixed points, we give the linearized recursion equations for our scaling fields :

$$(q_{l+1}^2\mu_{\parallel l+1})^{-1} = s^{-\eta_A^* + q^{*2}\mu_\parallel^* \frac{n}{12\pi}\rho^{*-\frac{1}{2}}}(q_l^2\mu_{\parallel l})^{-1} \qquad (2.34)$$

$$(q_{l+1}^2\mu_{tr,l+1})^{-1} = s^{-\eta_A^* + q^{*2}\mu_{tr}^* \frac{n}{12\pi}\rho^{*\frac{1}{2}}}(q_l^2\mu_{tr,l})^{-1} \qquad (2.35)$$

$$(q_{l+1}^2\mu_{\perp l+1})^{-1} = s^{-\eta_A^* + q^{*2}\mu_\perp^* \frac{n}{12\pi}(\rho^{*\frac{3}{2}}-4\rho^{*-\frac{1}{2}}(\rho^*-1)^2)}(q_l^2\mu_{\perp l})^{-1} \qquad (2.36)$$

$$\rho_{l+1} = s^{\eta_\perp^* - \eta_\parallel^*}\rho_l \qquad (2.37)$$

As before, the subscript "*" indicates the fixed-point values. We see that $\lambda_\rho = \eta_\perp - \eta_\parallel$ so the anisotropy is in particular marginal close to the isotropic fixed point. The recursion equations have the following fixed points [15]:

i) the Gaussian fixed point where $q = 0$, $\mu \to \infty$, $\rho = 1$ and $\beta = 0$, $\eta = \eta_\parallel = \eta\perp = 0$ and $\nu = \frac{1}{2}$.

ii) the Heisenberg fixed point : $q = 0$, $\mu \to \infty$, $\rho = 1$ and $\beta = \theta(\epsilon)$, $\eta = \eta_\parallel = \eta\perp = \theta(\epsilon^2)$ and $\nu = \frac{1}{2} + \frac{\epsilon(n+2)}{4(n+8)} + O(\epsilon^2)$.

iii) the isotropic superconducting fixed point : $\rho = 1$ and $\eta = \eta_\parallel = \eta_\perp = -\frac{18\epsilon}{n} + O(\epsilon^2)$, β and $\nu \in \mathbb{C}$ for n<n_c=365.9, while $q^2\mu_i = \frac{12\pi\epsilon}{n}, \forall i$. The non-reality of β and ν indicates a runaway to a first order phase transition [9].

iv) two new totally anisotropic superconducting fixed points where $\rho \to 0$ and $\rho^{-1} \to 0$ respectively. Considering the great anisotropy of the high-T_c superconductors ($\rho \simeq 10^{-2}$ to 10^{-3} and even more) it is expected that the RG trajectories we are interested in are influenced by the first of these fixed points. Setting $\rho = 0$ in the recursion equations, we get either $q^2\mu_i = 0$ or $(q^2\mu_i)^{-1} = 0$. From equations (2.21) to (2.30) we have then for the crossover exponents :

$$y_{q^2\mu_\parallel} = \nu_0(\eta_A^* - q^{*2}\mu_\parallel^* \frac{n}{12\pi}\rho^{*-\frac{1}{2}}) \tag{2.38}$$

$$y_{q^2\mu_{tr}} = \nu_0(\eta_A^* - q^{*2}\mu_{tr}^* \frac{n}{12\pi}\rho^{*\frac{1}{2}}) \tag{2.39}$$

$$y_{q^2\mu_\perp} = \nu_0(\eta_A^* - q^{*2}\mu_\perp^* \frac{n}{12\pi}(\rho^{*\frac{3}{2}} - 4\rho^{*-\frac{1}{2}}(\rho^* - 1)^2) \tag{2.40}$$

$$y_\rho = \nu_0(\eta_\perp^* - \eta_\parallel^*) \tag{2.41}$$

Close to the fixed point $\rho = 0$, $y_\rho = -\nu_0\eta_\parallel = \frac{3q^2}{2\pi}\nu_0\sqrt{\mu_\parallel\mu_{tr}}$ (the last equality follows from equation (2.30)) and is therefore positive. The anisotropy is thus relevant. The regions of the phase space of the couplings which are attracted by this fixed point are given by the conditions :

