Octupole Vibrations in Spherical Nuclei

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Abstract. A systematical study of the 3^- state in even-even nuclei $40 \leq A \leq 150$ is presented. The liquid drop model is applied to show the behaviour of $\hbar\,\omega_3$ against A. For the nuclei $Z = 50$ and $N = 82$ $\hbar\,\omega_3$ has been calculated using the pairing correlation model.

Introduction

A great number of 3^- states being observed in even-even spherical nuclei[1]), their systematical study is therefore possible. The collective nature of these states seems to be of no doubt whereas the measured $B\,(E\,3;\,3^- \rightarrow 0^+)$ are 20–100 times stronger than the single particle value.

If the collective motion of the nucleus is described in terms of shape oscillations of small amplitude, the 3^- state is interpreted as octupole vibrations around the spherical equilibrium shape[2]). This statement leads to the well known phonon model, in which the energy of the octupole phonon is simply:

$$\hbar\,\omega_3 = \sqrt{C_3/B_3} \tag{1}$$

C_3 is the restoring force parameter and B_3 the mass parameter. These both quantities depend on the nuclear structure. Our purpose is to compare the experimental values of $\hbar\omega_3$ with the predictions of the liquid drop model and of the pairing correlation model.

The liquid drop model

In a first attempt, the collective motion of the nucleons in the nucleus may be studied using collective degrees of freedom. For instance, if the nucleus is regarded as a liquid drop, the collective variables consist of the parameters describing the fluctuation of the nuclear surface. Such a model is based on very rough assumptions

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and does not provide any intrinsic explanation of the phenomena. However his rather simple form is helpfull to present the total set of experimental results. The nucleus being considered as an irrotationnal and incompressible liquid drop, B_3 and C_3 are given by

$$B_3 = \frac{1}{4\pi} A M R_0^2 \qquad C_3 = 10 R_0^2 S - \frac{3}{7\pi} Z^2 e^2 R_0^{-1} \tag{2}$$

The phonon energy depends only on the parameter S, the surface tension. Using $R_0 = 1{,}2 A^{1/3}$. 10^{-13} cm, the best agreement with experimental results [1] was found for $4\pi R_0^2 \bar{S} = 5{,}64\ A^{2/3}$ Mev (30–40% of the value of the semi-empirical mass formula). The results are plotted in figure 1. The model reproduces more or less the general behaviour of the 3^- energies against A.

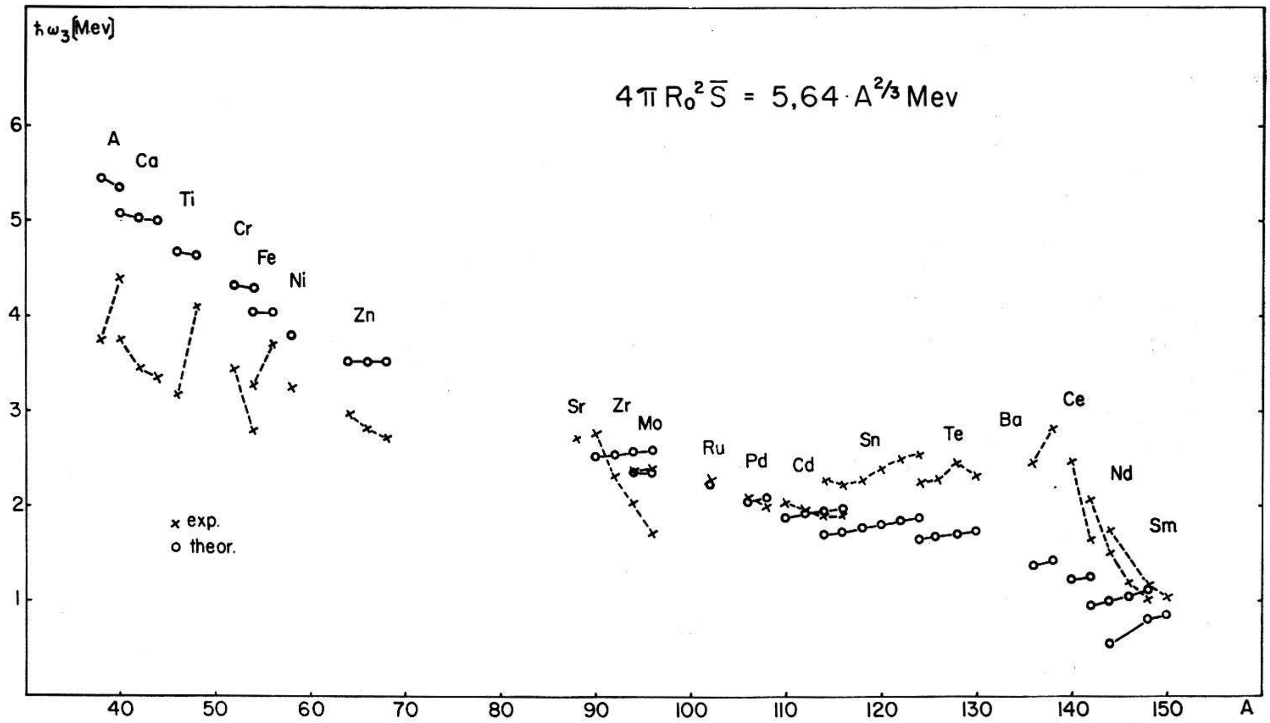

Figure 1

Energies of the 3^- states compared with the predictions of the liquid drop model.

