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Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **21 (1948)**

Heft V

PDF erstellt am: **10.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-111912

Morphology of aerial propulsion

by **F. Zwicky.**

(3. VI. 1948.)

Abstract.

The morphological method of solving scientific problems is described and applied to the field of aerial propulsion. The principal value of the method lies in the fact that it illuminates the basic phenomena which govern the solution of any given problem. It thus acts as a guide to systematic invention and development. The method also is useful for purposes of teaching complicated technical subjects.

The various ways are discussed in which aerial propulsion, based on the exploitation of energy from chemical reactions and of other available types of energy, can be achieved. Much stress is laid on very general performance calculations which allow the proper choice of propulsive power plants for given specific purposes. A universal thrust formula is derived which makes possible the determination of the thrust and of the specific propellant consumption of practically all propulsive power plants moving through various media such as the atmosphere, bodies of water, or the vacuum.

As quantitative illustrations, the universal thrust formula is applied to the aeroduct (ramjet), the aeroturbojet, the aeropulse, and the internal combustion engine-propeller combination. Graphs are added which illustrate the ideal performance values of the engines mentioned, as functions of the propellant parameter and the forward speed of the vehicle which is to be propelled. Finally, a method is indicated by which the universal thrust formula can be modified to include mechanical and thermal losses through the use of efficiency coefficients which are experimentally determined. Further refinements of the universal thrust formula can be obtained through exact integration or integration by successive approximations of the differential equations which govern the various processes taking place within and without the propulsive power plants.

Reference is also made to a most comprehensive mode of representation of the totality of calculated engine performances in special diagrams. This representation was worked out independently during the war by two groups of scientists; in Pasadena, and in Munich, Germany.

We propose to discuss here some applications of the *morphological method* to problems of aerial propulsion. The morphology of analysis, of synthesis, and of construction is a new method which, instead of attempting to solve individual and restricted problems as they turn up in the course of human endeavor, proceeds to solve whole classes of problems to which the restricted ones belong. The new method may be applied to any field of activity, practical or theoretical, material or spiritual.

Because of the peculiar circumstances brought about by the war emergency, the morphological method in its full extent and in all of its major implications was actually for the first time applied in the realm of *scientific engineering* and more specifically in the field of *jet propulsion*. The net result was the classification of propulsive power plants activated by positive jets[1]) which derive their energy from chemical reactions, or, in some of the best known cases, from chemical combustion or oxidation of proper fuels. This classification which has been described in other places[2]) reveals the existence of at least 576 basic pure-medium engines which may be used for the propulsion of all sorts of vehicles moving in a single medium (e.g. vacuum, air, water, earth). The method has proved its value in the invention and construction of many jet engines not previously known. It is now of historical interest only that it led to the invention, independently of the Germans, of the aeroresonator or V-1 buzz bomb engine[3]). It is, however, of future importance that the aeroresonator and the aeropulse are capable of performance (specific fuel consumption and specific cross-sectional thrust) equal or superior to that of the aeroturbojet.

In the present study, we shall apply the morphological method to the problem of aerial propulsion. We shall first briefly describe the method and then sketch in more detail some of its phases which have not previously been sufficiently emphasized.

It may be stated in advance that the new method not only serves as a sure guide for the navigation on the high seas of invention, but it also proves valuable in the field of technical education and of the comprehensive communication and dissemination of scientific and technical knowledge. This dissemination now is bogged down in a complicated network of individual knowledge which is not organically systematized for purposes of teaching. A morphological analysis of the theory and practice of education is therefore in order, and will be discussed in another place.

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[1]) In a positive jet matter is being expelled away from the vehicle.

[2]) F. Zwicky, "Morphology of Jet Engines" paper presented at the International Congress for Applied Mechanics in Paris, September 1946 and appearing in the Proceedings of the Congress.

[3]) *Aviation Week*, December 1, 1947, p. 26.

The morphological method.

The new method of solving technical problems proceeds by the following steps.

a) Formulation of some specific problem. This problem is immediately generalized as far as it appears possible and advantageous.

b) Analysis of the proposed generalized problem and close examination of all of the significant parameters which might enter its solution.

c) Schematic deduction of the totality of the solutions of the generalized problem in terms of the determining parameters.

d) Determination of the *ideal performance* or the *ideal value* of all of the solutions of the given problem, followed by a realistic evaluation of the expected *practical performance* after inefficiencies and losses have been taken into account.

e) Inter-comparison of all of the solutions in terms of relative value with respect to the required purposes. Choice of those solutions which are most suited to the realization of those special requirements.

f) Detailed analysis, design, and construction of the special solutions chosen. This in general requires a secondary (submorphological) analysis of some of the solutions of the original general problem.

We now proceed to discuss one by one and in greater detail each of the six steps mentioned in the preceding. For purposes of definite illustration we choose as our subject the field of aerial propulsion. But even with this restriction our problem is immense and we shall go into greater details only on point (d) concerning the ideal performance of a few selected propulsive power plants.

1. *Formulation of the problem of aerial propulsion.*

The task of propelling a material vehicle[4]) of macroscopic dimensions through the air has two aspects. In the first place there are features of this problem which are specifically related to the properties of the air or more generally the earth's atmosphere and its various characteristics. The morphology of aerial propulsion is concerned with these properties to the exclusion of those related to other media, such as bodies of water and the earth. If both the air and water or the earth are involved, we enter the field of *interfacial*

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[4]) The propagation of signals, acoustical, electromagnetic or otherwise in nature represents another fascinating field for a morphological analysis which as yet has not been carried out.

devices, that is, vehicles which consume air and travel on the surface of the earth or on the sea. We are not concerned here with such interfacial propulsive power plants, of which there is a great multitude. In the second place, many additional problems are to be solved in connection with the propulsion of vehicles through the air. These problems involve the materials and the types of construction, structural analysis, auxiliary equipment of the aircraft, instruments of navigation, controls and other items which may be of a more general nature than that specifically related to aerial propulsion. We shall limit the discussion of these auxiliary problems to a few remarks at the end of this study.

We now formulate the problem of aerial propulsion. The task before us may at first be a more or less restricted one. For instance, we may wish to transport loads through the air with the least expenditure. Or we may want to propel given loads over the greatest possible distances regardless of cost. There are in practice a great many such requirements which in the past have usually been tackled one by one, a method which has resulted in remarkable achievements. We now know, however, that these achievements have been reached at an exorbitant cost, both in material and in spiritual effort, and that the best solutions have presumably still escaped our attention. To fill in the gaps and to get leads on where to look for new inventions, the morphological method has been conceived. This method, instead of tackling every individual problem, starts from the most general formulation of the problem of aerial propulsion. Only after the solution of the general problem has been schematically derived shall we occupy ourselves with the elaboration of special cases.

The general problem of aerial propulsion may be formulated as follows. (We shall not here consider the motion of light quanta, elementary particles or microscopic bodies.)

Problem: Any kind of material, macroscopic body is to be propelled through the air by whatever means we have at our disposal.