$$q^2\mu_\parallel > \frac{12\pi}{n}\eta_A\sqrt{\rho} \tag{2.42}$$

$$q^2\mu_{tr} < \frac{12\pi}{n}\eta_A\sqrt{\rho^{-1}} \tag{2.43}$$

$$q^2\mu_\perp \to 0 \tag{2.44}$$

where according to equation (2.20), it is natural to set $\eta_A = \epsilon$. Perpendicular to the CuO-plane, the magnetic permeability is thus very small. The material tends to expel the magnetic field in this direction. We will come back to this point later.

Close to this fixed point the transition is second order with

$$\beta = O(\epsilon) \tag{2.45}$$

$$\eta = \eta_\parallel = \eta\perp = O(\epsilon^2) \tag{2.46}$$

$$\nu = \frac{1}{2} + \frac{\epsilon(n+2)}{4(n+8)} + O(\epsilon^2) \tag{2.47}$$

i.e. the same values as for the Heisenberg fixed point. Beside the decrease of the magnetic permeability in the z-direction, the only observable effect of this fixed point would therefore be to reduce the size of the first order phase transition. This point deserves some comments. The Ginzburg criterion (equation (1.2)) determines the size of the critical region, i.e. the interval of temperature around the mean-field critical temperature where the fluctuations of the order parameter become so important as to produce the breakdown of the mean-field theory. Inside this interval of temperature, the behaviour of the model is determined by the nature of the flow around the fixed points of the RG recursion equations. As we perform RG iterations, the flow approaches successively different fixed points thus determining different critical behaviours. We can therefore observe different critical exponents in the interval of temperature defined by equation (1.2). Since the new anisotropic fixed points have the same critical exponents as the Heisenberg fixed point, we cannot distinguish their respective region of influence. However, it is possible that the new fixed points reduce the size of the interval of temperature around T_c influenced by the complex fixed point. That is what we mean by "reducing the size of the first order phase transition".

Before we turn our attention to the effect of this fixed point on the criticality of our model, let us mention that the other fixed point is characterized by the same critical exponents, $\beta = O(\epsilon)$, whereas the "attractivity condition" (see eq. (2.38) to (2.40)) reads :

$$q^2 \mu_\parallel \quad < \quad \frac{12\pi}{n} \eta_A \sqrt{\rho} \tag{2.48}$$

$$q^2 \mu_{tr} \quad > \quad \frac{12\pi}{n} \eta_A \sqrt{\rho^{-1}} \tag{2.49}$$

$$q^2 \mu_\perp \quad \to \quad \infty \tag{2.50}$$

3   Numerical Study Of The Flow

The recursion equations we have derived exhibit two new fixed points. One of them attracts our attention for the two following reasons in connexion with the properties of the high-T_c superconductors : First, it is highly anisotropic, the RG trajectories that pass close to it describe almost decoupled planes, and secondly, it exhibits a large magnetic permeability parallel to the CuO-planes, while perpendicular to these planes, the permeability is very small. In order to determine the influence of this fixed point on the flow, we numerically study the evolution of the "effective critical exponents" defined in (2.29) and (2.30) and of the magnetic permeabilities given by (2.21) to (2.23).

As as already been said, every RG iteration integrates out fluctuations with wave vectors $\frac{\Lambda}{s} \leq k \leq \Lambda$. At this point we must recall that the physical justification for this choice of RG transformation is that, in approaching the critical temperature, the correlation length grows more and more. As a consequence, fluctuations with long wave vectors lose their importance and can be integrated out. Therefore, the more integration , i.e. the more RG iteration, we perform, the closer we are to the critical temperature. In other words the successive free energy functionals that are generated by the RG transformation define a flow in the

coupling space that reflects the critical behaviour of the model as we approach the critical temperature in the sense that it defines a sequence of correlation lengths

