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## On the Electronic Mean Free Path in Aluminum

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*Abstract.* We find experimentally that the electronic mean free path  $l$  in aluminum is given by  $\sigma/l = (1.48 \pm 0.02) \times 10^{11} \Omega^{-1} \text{ cm}^{-2}$  where  $\sigma$  is the conductivity. This corresponds to a Fermi surface area which is 59% of that expected from the free electron model. One can understand this low value on the assumption that small angle scattering makes the free path very small in the higher Brillouin zones.

The Fermi surface of aluminum has been the subject of a number of detailed studies, and its shape is now very well known. Aluminum is therefore an admirable metal for use in testing various aspects of transport theory.

We have recently been especially interested in the question of how well the area of the Fermi surface can be estimated from the mean free path of the electrons in a metal making use of the expression

$$\sigma/l = \frac{e^2}{12 \pi^3 \hbar} S, \quad (1)$$

where  $S$  is the free area of the Fermi surface in  $k$ -space, and  $l$  is given by:

$$l = \frac{1}{S} \iint dS \tau(k) |v(k)|. \quad (2)$$

Expression (1) holds irrespective of the anisotropy of  $l$  for single crystals with energy surfaces of cubic symmetry or for polycrystals of such a material.

Values of  $\sigma/l$  for several metals obtained from size effect observations have been published. The results found by different observers vary widely. Such measurements involve the investigation of a range of samples of different dimensions. Evaluation of the results depends upon the physical purity of all the samples being equal. Thus the careful measurements of FØRSVOLL and HOLWECH<sup>1)</sup> on aluminum films show considerable scatter in the derived values of  $\sigma/l$  because of this difficulty.

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This problem can be avoided by making use of an eddy current method recently described by one of us<sup>2)3)</sup>. The resistivity  $\varrho_\tau$ , determining the decay time  $\tau$  of eddy currents induced in thin plates of thickness  $d$  by the removal of a small external magnetic field  $H$  parallel to the plane of the plate, has a different dependence upon  $l/d$  from that of the D.C. resistivity  $\varrho_F$  of such plates. Measurements of  $\varrho_\tau$  and  $\varrho_F$  in a single plate therefore allow a determination of the mean free path  $l$ . Work on indium showed that very reproducible results for  $\sigma/l$  could be obtained by this method. These values are in good agreement with an extensive series of ordinary size effect measurements on indium carried out recently in this laboratory<sup>4)</sup>.

It seemed desirable, therefore, to make use of this method in other metals of cubic, or almost cubic, symmetry having simple Fermi surfaces. The extension of this technique to lead is problematical because the magnetic field required to suppress superconductivity introduces serious errors due to magnetoresistive size effects. Work on copper is complicated by the relatively short mean free path found even in the purest samples obtainable. Aluminum, however, fulfills the requirements of our investigation.

We have now made use of this eddy current method to measure  $\sigma/l$  in aluminum. Pure zone refined aluminum specimens rolled into thin strips  $0.5 \times 10$  cms and of the thicknesses given in Table I were investigated. Each specimen was examined in the unannealed and in the annealed state. All measurements were made at 4.2° K.

Table I  
The experimental results

Specimen	thickness $d$ (mm)	$\varrho_\tau$ ( $10^{-9} \Omega$ cm)	$\varrho_F$ ( $10^{-9} \Omega$ cm)	$l/d$	$l$ (mm)	$\sigma = 1/\varrho$ ( $10^{10} \Omega^{-1} \text{cm}^{-1}$ )	$\sigma/l$ ( $10^{11} \Omega^{-1} \text{cm}^{-2}$ )
Al 1	0.193	1.123	0.792	0.55	0.106	1.58	1.49
Al 2	0.106	1.85	1.230	0.66	0.070	1.06	1.52
Al 1a	0.193	1.473	1.238	0.32	0.062	0.92	1.50
Al 2a	0.106	2.46	1.88	0.42	0.045	0.63	1.42

Sample Al 1a and Al 2a before, and sample Al 1 and Al 2 after annealing.