2. *Analysis of the problem of aerial propulsion.*

A number of fundamental characteristics of propulsion through air immediately present themselves. Some of the important aspects or qualitative parameters are as follows.

a) Nature of the propulsive force.

b) Nature of the energy which is used to generate the propulsive force.

c) The duration of the propulsion.

d) The distinction between craft lighter than air and those heavier than air.

e) The free-flight characteristics of the craft.

f) The type of propulsive power plant used and the choice of the propellants.

g) The distinction between manned and unmanned vehicles with implications on the controls and the purpose of the craft.

3. *Schematic deduction of the totality of possible solutions.*

This totality may be arrived at through a discussion of the fundamental aspects mentioned in the previous section. Analyzing these aspects, we find:

a) There are four different types of propulsive forces which may be characterized by the following scheme:

Long Distance Forces	Contact Forces
Attraction	Push < Positive Jets / Negative Jets
Repulsion	Pull

Gravitation and electromagnetic forces represent the category of long distance forces. Matter which is expelled from, or which impinges on the propelled vehicle is characteristic of the positive and negative jets. For a more detailed discussion we refer to a previous publication[2]).

b) Energy is available to us in many forms, which also have been listed previously[2]), where it was shown that nine categories of energy may clearly be distinguished. Of these the gravitational, nuclear (atomic), and chemical energy are currently of the greatest practical importance.

c) The duration of propulsion covers the continuous range from practically zero time (instantaneous forces) to intervals limited only by the propellant supply. The extremes are instantaneous launching, represented respectively by the firing of projectiles from guns and the sustained propulsion of manned aircraft.

d) The distinction between craft lighter than air and heavier than air is of course an important and well-known one, with the historical emphasis having shifted in our time from lighter than air to heavier than air.

e) The free flight characteristics of the aircraft are the subject of the very complicated field of aerodynamics. While the science of flight at subsonic speeds has reached a high state of development, the investigations of supersonic flight and of the interaction of aerodynamic forces with the propulsive forces of positive jets, for instance, are in their infant stages and will be the subject of determined future research.

f) The morphology of the propulsive power plants and of the activating propellants is a problem of extreme interest. While the morphology of propulsion based on the energy from chemical reactions has, during the past decade, been worked out in considerable detail, the morphology of propulsion based on nuclear energy represents one of the biggest unsolved problems. We only wish here to call attention to an error which may easily be committed in seeking this solution. This error made its appearance when propulsion by chemical means was first attempted and achieved via the detour of stationary power plants such as internal combustion engines, with some thrust-producing device such as propellers attached to these engines. As we have now learned, the direct transformation of chemical energy into translational energy through the use of positive jets represents a more organic mode of attack. Going one step farther, we are now likely to make the mistake of trying to achieve nuclear propulsion by associating nuclear energy with the positive jets which proved so successful an instrument when associated with chemical energy. However, already the fact that in this manner only an infinitesimal fraction of nuclear power can be transformed into translational power should make us pause and reconsider the problem of generating propulsion from nuclear energy entirely detached from previously known propulsive power plants. In order to break the shackles of conventional thinking, we only wish here to make the suggestion that instead of propelling a craft through the air we might reverse the concept and think of propelling the air past the vehicle. In other words, one might generate artificial winds and propel the aircraft with the reaction of *negative jets*, a procedure which may well insure a more efficient exploitation of nuclear energies than can be attained through the action of positive jets.

As already mentioned, the morphology of chemical propellants and of propulsive power plants activated by them has been most extensively investigated during the past decades. The scheme of all propulsive power plants useful respectively in vacuum, air, water, and earth as the media which are to be traversed has been previously discussed[2]). We here repeat in greater detail that part of the scheme which applies to the air. Limiting ourselves strictly to chemical propellants for purposes of illustration, we perceive that we have two large classes of engines, depending on whether we use self-contained propellants or whether we wish to employ substances (fuels) which are being reacted with the oxygen or other components of the air flowing through the engine from the surrounding atmosphere. Accordingly, we distinguish between:

A) Engines propelled by self-contained propellants; and

B) Engines with free air intake.

The former engines, which are pure rockets, also operate in vacuum, not being restricted to the presence of air.

The devices of the class B) we designate with the prefix "aero", in contradistinction to devices with intake of free water or earth which we characterize by the prefixes "hydro" and "terra".

The free air which is flowing through the aero-engines may serve two different purposes; namely, the chemical reaction and the thrust augmentation.

In the first place certain components of the air such as the oxygen and the nitrogen may be reacted with suitable chemicals such as gasoline and lithium respectively. In addition to O_2 and N_2 certain minor components of the air, for instance water vapor, water droplets, ozone in the upper atmosphere and others might be used in the reactions taking place in the combustion chambers of the engine. One might even think of the reaction of oxygen and nitrogen of the air which can occur under certain conditions of external pressure and temperature. In this connection a remark may not be superfluous which will be immediately clear to chemists although it may sound strange to power plant engineers. This remark concerns the difference between the enthalpy (heat of reaction or combustion) of a reaction and the *free enthalpy* ($\Phi_p = \varepsilon_i - TS + pV$, where ε_i, S, T, p, V are the internal energy, entropy, temperature, pressure and specific volume). What is actually available for transformation into propulsive power is the free enthalpy and not the enthalpy. Thermodynamic or thermopropulsive efficiency should strictly be expressed in terms of the free enthalpy. In the ordinary combustion processes the difference between enthalpy and free enthalpy is usually small and can therefore be neglected in a first approximation. In reactions such as those between nitrogen and oxygen the difference may be large. In fact the heat of reaction or combustion may be negative and energy may still be available because of a positive change in free enthalpy. Propulsive power plants can therefore in principle be activated by reactions which are characterized by negative heat and which make up for the deficit by extracting heat from the surrounding medium, air, water or whatever it may be. These interesting possibilities deserve a separate analysis which will be given elsewhere.

In the second place the air flowing from the outside atmosphere through or around an aero-engine acts as an augmentor of the thrust, either internal or external.

In addition to the type of propellant used, several other parameters radically influence the character of an aero-engine. Some of these other parameters are:

α) The physical phase of the propellants which are gaseous, liquid or solid.

β) The reaction speed of the propellants, which may be from infinite to zero (such that artificial ignition is necessary).

γ) Most important is the mechanical character of the motion of the engine parts relative to the propellants and to the working fluid. Four possibilities present themselves, namely: translation, rotation, oscillation or no motion at all.