Pairing Correlation Model

Obviously a rigorous theory of the nuclear motion requires the solution of a many — body problem. The pairing correlation model constitutes a first step in this direction.

The model being well extended in many papers*) we only want to sketch the method. The start point is the hamiltonian

$$H = H_0 + H_{pairing} + H_{long\ range} \tag{3}$$

*) We refer us principally to the papers of KISSLINGER and SØRENSEN [3] and of YOSHIDA [4]. All the symbols used here may be found in these two articles.

H_0 is the hamiltonian of the spherical average field, here the shell model hamiltonian. $H_{pairing}$ is associated with the short range forces and provides a strong coupling between the nucleons coupled to $J = 0$. It is responsible for the gap of the even-even nuclei and for a strong configuration mixing.

Each pair of nucleon is on the average spherical symmetric, but it undergoes fluctuations in which happen, momentarily, non-spherical configurations. The long range term gives rise to the coherent addition of the pair fluctuations.

$H_{pairing}$ is treated with the help of the quasi-particle formalism. Following KISSLINGER and SØRENSEN[3]) we simulate the long range force by a P_3 force. We then apply the adiabatic perturbation statement and we get for the energy:

$$\hbar\,\omega_3 = \left[\frac{\sum_{jj'} \frac{\langle j' \mid r^3\, Y_3 \mid j\rangle^2\, u_{jj'}^2}{E_j + E_{j'}}}{\sum_{jj'} \frac{\langle j' \mid r^3\, Y_3 \mid j\,\rangle^2\, u_{jj'}^2}{(E_j + E_j)^3}}\right]^{1/2} \cdot \left[1 - \chi \sum_{jj'} \frac{\langle j' \mid r^3\, Y_3 \mid j\rangle^2\, u_{jj'}^2}{E_j + E_{j'}}\right]^{1/2} \tag{4}$$

where

$$\langle\, j' \mid r^3\, Y_3 \mid j\,\rangle^2 = \frac{1}{4\,\pi}\,(2\,j + 1)\,(2\,j' + 1)\,(j'\,\tfrac{1}{2}\,j - \tfrac{1}{2} \mid 3\,0)^2 \,\langle\, N'\,l' \mid r^3 \mid N\,l\,\rangle^2 \tag{5}$$

N, l and j denote the number of quanta of the harmonic oscillator, the orbital and the total angular momentum of the shell model states. The radial matrix elements have been tabulated by YOSHIDA[4]). E_j is the quasi-particle energy and χ gives the interaction strength. Using the values of KISSLINGER and SØRENSEN for ε_j, λ and Δ, we calculated the $\hbar\,\omega_3$ for the nuclei $Z = 50$ and $N = 82$. The interaction was supposed to be effective only between the extra-core nucleons. Thus the 3^- state is obtained

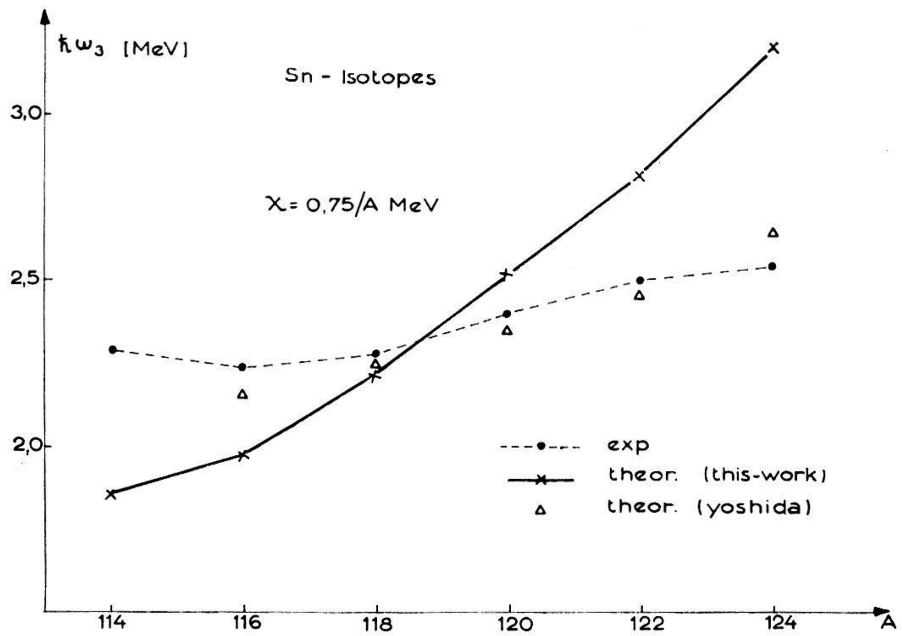

Figure 2

Energies of the 3^- states in Sn-isotopes.