$$\xi_l \sim s^l \sim \frac{|T_l - T_c|}{T_c}^{-\nu_0} \tag{3.1}$$

Thus there is a relation between the temperature T_l and the number l of iterations performed [16]:

$$\frac{|T_l - T_c|}{T_c} \sim s^{-\frac{l}{\nu_0}} \tag{3.2}$$

in term of the critical exponent ν_0 of the uncharged model. This allows us to compute the size of the critical region governed by the complex superconducting fixed point, i.e. the size of the first order phase transition, by determining at which iteration the couplings and the critical exponents start to move away from one value to a new one. Let us remark that the intermediate regime between the two fixed-point describes in no way any physical behaviour. Only the fixed-point values of the critical exponents are susceptible of being observed.
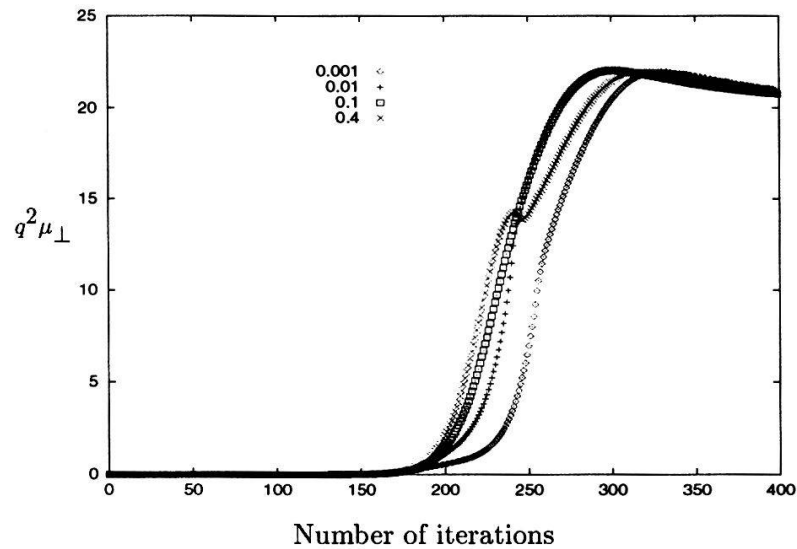
Figure 9 : **Evolution of the perpendicular magnetic permeability under the RG iteration for initial anisotropies** $\rho = 0.001, 0.01, 0.1$ **and** 0.4.

In figures 9 to 12 we give the results of our numerical investigations. All the figures have been obtained with the values n=2, ϵ=1, supposedly adequate for high-T_c superconductors,and $s = 1.1$. Simulations performed with different rescaling factors corroborate our conclusions. In figure 9 we plot the evolution of the crossover scaling field $q^2\mu_\perp(l)$ for various initial anisotropies. As a consequence of the anisotropy, the model shows a certain opposition to the apparition of charge, which we interpret as a direct consequence of the new totally anisotropic fixed-point $\rho = 0$. The extreme anisotropy of this fixed point almost confines the charge carriers to the XY-plane. As a consequence they can only react to a field in z-direction, i.e. $\mu_\perp$ is much more affected by this fixed point than $\mu_\parallel$. This interpretation is corroborated by figure 10 where we plot $q^2\mu_\parallel$ against $q^2\mu_\perp$ for various initial anisotropies.

The effect of the new fixed point is obvious. There is thus no doubt that the flow is drastically influenced by the anisotropy.
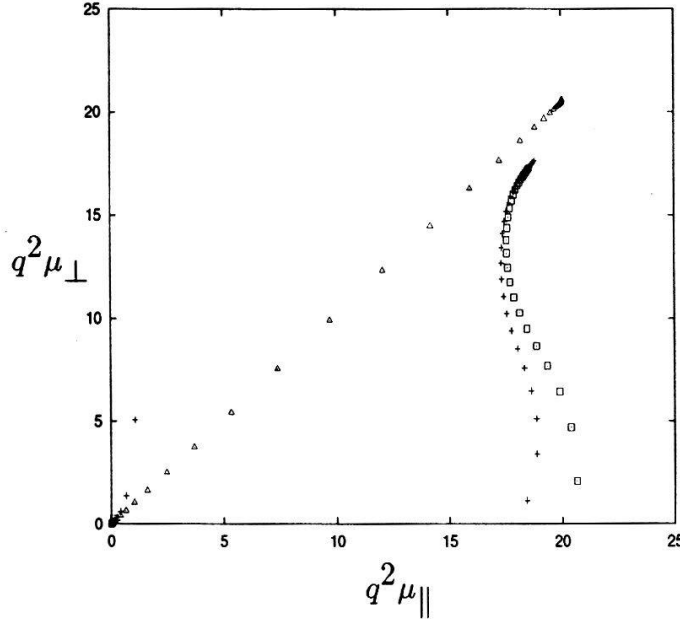