δ) We have already mentioned that the air also plays the role of a thrust augmentor. This thrust augmentation may be internal or external depending on whether the excess air (whose oxygen does not enter the chemical reaction) flows through the engine, like in the aeroturbojet, or whether it does not, like in the case of a propeller. A more strictly formulated statement is that we have external thrust augmentation if the thermal efficiency

$$\eta_{th} = \frac{\Delta\varepsilon}{\Delta\Phi_p} \tag{1}$$

($\Delta\varepsilon$ = total mechanical energy transmitted to the vehicle and to the air, $\Delta\Phi_p$ = available change in free enthalpy) is *independent* of the forward velocity u_0. On the other hand, in the case of internal thrust augmentation, the thermodynamic efficiency η_{th} depends in general on u_0. In the latter case the overall thermopropulsive efficiency

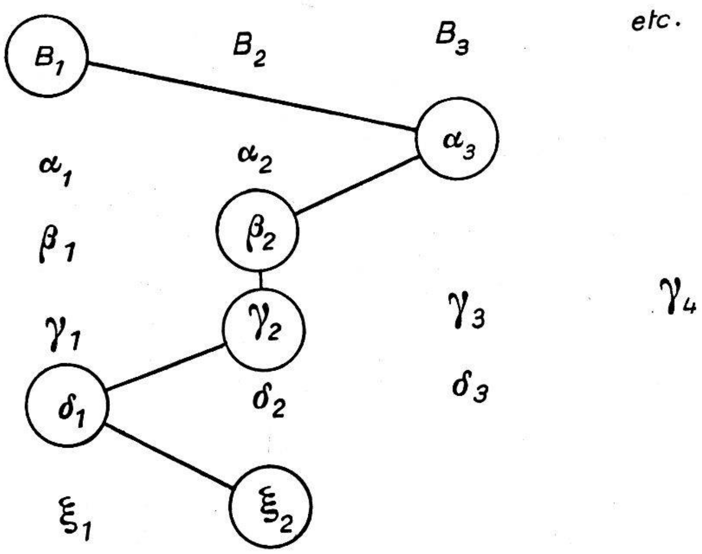
$$\eta = F u_0/\Delta\Phi_p \tag{2}$$

(F = thrust) is made up (in a rather complicated way) of the thermal efficiency and the propulsive efficiency and is not expressible as a simple product of the two. Things become still more complicated when the action of the positive jet also directly influences the free flight characteristics; that is, the drag and lift of the vehicle, as was first demonstrated by the Germans in wind tunnel experiments on models of the V-2 missile.

In addition to external and internal thrust augmentation we must admit the case of no thrust augmentation as a separate one, since often it cannot be obtained as some limit of the other two more general cases.

ξ) Finally, it is of importance whether the engine operates intermittently or continuously.

If we restrict ourselves to propulsive power plants which make use of free air we may represent the possible number of these power plants through the matrix scheme:



Where B_1 stands for engines which react the oxygen of the air, B_2 the engines which react the nitrogen, B_3, etc., the ozone, water of the air, etc. The other matrices α, β, etc., contain the number of elements previously described in sections α, β, etc. If we circle one element of each matrix and connect them we obtain one possible aero-engine. The engine actually picked out of the diagram is an aeroturbojet of continuous operation fed by a solid propellant which is not spontaneously ignitable in air. All combinations of this sort make for possible theoretical engines provided that no internal contradiction is encountered and no fundamental law of physics stands in the way of its realization.

The total number of possibilities in our scheme is

$$n \times 3 \times 2 \times 4 \times 3 \times 2 = 144\, n \tag{4}$$

if n is the number of elements in the matrix B. If we restrict ourselves to propellants or fuels which react with oxygen and nitrogen we should have $n = 2$; that is, 288 basic aero-engines. If we admit combination engines making use of more than one element in each matrix the possible number of aero-engines becomes very great. If we further drop the restriction to chemical propellants and admit all of the cases a), b), c), etc. of section 2) the number of aero-engines and means of aerial propulsion becomes perfectly immense and a special treatise will be necessary to classify them, even schematically. There is thus open to us a very extended field for further investigation.

Among the aero-engines already built (underlined) or awaiting future construction are the

aeroduct (ramjet)
staggered aeroduct
valveless aeroresonator } intermittent aeroducts
valveless aeropulse
aeroresonator or pulsejet (with valves)
aeropulse
aeroturbojet
aeropistonjet
aeroturbopropeller
aeropistonpropeller

and many others. We propose to add the prefix "aero" to all of these engines because in the field of underwater propulsion there are *hydro-engines* potentially possible which, with certain characteristic differences, are the analogues of the aero-engines listed. There are thus hydroducts, hydroresonators, hydropulses, hydroturbojets, and so on.

4. *Determination of ideal performance.*

In the search for the most rational solution of a given propulsion problem, it is absolutely necessary to establish the general scheme or *morphological manifold* of all propulsive power plants, following the approaches which we have described in the preceding. If the number of possible solutions were small, one might explore them experimentally one by one. However, in view of the immense number of potential solutions it is essential to develop powerful means of judging the performance of all engines a priori since no industrial group or state is rich enough in financial, material, and human resources to experiment with all of them. What one must seek therefore are general formulae which will allow us to estimate the performance of whole classes of devices. As an example of such a formula we shall here discuss a

Universal thrust formula

and apply it to the calculation of the performance of a few aero-engines. Such a formula may be derived from first principles of mechanics and thermodynamics if we neglect some of the refinements in the nature of the fluid flow through our engines. We assume as a first approximation that the velocity profiles in all ducts may be considered constant over their cross sections. This assumption makes possible the solution of all propulsion problems in terms of so-called first principles such as the laws of conservation of energy, momentum, moment of momentum and so on which contain only first derivatives rather than second derivatives. These latter of course appear in the differential equations which must be integrated if one wishes to derive exact solutions for the thrusts generated by various propulsive power plants. Such integrations are usually mathematically very difficult.

Proceeding in successive approximations we limit ourselves here to the application of first principles.

If we denote with M_a the mass of air which per second flows through our power plant and with M_f the corresponding mass of the propellant or fuel, then

$$M = M_a + M_f \tag{5}$$

represents the total mass which per second flows through the engine. We define the ratio

$$\beta = M_f/M \tag{6}$$

as the fuel parameter.

We assume further that per unit mass of the propellant at rest the energy $\Delta\varepsilon$ is actually available for transformation into useful mechanical energy. Since the surrounding atmosphere may be assumed to possess constant pressure p_0 and T_0 during the reaction the theoretically available energy would be the difference $\Delta\Phi_p$ in free enthalpies of the reagents and the reaction products and the thermodynamic efficiency of our reaction is given by equation (1). However, the propellant is at rest with respect to the vehicle moving with the velocity u_0. The total available energy per unit mass therefore is

$$\Delta\,\varepsilon_t = \Delta\,\varepsilon + u_0^2/2 \tag{7}$$

From the conservation of energy it follows that

$$\Delta E_t = M_f\,\Delta\,\varepsilon_t = F\,u_0 + M\,\Delta\,u^2/2 \tag{8}$$

where

$$\Delta\,u = u_{\text{exit}} - u_0 \tag{9}$$

and u_{exit} is the average velocity of the positive jet of material which is being expelled. From these relations we obtain the universal thrust formula

$$F = M_a \Delta u + M_f u_{\text{exit}} = M u_0 \left[\beta - 1 \pm \sqrt{1 - \beta + 2\beta \Delta\varepsilon/u_0^2}\right] \tag{10}$$

The effective exhaust velocity u^* and the specific impulse I_{sp} as commonly defined*)

$$u_{\text{EFF}} = u^* = F/M_f = \frac{u_0}{\beta}\left[\beta - 1 \pm \sqrt{1 - \beta + 2\beta \Delta\varepsilon/u_0^2}\right] \tag{11}$$

$$I_{sp} = u^*/g \tag{12}$$

where g is the acceleration of gravity. The positive and negative signs of the square root apply respectively to the jet being ejected backward and forward. We shall retain only the positive sign. That is, we are here interested only in jets which are directed exactly opposite to the direction of motion.

There are two cases for which we must rewrite the universal thrust formula.