In Table I the specimens and the values obtained for  $\varrho_\tau$  and  $\varrho_F$  are listed together with the values of  $\sigma/l$  deduced. The values calculated for  $\sigma/l$  for the unannealed specimens are very sensitive to small errors, and the work on annealed specimens must be regarded as the more reliable. It will be seen that these specimens give results of good consistency.

The average value of  $\sigma/l$  so obtained is

$$\sigma/l = (1.48 \pm 0.02) \cdot 10^{11} \text{ ohm}^{-1} \text{ cm}^{-2}.$$

The probable error in these measurements is much smaller than in the work of FØRSVOLL and HOLWECH<sup>1)</sup> and of MONTARIOL and REICH<sup>5)</sup>. Their values of  $\sigma/l$  and

values using other experimental methods and from theoretical considerations are listed in Table II. It will be seen that all the size effect values for  $\sigma/l$  are considerably smaller than the results derived from anomalous skin effect measurements or from theory assuming an almost free electron model.

Table II  
Values of  $\sigma/l$  for aluminum obtained by various methods

Method	$\sigma/l$ ( $10^{11} \Omega^{-1} \text{ cm}^{-2}$ )	Investigator	S using Equation (1) (in % of $S_0$ )
D.C. – size effect	0.93	MONTARIOL and REICH <sup>5)</sup>	37%
D.C. – size effect	1.43	FØRSVOLL and HOLWECH <sup>1)</sup>	57%
D.C. – and eddy current size effect	1.48	our work	59%
Anomalous skin effect	2.51	FAWCETT <sup>10)</sup>	100%
Anomalous skin effect	2.04	CHAMBERS <sup>11)</sup>	81%
Theoretical assuming three free electrons per atom	2.51		100%
Theoretical using Equation (1) and considering only the second zone Fermisurface <sup>a)</sup>	1.70		68%

$S_0$  = Fermi surface calculated for three free electrons per atom ( $S_0 = 38.4 \cdot 10^{16} \text{ cm}^{-2}$ ).

<sup>a)</sup> After an estimate of FØRSVOLL and HOLWECH<sup>1)</sup>.

A similar but even more striking phenomenon occurs in indium where work by one of us using the same D.C. – and eddy current technique<sup>2)3)</sup> – yields a  $\sigma/l$  value which corresponds to only 43% of the three free electrons per atom value. It is highly unlikely that the Fermi surface in these two metals can be so very much smaller than the free electron theory surface, and this view is clearly supported by the anomalous skin effect results.

For complicated Fermi surfaces the mean free path deduced from size effect measurements can not be regarded as the mean value defined by (2), but will mainly allow for those parts of the Fermi surface having the longest free paths. This phenomenon has been discussed for a two band model for the case of thin wires by one of us<sup>6)</sup>. Thus we must suppose that our size effect measurements determine the area of those parts of the Fermi surface having the longest mean free paths, and that the remaining parts of the surface contribute only negligibly to the current. It is interesting to note that our value for  $S$  is somewhat smaller than the value ascribed to the closed surface in the second Brillouin zone<sup>1)</sup>.

KLEMENS<sup>7)</sup> has pointed out that small angle scattering may be particularly efficient in reducing the relaxation time for the strongly distorted parts of the Fermi

surface in higher zones, and that the cross section for such small angle scattering may be much larger than that for large angle scattering even in the absence of phonons. The anisotropy of the conduction properties of tin investigated by KLEMENS VAN BAARLE and GORTER<sup>8)</sup> appear interpretable along similar lines. It is also clear from TAYLOR's<sup>9)</sup> work on copper that conditions of small angle scattering lead to remarkable anisotropies in  $\tau$  even in this simple metal.

We conclude from the present measurements and those on indium that the mean free path anisotropy in the residual resistance region is much greater than previously realized and that it may lead to a quenching of the conductivity of large parts of the Fermi surface even in metals with energy surfaces close to those of the free electron models.

This work was carried out while one of us (E. M. FRYER) was on a visit to the Institut für kalorische Apparate und Kältetechnik. We wish to thank Professor P. G. KLEMENS for a very interesting discussion on the effects of small angle scattering. Our thanks are due to Professor Dr. P. GRASSMANN, the Director of the Institute, for his interest and encouragement. This work was in part supported financially by the Schweizerischer Nationalfonds.

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