Figure 10 : **Plot of the perpendicular vs. the parallel magnetic permeability for initial anisotropy $\rho = 0.9$ (triangles), 0.05 (crosses), 0.001 (squares). As a consequence of the anisotropy, $q^2\mu_\perp$ is switched on later than $q^2\mu_\parallel$.**

In figure 11 we plot the "effective critical exponent" (equations (2.29)) for various initial anisotropies. This plot shows definite evidence of the delay of the runaway to the first order transition. Everytime we lower the anisotropy by a factor ten, the crossover of $\eta_\perp$ from $\eta_\perp = 0$ to $\eta_\perp = -18\frac{\epsilon}{n}$ ($-18\frac{\epsilon}{n} = -9$ for superconductors since they have n=2 and d=3, i.e. ϵ=1) is retarded by 15 to 20 iterations. This corresponds to a reduction of the first order phase transition :

$$\delta(\rho) \doteq \frac{\Delta T(\rho)}{\Delta T(1)} = \frac{s^{-\frac{l(\rho)}{\nu}}}{s^{-\frac{l(1)}{\nu}}} = s^{\frac{\Delta l(\rho)}{\nu}} \tag{3.3}$$

where $\Delta T(\rho)$ is the size of the first order phase transition, and $\Delta l(\rho) \doteq l(\rho) - l(1)$ is the delay of the crossover. We furthermore notice that there is no intermediate value of η since the Heisenberg fixed point and the anisotropic one have the same critical exponents. In figure 12, we plot $\Delta l(\rho)$ versus the logarithm of the initial anisotropy. The straight line is given by $\Delta l(\rho) = 18.42 \ln(\rho)$. The accuracy of the fit naturally leads us to conclude that, at least in the range of anisotropy we have studied, the reduction is in a good approximation given by :

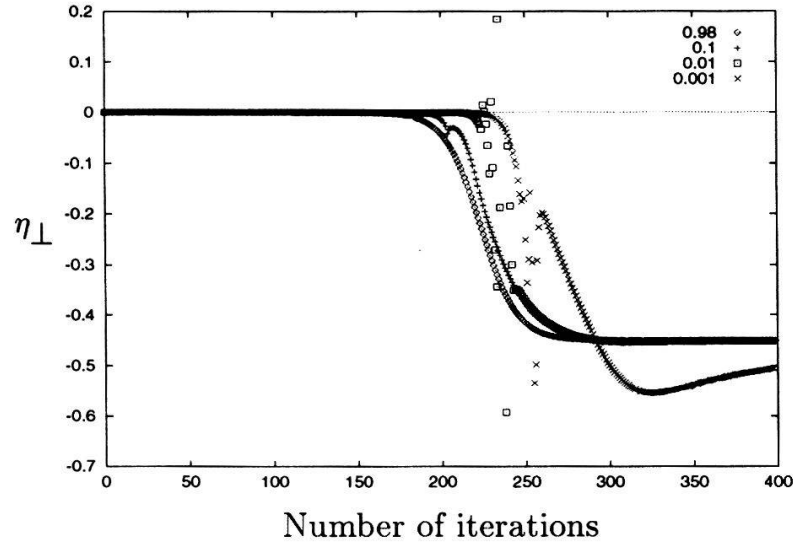$$\delta(\rho) \approx \rho^{18.42\ln(1.1)\nu_0} \approx \rho^{1.053} \tag{3.4}$$