The *first case* concerns modes of adding energy ΔE_t per second to the working fluid without at the same time adding mass, that is, $M_f = 0$, or nearly so. If we wish, we might of course retain the equations (7) through (11) if we ascribed to ΔE_t the relativistic mass

$$M_f = \Delta E_t/c^2 \tag{13}$$

which makes

$$\Delta\varepsilon_t = c^2 \tag{14}$$

where c is the velocity of light. The effective velocity and the specific impulse in this case assume extremely large and inconvenient values. We therefore prefer to dispense with the use of I_{sp} and to express the thrust F simply in terms of the power added, and we obtain, with $\beta = 0$

$$F = M_a u_0 \left[-1 + \sqrt{1 + 2\Delta E_t/M_a u_0^2}\right] \tag{15}$$

The *second case* refers to *external thrust augmentation* such as it is present in the case of a reciprocating engine with propeller at-

*) It is unfortunate that the definition (12) for specific impulse has been so widely adopted that it will be difficult to change. Actually g should not enter any of the formulae of phenomena of propulsion as those described here, which have no connection whatever with the gravitational field of the earth. The true specific impulse would have been $u^* = F/M_f$. This is the so-called effective exhaust velocity which the mass M_f would have to possess if the thrust F were to be achieved by exhausting the propellant alone rather than, as is actually done, the mass M.

tached. In this case we conveniently calculate the thrust in two steps, namely,

$$F = F_1 + F_2 \tag{16}$$

where F_1 originates in the exhaust of the engine and is given by (10). F_2 refers to the propeller with ΔE_{tot} equal to the shaft horsepower in the ideal case of no losses. The formula (15) is then applicable for the partial thrust F_2.

For purposes of illustration, we now apply the universal thrust formula to four specific aero-engines for which we choose the aeroduct (ramjet), the aeroturbojet, the aeroresonator or aeropulse, and the combustion engine-propeller combination. These illustrations should demonstrate the power and elegance of our general method.

As regards the consideration of losses and inefficiencies in propulsive power plants we call attention to the following important facts. There are two different types of losses, namely,

α) Losses which are unalterably related to the intrinsic characteristics of the various power plants. These losses are of two kinds. In the first place no aerial engine can make full use of the chemical energy available unless infinite ratios T_c/T_0 or p_c/p_0 are achieved, were T_c, p_c are the explosion temperature and pressure while T_0, p_0 are the temperature and pressure of the surrounding air. Every engine therefore operates with a characteristic thermodynamic efficiency, which in general depends on forward speed, as we shall demonstrate in the special examples. In the second place, even if all of the available chemical energy were transformed into mechanical energy only one part of this is useful propulsive energy since some kinetic energy is of necessity lost to the surrounding air unless the ratio of air to fuel used in the engine is infinite. Every engine therefore possesses a characteristic propulsive efficiency

$$\eta_p = F u_0/\Delta E_t \tag{17}$$

which also depends on forward velocity.

β) There are losses which are of a more parasite nature. For instance, the combustion may proceed too slowly and may therefore not go to completion. Or there are losses because of internal friction, shock waves and heat transfer to the surrounding medium.

We shall disregard losses of the type β for the present in order to derive the performance of ideally operating propulsive power plants. We thus set all efficiencies equal to unity except the unalterably fixed thermodynamic and propulsive efficiency.

A. The Aeroduct.

The aeroduct (ramjet) is perhaps the simplest of all aero-engines. The air passes through a simple duct as shown in Fig. 1.

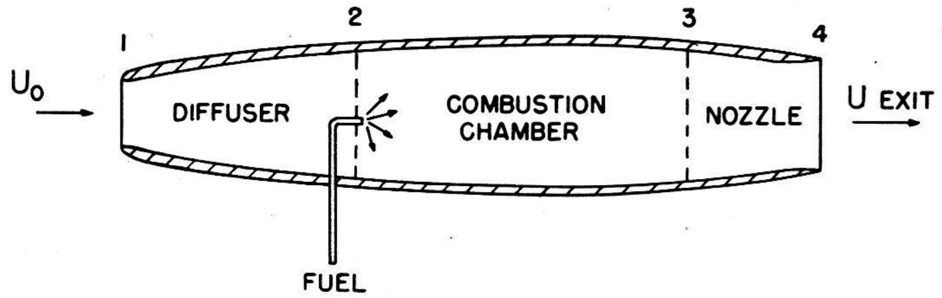


Fig. 1.
Aeroduct.

The propellant is added behind section 2 and the reaction takes place in the stagnation section 2,3. The air is precompressed by ram in the diffusor (1,2). The reaction products expand in the nozzle (3,4) so that the thermodynamic cycle is given by Figure 2.

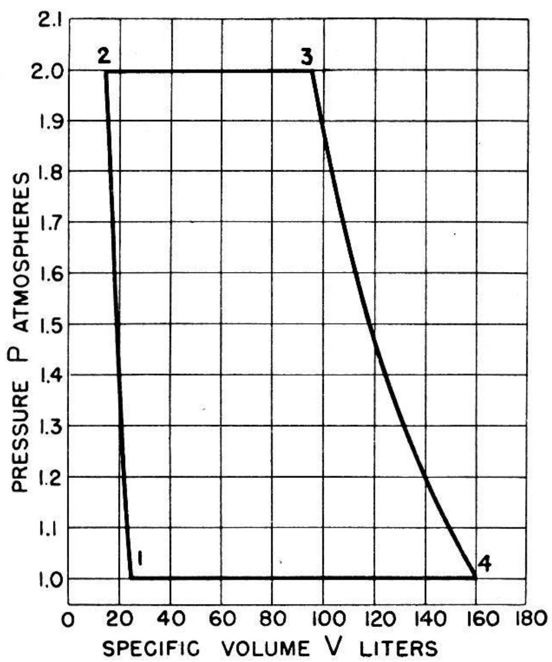


Fig. 2.
Thermodynamic Cycle of the Aeroduct.
Pressure vs. Volume for one Mole Air Fuel: Gasoline-air at Stoichiometric Ratio.

Adding the heat of combustion Δh at the constant pressure $p = p_2 = p_3$ raises the temperature of the air from T_2 to T_3 so that

$$T_3 - T_2 = \Delta h / c_p \tag{18}$$

where c_p is the specific heat of the air, or rather, of the air plus the combustion products, at constant pressure. The available useful mechanical energy $\Delta\varepsilon$ is equal to the area enclosed in the cycle (1, 2, 3, 4). It is

$$\Delta\varepsilon = \int_4^3 v\,dp - \int_1^2 v\,dp \tag{19}$$

Assuming isentropic compression (1, 2) and expansion (3, 4) we obtain

$$\Delta\varepsilon = c_p[(T_3 - T_4) - (T_2 - T_1)] \tag{20}$$

Since

$$p_1/p_2 = p_4/p_3 \text{ and } T_1/T_2 = T_4/T_3 \tag{21}$$

we obtain for the thermodynamic efficiency of the cycle

$$\eta_{th} = 1 - T_1/T_2$$

or

$$\eta_{th} = 1 - (p_1/p_2)^{\frac{\gamma-1}{\gamma}} \tag{22}$$

where $\gamma = c_p/c_v$ is the ratio of the specific heats. γ is assumed to be constant in the first approximation and for simplicity. Admitting no diffusor losses we have for the *maximum* stagnation or ram pressure