with the two configurations $(h_{11/2}\, d_{5/2})$ and $(h_{11/2}\, g_{7/2})$. The best agreement with the experimental results corresponds to an interaction constant of

$$\chi = 0{,}75 \,/\, A \ \text{Mev.}$$

The Sn 3^- states were also calculated by YOSHIDA[4]), using the method of linearized equations of the motion proposed by BARANGER[5]). In his computation, he assumes an effective octupole interaction between all the nucleon pairs, which gives much more possible configurations. This last point seems to modify greatly $\hbar\,\omega_3$ (A).

The results are presented in figures 2 and 3. As pointed out the number of included configurations seems to be of great importance. More specificly: for two configurations $\hbar\,\omega_3$ is sensitive to the parameters ε_j and λ.

EL-Transition Probabilities

Beside the energy, the reduced transition probability provides a good test for nuclear models. In our case we have for the ground state transition

$$B\,(E\,3) = \frac{7}{2}\, e^2_{eff.}\, \frac{1}{\sqrt{B\,C}} \tag{6}$$

e^2_{eff} is the effective charge. Considering this quantity equal to 1 we calculated $B\,(E\,3)$ for six measured transitions. The results are shown in the following table.

	$B(E\,3)_{theor.}$	$B(E\,3)_{exp}$	$B(E\,3)_{exp}/B(E\,3)_{sp}$
Sn^{114}	0,50	0,54	90
Sn^{116}	0,45	0,35	57
Sn^{122}	0,40	0,73	107
Sn^{124}	0,36	0,68	95
Ce^{140}	0,20	0,84	93
Nd^{142}	0,36	0,49	52

Here we have $B(E\,3)_{sp} = 4{,}6 \cdot 10^{-7}\, A^2\, e^2 \cdot 10^{-72}\ \text{cm}^6$

On another hand it is well known that the 3^- states are decaying mainly by $E\,1$ transitions to the first 2^+ levels, rather than by $E\,3$ transitions to the ground state. These $E\,1$ are strickly forbidden in a pure phonon model, as well as in our two quasiparticle approximation, as long as the configurations of the core nucleons are not taken into account. Thus we should expect strongly reduced $B\,(E\,1)$. In the case of Nd^{144} the $B\,(E\,1)$ may be deduced from the $B\,(E\,3)$ measured by HANSEN and NATHAN[1]), the intensity ratio $I\,(E\,1;\, 3^- \rightarrow 2^+)/I\,(E\,3;\, 3^- \rightarrow 0^+)$ being known. We get

$$B\,(E\,1) = 2{,}4\ 10^8$$

If we refer to the systematic established by PERDRISAT[6]), this value is to be compared with three $B\,(E\,1)$ measured in A^{38}, Ni^{58} and Ni^{60} respectively. These three cases are also $3^- \rightarrow 2^+$ transitions. It is the slowest transitions we know in the spherical even-even nuclei. Their systematical study would be of great interest.

The same conclusions are more or less applicable to the El transitions connecting the 3^- and the 4^+ levels. Particulary, the comparison between B ($E1$; $3^- \rightarrow 4^+$) and B ($E1$; $3^- \rightarrow 2^+$) may supply informations on the nature of the 4^+ level.

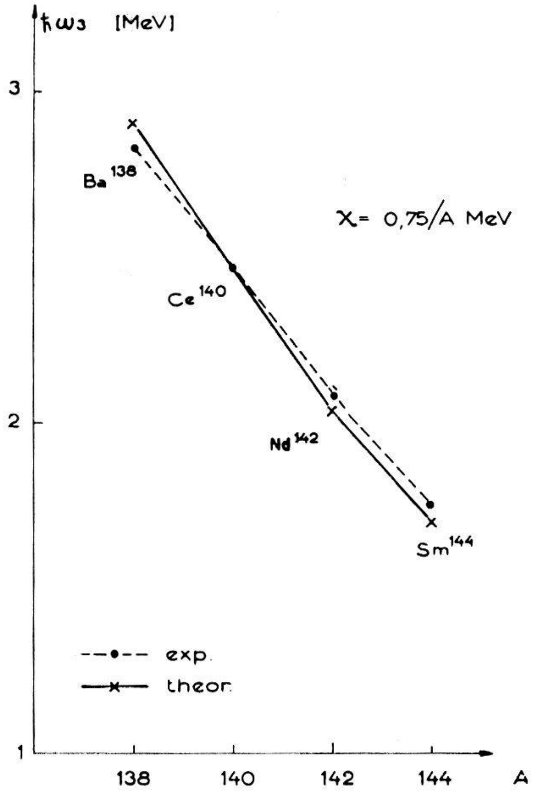

Figure 3

Energies of the 3^- states for the nuclei $N = 82$.

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