Figure 11 : **Evolution of the effective perpendicular critical exponent $\eta_\perp(l)$. The delay of the crossover is clearly noticeable and reaches 15 to 20 iterations for a lowering of a factor 10 of the anisotropy.**

This result can be expressed in term of a crossover function ($\epsilon=1$)[17]:

$$\eta(\rho,t) = t^x f(\frac{t}{\rho^\phi}) = t^0 f(\frac{t}{\rho^{1.053}}) = f(\frac{t}{\rho^{1.053}}) \tag{3.5}$$

where t$=\frac{|T-T_c|}{T_c}$ is the reduced temperature. The size of the first order transition in an isotropic material is approximated by [4]: $|T - T_c| \lesssim \epsilon_c T_c \kappa^{-3}$ in term of the Ginzburg-Landau parameter $\kappa = \frac{\lambda}{\xi}$,where λ is the penetration depth, ξ the coherence length, and ϵ_c is defined in equation (1.2). This is typically of the order of 10^{-5} if we insert values corresponding to high-T_c superconductors. So the size of the first order phase transition, reduced by a factor 100 to 1000 by the high anisotropy of the HT$_c$ is at best of the order $|T - T_c| \lesssim 10^{-7}$ hardly experimentally observable, though theoretically not completely destroyed.
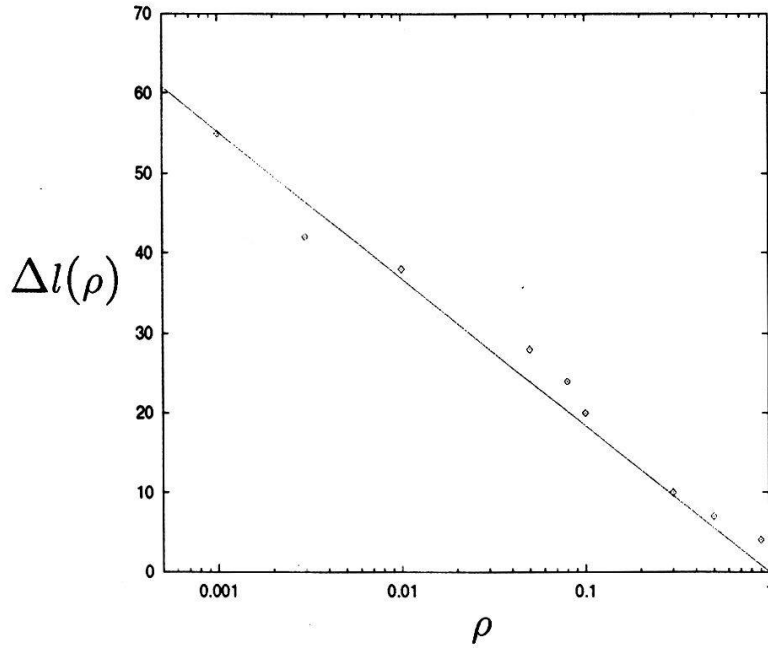
Figure 12 : **Reduction of the size of the first order phase transition. The straight line is given by $\Delta l(\rho) = 18.42 \ln\rho$.**

4   Conclusions

In this paper, we have presented a phenomenological theory of the superconducting phase transition. We started from the Ginzburg-Landau theory, put emphasis on the anisotropy of the Copper Oxides Superconductor, and introduced a minimal coupling between the order parameter which describes the superconducting condensate and the intrinsic magnetic field produced by the charge carrier. Furthermore, we used a treatment which takes care of the fluctuations of both the order parameter and the gauge field. We saw that such a treatment was imposed by the small coherence length and the large critical temperature of the HT_c. Our results were obtained using the Wilson-Fisher ϵ-expansion up to first order in ϵ=4-d. Our main purpose was to determine the order of the phase transition. Our results are the following :

The anisotropy modifies the RG recursion equations in that it introduces two new totally anisotropic fixed points which modify drastically the shape of the RG flow. As a consequence, the runaway to the first order phase transition is delayed sensibly, and the size of the first order phase transition is reduced by a factor

$$\delta(\rho) \doteq \frac{\Delta T(\rho)}{\Delta T(1)} \approx \rho^{1.053} \qquad (4.1)$$