$$p_2 = p_1\left[1 + \varrho_1\frac{u_1^2}{2}\frac{\gamma-1}{\gamma}\Big/p_1\right]^{\frac{\gamma}{\gamma-1}} \tag{23}$$

from BERNOULLI's equation, where $\varrho_1 \cong \varrho_0$ stands for the mass density of the air at the entrance. If we consider a straight combustion, with heat Δh available per unit mass, we obtain

$$\Delta\varepsilon = \eta_{th}\Delta h \tag{24}$$

With p_2/p_1 given by (23) and ignoring any difference between u_1 and u_0, it is

$$u^* = gI_{sp} = \frac{u_0}{\beta}\left\{\beta - 1 + \sqrt{1 - \beta + 2\beta\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]\frac{\Delta h}{u_0^2}}\right\} \tag{25}$$

or

$$u^* = \frac{u_0}{\beta}\left\{\beta - 1 + \sqrt{1 - \beta + 2\beta\left[1 - \left(1 + \frac{\varrho_0 u_0{}^2}{2}\frac{\gamma-1}{\gamma}\Big/p_0\right)^{-1}\right]\frac{\Delta h}{u_0^2}}\right\} \tag{26}$$

For velocities small compared with the velocity of sound a, that is

$$u_0 \ll a_0 = (\gamma p_0/\varrho_0)^{\frac{1}{2}} \tag{27}$$

we obtain

$$u^* \cong \frac{u_0}{\beta}\left\{\beta - 1 + \sqrt{1 - \beta + \beta\frac{\varrho_0}{p_0}\frac{\gamma-1}{\gamma}\Delta h}\right\} \tag{28}$$

In practice we always have $\beta \ll 1$ so that (28) reduces approximately to

$$u^* \cong \frac{u_0}{2}\left\{1+\frac{\varrho_0}{p_0}\frac{\gamma-1}{\gamma}\,\Delta h\right\} = \frac{u_0}{2}\left\{1+(\gamma-1)\,\Delta h/a_0^2\right\} \qquad (29)$$

On the other hand for $u_0 \gg a_0$, and also $u_0^2 \gg \Delta h$ we obtain

$$u^* \cong \frac{u_0}{\beta}\left\{\beta - 1 + \sqrt{1-\beta}\right\} \qquad (30)$$

and for $\beta \ll 1$, that is, large excess of air

$$u^* = g\, I_{sp} \cong u_0/2 \qquad (31)$$

All of these values are of course ideal limits, since for supersonic flow considerable shock losses prevent the ideal conversion of speed into pressure and vice versa. These losses become especially great in most of the actual aeroducts, since it is very difficult to adjust the cross sections of the diffusor and of the nozzle so as to give maximum efficiencies at all forward speeds u_0.

The ideal performance for a number of values of the fuel parameter β is plotted in the figures 3, 4 and 5, taking as the fuel octane with $\Delta h = 11.2$ Kcal/gram.

The minima in the curves for I_{sp} and u^* and the subsequent rises towards infinity are of course due to the fact that the kinetic energy $u^2{}_0/_2$ invested in the fuel, because of acceleration of the vehicle, begins to outweigh the heat of combustion Δh and thus becomes the determining factor.

Elaborating on our fundamental analysis there is no difficulty in extending our formalism to include the influence of losses through the introduction of diffusor efficiencies, nozzle coefficients and so on. In general the losses grow with increasing forward speed u_0.

Starting from the simple aeroduct a number of more sophisticated propulsive power plants may be visualized, such as the *staggered aeroduct* and the *intermittently operating aeroduct*. The latter may be either a *valveless aeropulse* or a *valveless aeroresonator*.

The staggered aeroduct attempts to achieve an ultimate stagnation pressure superior to that of formula (23) by burning only part of the fuel at the first stagnation point (2) and by reconverting the velocity achieved at (3) into higher pressure at which the fuel can be burned with an increase in thermodynamic efficiency.

The intermittently operating valveless duct engines involve at first sight some losses which do not occur in the continually operating ducts. These losses have their origin in the fact that on pulsating cycles there are periods at which the pressure expansion ratio for the hot reaction products is small and the efficiency of transformation of enthalpy into kinetic energy of the jet is low. However, this low efficiency would appear to be compensated and even a gain may be expected because of the following advantages. With proper design of the duct instantaneous explosion pressures can effectively be used which are much greater than the stagnation pressure p_2. This in turn results in over-expansion and the thermodynamic cycle operates more efficiently between the higher peak pressure and the lower value of the minimum pressure achieved. This under-

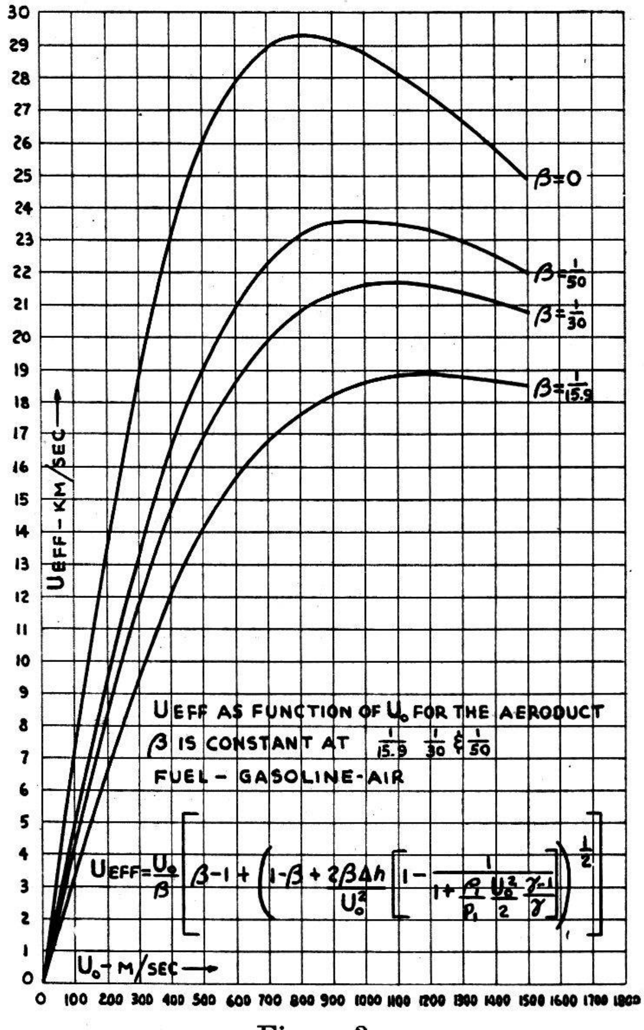


Figure 3.

pressure also makes possible the scavenging of the duct at zero forward speed making for a finite thrust at zero speed with the additional possibility of lateral air intake.

The two main defects of the operation of the aeroduct are its dependence on forward speed for precompression of the working fluid and in the use of constant pressure expansion along which the thermal energy from the combustion is fed in. This *constant pressure expansion is not a thermodynamically reversible process* and

*

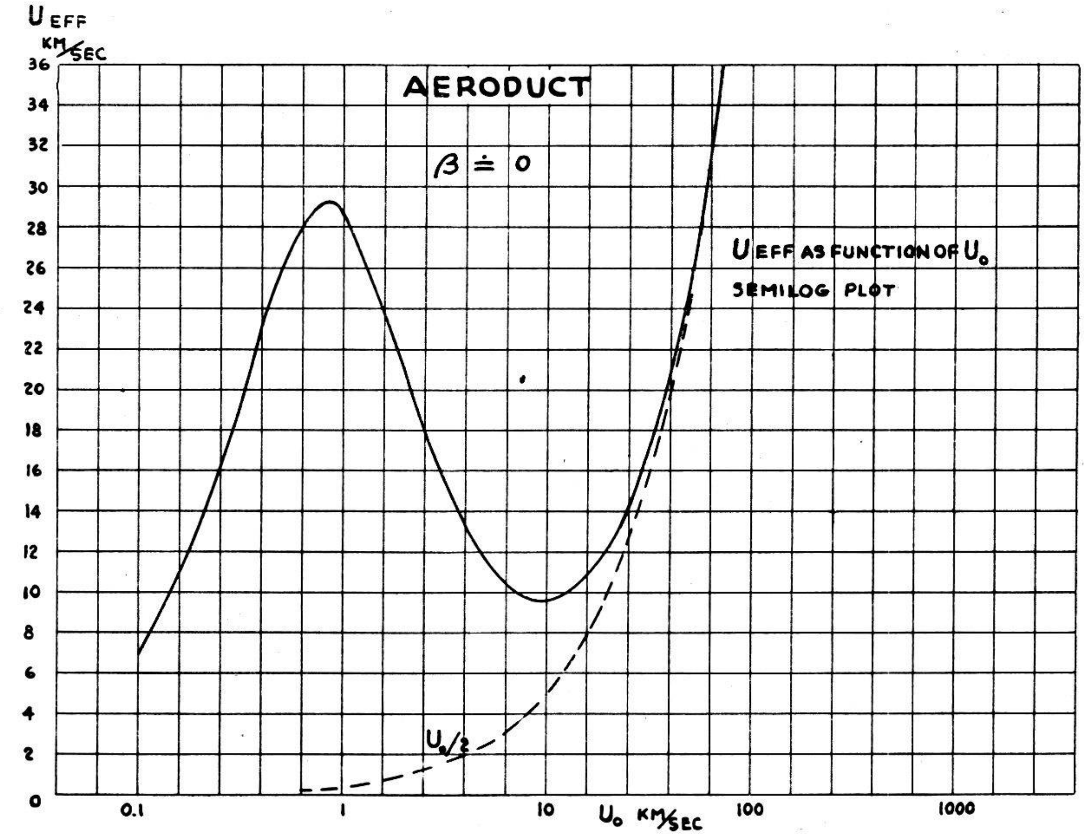


Figure 4.

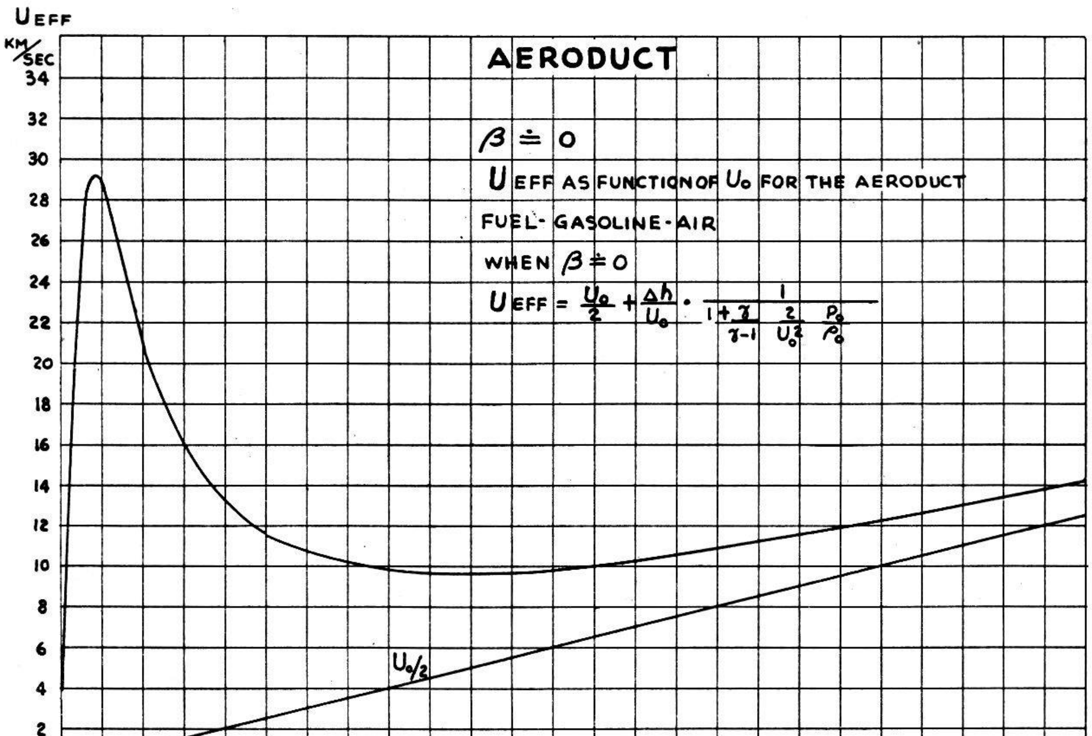


Figure 5.

it is therefore of lower efficiency than isothermal or adiabatic volume changes.

As already mentioned, there are engines possible which do away with the defects of the aeroduct in a yet more successful manner. We have already referred to the staggered aeroduct and to the intermittently operating valveless aeroresonator and the aeropulse. In the following two sections the universal thrust formula is applied to the aeroturbojet and to the aeropulse. These are two of the engines which partly overcome the shortcomings of the aeroduct.

B. The Aeroturbojet.

In the aeroturbojet an axial or centrifugal compressor is built into the entrance diffusor of the aeroduct, which serves to step up the stagnation pressure from p_2 to p_3. This improves the thermodynamic efficiency and it also provides for thrust at zero forward speed of the engine. The compressor is driven by a turbine which derives its power from the gaseous reaction products expanding

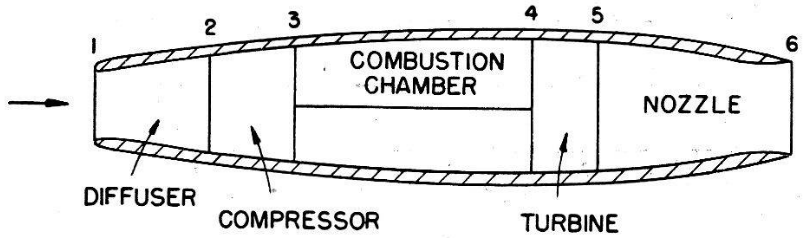


Figure 6.

The Aeroturbojet.

from p_4 to p_5. The remaining expansion from p_5 to p_6 generates the thrust.