This reduction lowers the size of the first order transition at least down to $|T - T_c| \lesssim 10^{-7}$ and thus destroys any hope of its experimental observation in the HT_c. Furthermore and following the already mentioned papers of Halperin et. al.[4] and of Chen et. al.[9], Kolnberger

and Folk [18] performed a few years ago an expansion up to second order in $\epsilon=4$ - d. This expansion lowered n_c, the critical dimension of the order parameter under which the transition is first order, from $n_c = 365.9$ down to 2.4, i.e. in a physically attainable range, thus suggesting a suppression of the first order transition. It is thus rather unlikely that introducing the anisotropy in a second order expansion will favour a first order transition. It is however possible that our results are not applicable to high-T_c superconductors, since setting $\epsilon=1$ in our formula could put us outside of the convergence radius of our expansion. (Even inside the radius of convergence, the precision of this expansion is hard to estimate !) Moreover, it is possible that the anisotropy should be introduced in a different way, leading to different results.

The appearance of the new totally anisotropic fixed point, especially the associated increase in the perpendicular magnetic permeability, seems to have a strong relation with some experimental properties of the high-T_c superconductors. It would be in particular interesting to study the modifications due to the presence of an external field, since they could enlighten some aspects of the vortex formation, though a more careful consideration of the anisotropy could be necessary [19]. In another direction, a $\frac{1}{n}$ - expansion could give further hints on the critical behaviour of this model. Finally, as has been pointed out by Lawrie [20], a more careful study of the actual form of the thermodynamic functions could be necessary in order to determine the exact order of the transition.

Appendix A

We explain in this section the lines of derivation of equations (2.20) to (2.27). The general method has already been described in section 2.2. In order to get the recursion equations for the various couplings, we have to compute the contributions arising from the figures 3 to 6. Comparison of these contributions with the original free energy functional leads us to the recursion equations.

As an example, we derive explicitly equation (2.23). The graph to be computed is the rightest on figure 3. Its contribution is :

$$-2q^2 n \gamma_\perp \frac{\int d^d k}{(2\pi)^d} \frac{(\vec{k}_\perp \vec{A}_{\vec{k}}^\perp)^2}{(\gamma_\perp \vec{k}_\perp^2 + \gamma_\parallel \vec{k}_\parallel^2 + \alpha)(\gamma_\perp (\vec{k}_\perp + \vec{k}_\perp')^2 + \gamma_\parallel (\vec{k}_\parallel + \vec{k}_\parallel')^2 + \alpha)}$$

where d is the dimension, d=4-ϵ, and n is the number of component of the order parameter. Since we restrict ourselves to an expansion in first order in ϵ, we perform the calculation setting d=4 and α=0. This comes from the fact that the term we are interested in has a coupling $(8\pi\mu_0)^{-1}$. As a consequence our correction will be proportional to $q^2\mu_0 = O(\epsilon)$. We must thus compute :

$$\frac{-2q^2 n \gamma_\perp}{(2\pi)^4} \int_{\frac{\Lambda}{s}}^{\Lambda} dk \int_0^{2\pi} d\phi_1 \int_0^\pi d\phi_2 \int_0^\pi d\phi_3 \frac{k^3 (\vec{A}_{\vec{k}}^\perp)^2 \sin(\phi_2) \sin^2(\phi_3) \cos^2(\phi_3)}{(\gamma_\perp \cos^2(\phi_3) + \gamma_\parallel \sin^2(\phi_3))(\gamma_\perp (\vec{k}_\perp + \vec{k}_\perp')^2 + \gamma_\parallel (\vec{k}_\parallel + \vec{k}_\parallel')^2)}$$