In the ideal case of 100% efficiency in both the turbine and the compressor we should have

$$\Delta\varepsilon_{32} = \Delta\varepsilon_{45} = \int_2^3 v\,dp = \int_5^4 v\,dp = c_p(T_3 - T_2) \tag{32}$$

which is the isentropic work of compression done on the air by the compressor and delivered to the latter by the turbine. Here c_p is the average specific heat of the air or of the air plus the reaction products at constant pressure and $T_3 - T_2$ is the difference in absolute temperatures between the points 2 and 3. The work H is the difference in enthalpy between points 2 and 3, rather than the difference in internal energy. This is due to the fact that the air is not being compressed at rest but is moving. The work done on the flowing air is therefore

$$H = -p_2 v_2 + p_3 v_3 - \int_2^3 p\,dv = \int_2^3 v\,dp \tag{33}$$

The ideal energy balance therefore presents itself as follows.

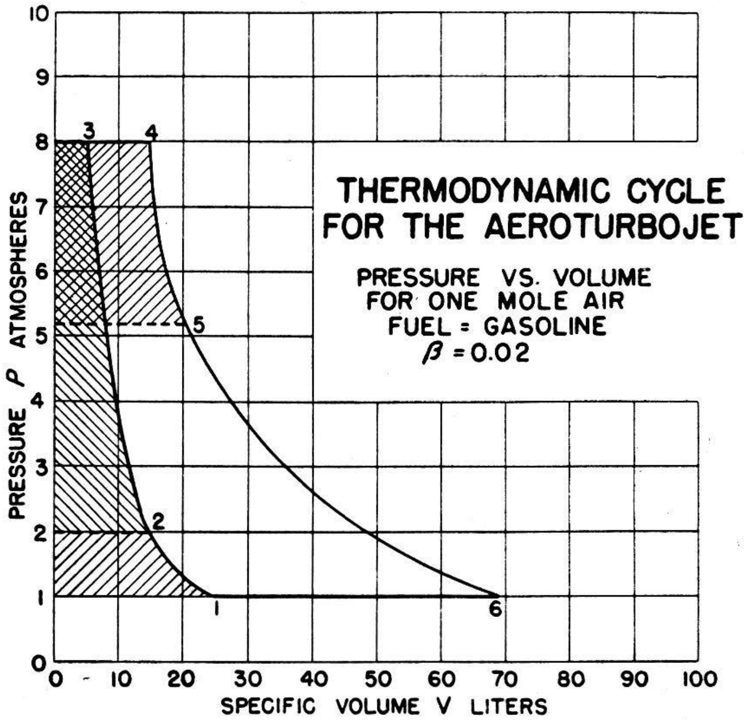


Figure 7.
Thermodynamic Cycle for the Aeroturbojet.

The total mechanical energy ideally available from the isentropic expansion of the combustion products is

$$\Delta\varepsilon_{46} = c_p (T_4 - T_6) \tag{34}$$

From this we must subtract both $\Delta\varepsilon_{45} = \Delta\varepsilon_{32}$ which is fed into the turbine and

$$\Delta\varepsilon_{21} = \int_1^2 v\,dp = c_p (T_2 - T_1) \tag{35}$$

which the duct entrance diffusor transmits to the entering air in compressing it isentropically from 1 to 2. The net remaining available mechanical energy therefore is

$$\Delta\varepsilon = c_p [(T_4 - T_6) - (T_3 - T_1)] \tag{36}$$

which is equal to the area enclosed between the heavy lines connecting points 1, 3, 4 and 6 in Figure 7. As proved in the section on the aeroduct, this area is also equal to

$$\Delta \varepsilon = \left[1 - (p_1/p_3)^{\frac{\gamma-1}{\gamma}}\right] \Delta h \tag{37}$$

where Δh is the heat of combustion per unit mass of the fuel used. Formula (37) is thus identical in form to the formula (24) which expresses the energy available in the aeroduct. The difference between the ideal aeroduct and the ideal aeroturbojet becomes apparent only if one rewrites the ratio p_1/p_3 for the turbojet. It is

$$\frac{p_1}{p_3} = \frac{p_1}{p_2}\,\frac{p_2}{p_3} = \frac{1}{K}\,\frac{p_1}{p_2} \tag{38}$$

where, by arbitrary choice, the compression ratio achieved by the compressor

$$K = \frac{p_3}{p_2} = \text{constant} \tag{39}$$

The maximum stagnation pressure p_2 again is given by

$$p_2 = p_1 \left[1 + \varrho_0 \frac{u_0^2}{2} \frac{\gamma-1}{\gamma} \Big/ p_0\right]^{\frac{\gamma}{\gamma-1}} \tag{40}$$

For the aeroduct it is

$$K = 1 \tag{41}$$

while for the aeroturbojet

$$K \neq 1 \tag{42}$$

The ideal effective exhaust velocity for the latter is thus

$$u^* = g\,I_{sp} = \frac{u_0}{\beta}\left\{\beta - 1 + \sqrt{1 - \beta + 2\,\beta \frac{\Delta h}{u_0^2}\left[1 - \left(\frac{p_1}{p_3}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right\} \tag{43}$$

For zero forward velocity we have

$$u^* = \sqrt{\frac{2\,\Delta h}{\beta}\left[1 - K^{\frac{1-\gamma}{\gamma}}\right]} \tag{44}$$

The aeroturbojet therefore produces finite thrust at zero forward speed, in contradistinction to the aeroduct which with $K = 1$ and $u_1 = 0$, has $u^* = g \times I_{sp} = 0$ according to formula (44).

In Figures 8 and 9, the performance of the ideal aeroturbojet is indicated for several values of the fuel parameter β. Actual turbojets use fuel parameters which lie in the range $^1/_{50} > \beta > {^1/_{100}}$. In diagrams 8 and 9 the value $K = p_3/p_2 = 4$ is chosen for the compression ratio.

In contradistinction to the aeroduct, the effective exhaust velocity and the specific impulse at zero forward speed $u_0 = 0$ are