The term we are interested in is proportional to $\vec{k'}_\perp^2 \vec{A}_{\vec{k}}^{\perp\,2}$. In order to get this we expand the second term in the denominator of the integrand in a Taylor series in $\vec{k'}$. The relevant contribution is then :

$$\frac{-2q^2 n \gamma_\perp}{(2\pi)^4} \int_{\frac{\Lambda}{s}}^{\Lambda} dk \int_0^{2\pi} d\phi_1 \int_0^\pi d\phi_2 \int_0^\pi d\phi_3 \frac{(\vec{A}_{\vec{k'}}^\perp)^2 \sin(\phi_2)\sin^2(\phi_3)\cos^2(\phi_3)}{k(\gamma_\perp \cos^2(\phi_3) + \gamma_\| \sin^2(\phi_3))} \times$$

$$\times \left[\frac{4}{(\gamma_\perp \cos^2(\phi_3) + \gamma_\| \sin^2(\phi_3))} \left[\gamma_\perp^2 \cos^2(\phi_3)k_\perp'^{\,2} + \gamma_\|^2 \sin^2(\phi_3)(\sin^2(\phi_2)\cos^2(\phi_1)k_x'^{\,2} + \right. \right.$$

$$\left. \left. + \sin^2(\phi_2)\sin^2(\phi_1)k_y'^{\,2} + k_t'^{\,2}\cos^2(\phi_2) \right] - (\gamma_\perp k_\perp'^{\,2} + \gamma_\|^2 k_\|'^{\,2}) \right]$$

$$= -\frac{q^2 n}{16\pi^2} \vec{A}_{\vec{k'}}^{\perp\,2} \ln(s) \left[\frac{1}{2}k_\perp'^{\,2}\rho^{\frac{3}{2}} - \frac{1}{6}k_\|'^{\,2}\rho^{\frac{1}{2}} \right]$$

where since d=4, $\vec{k'} = (k_x', k_y', k_t', k_\perp')$. The other graphs on figure 3 are treated in the same way. The choice of the coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$ allows us to transform terms like $k_x' k_y'$ in relevant terms. We end up with the contribution :

$$\frac{q^2 n \ln(s)}{96\pi^2} \left[\left[\rho^{\frac{3}{2}} - 4\rho^{-1}(\rho-1)^2 \right] k_\perp'^{\,2} \vec{A}_{\vec{k'}}^{\perp\,2} + \vec{k}_\|'^{\,2} \vec{A}_{\vec{k'}}^{\|\,2} \rho^{-\frac{1}{2}} + (k_\|'^{\,2} \vec{A}_{\vec{k'}}^{\perp\,2} + k_\perp'^{\,2} \vec{A}_{\vec{k'}}^{\|\,2})\rho^{\frac{1}{2}} \right]$$

Comparison with the right part of (2.18) leads us directly to equations (2.21) to (2.22). The other couplings are treated in the same way.

Let us finally add that these recursion equations are linearized equations. They are built with the first two terms of a power expansion in ϵ. In that sense we can rewrite equation (2.24) :

$$\gamma_{\perp,l+1} = \gamma_{\perp,l}(1 - \eta \ln s)\left[1 - \frac{3}{2\pi}q^2\gamma_{\perp,l}\ln s \left(\frac{\mu_{tr,l}^{\frac{3}{2}}\mu_{\perp,l}^{\frac{1}{2}} - \mu_{tr,l}\mu_{\perp,l}\rho_l^{\frac{1}{2}}}{\mu_{tr,l}\gamma_{\|,l} - \mu_{\perp,l}\gamma_{\perp,l}} \right) \right]$$

$$= \gamma_{\perp,l}(-\eta - \frac{3}{2\pi}q^2\gamma_{\perp,l} \frac{\mu_{tr,l}^{\frac{3}{2}}\mu_{\perp,l}^{\frac{1}{2}} - \mu_{tr,l}\mu_{\perp,l}\rho_l^{\frac{1}{2}}}{\mu_{tr,l}\gamma_{\|,l} - \mu_{\perp,l}\gamma_{\perp,l}})\ln s + O(\epsilon^2)$$

The fixed-point condition $\gamma_{\perp,l+1} = \gamma_{\perp,l}$ leads us then to equation (2.29). Equation (2.30) is obtained in the same way from equation (2.25).

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