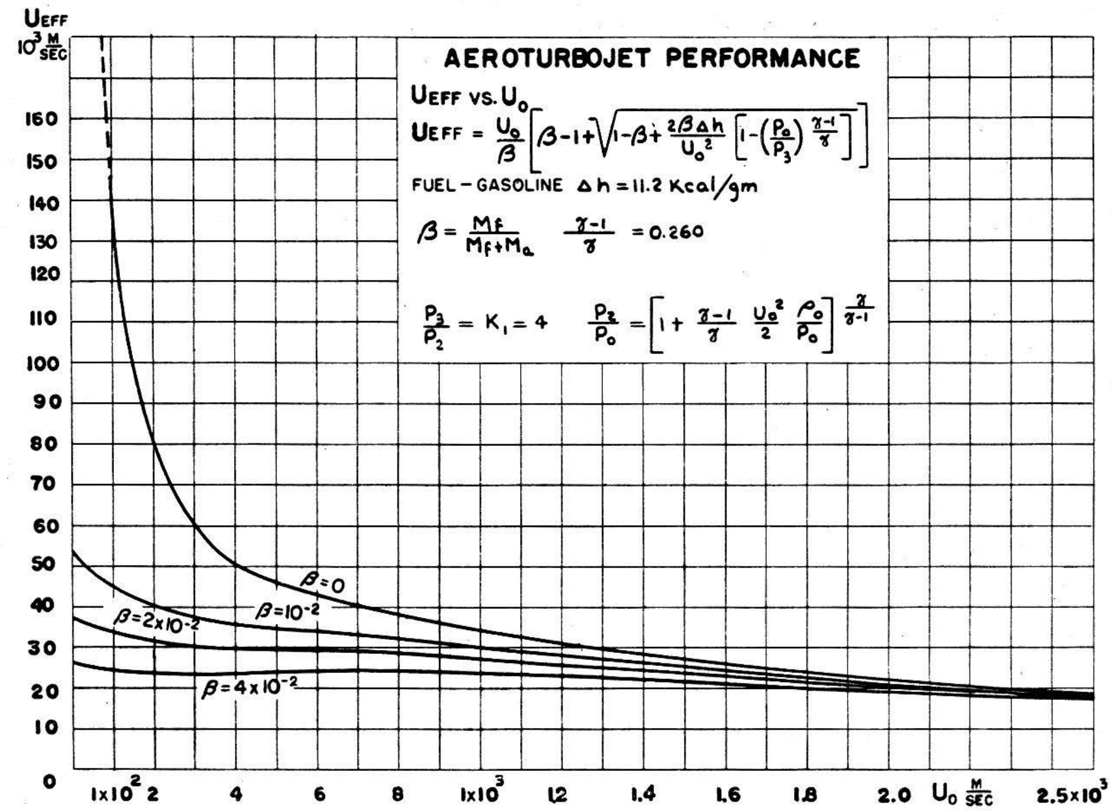


Figure 8.

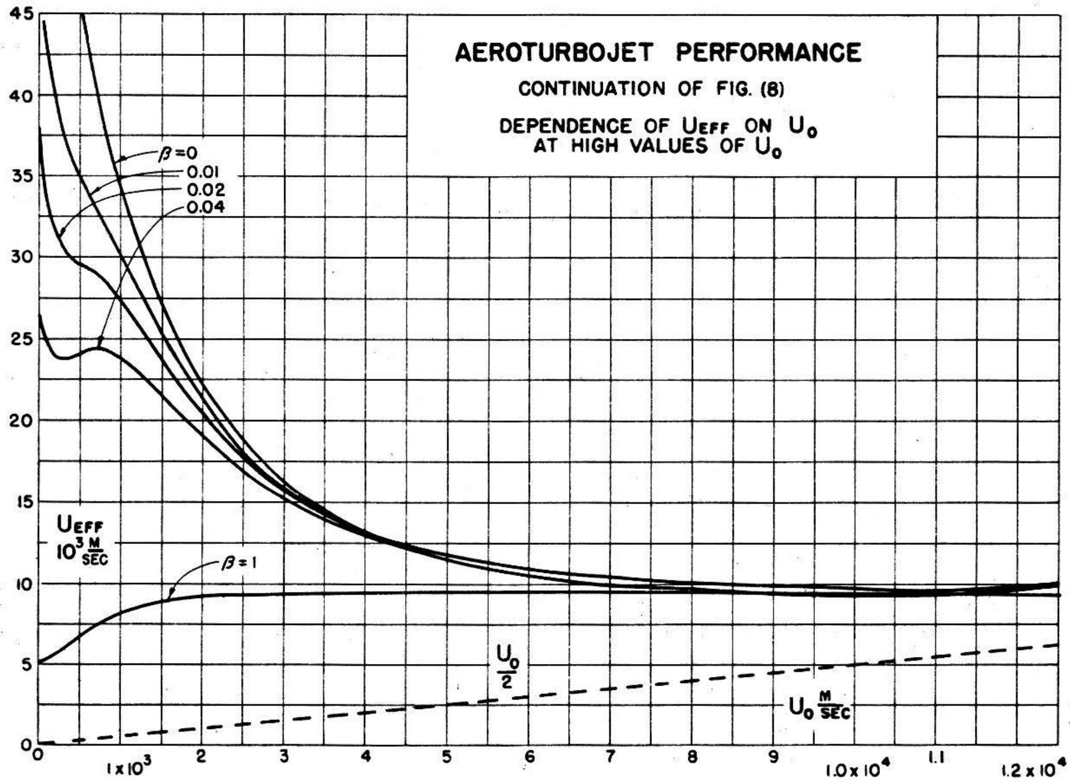


Figure 9.

different from zero and are the larger the greater K and the smaller the fuel parameter β.

An interesting feature of the curves of the effective exhaust velocity u^* versus u_0 in the ideal aeroturbojet is the appearance of a minimum and a maximum at higher values of β and at low speeds u_0. These minima and maxima have their origin in the deficiency of the aeroduct of having zero thrust at zero forward speed. This deficiency is in part carried over to the aeroturbojet and becomes very apparent if we approach the value $\beta = 1$. This of course cannot be done with octane as a fuel, since there would not be enough air to burn it completely. The curve $\beta = 1$ is therefore drawn for the imagined operation of the aeroturbojet with very little air and a major portion of a hypothetical self-contained propellant (fuel plus oxidizer) which is assumed to produce per gram the same heat of reaction as one gram of octane burned in air. This limiting case of the aeroturbojet is different from an ordinary rocket motor only insofar as the chamber pressure is not constant, but the small air intake through its ram determines the pressure p_2. The result is the curve $\beta = 1$. u^* is different from zero at $u_0 = 0$ but increases with increasing u_0.

Assuming no losses in the compressor and in the turbine, the aeroturbojet would be a propulsive power plant of absolutely ideal performance, especially for $\beta = 0$. By literally pulling itself up on its own bootstraps; that is, by feeding energy from the expansion process into the turbine and thence into the compressor for purposes of precompression of the air, the relative loss inherent in the thermodynamically irreversible process of constant pressure expansion is minimized. In practice, however, the value of the precompression K can not bedriven too high. As we step up the ratio K we run into shock losses in the compressor which also needs more stages and becomes cumbersome. At the same time the temperature T_3 is increased. This means increased heat losses. Also T_3 and T_4 become so high that no materials for the turbine blades are available which will withstand these temperatures. All of these factors prevent us from making full use of the excellent intrinsic characteristics of the ideal aeroturbojet. It is therefore obvious that the practical shortcomings of the aeroturbojet must be minimized through the use of better materials. Or better yet, the turbine and the compressor should be eliminated and the precompression should be achieved without the use of auxiliary machinery. This in principle can be done by cascading the pressures in the gaseous combustion process, a possibility which may be realized in the aeropulse.

C. The Aeropulse.

Propulsive power plants of the type of the aeropulse were proposed on various occasions during the past decades. The first successful realization however was found in the *V*–1 buzz bomb engine originally developed by Paul Schmidt in Munich, Germany. The buzz bomb engine, strictly speaking, is an aeroresonator for which the ambiguous and historically incorrect designation "pulsejet" is now in wide use. In the aeroresonator, whose mode of operation has been described in many places, the frequency of cycling adjusts itself automatically after the fashion of resonance in an organ pipe. Conditions are complicated only because of the presence of combustion processes and the non-linearity of the resulting operations. While the *aeroresonator* derives its thrust from a *natural oscillation* of a valved aeroduct, the operation of the *aeropulse* is based on a *forced oscillation* which is achieved either through timed air valving,

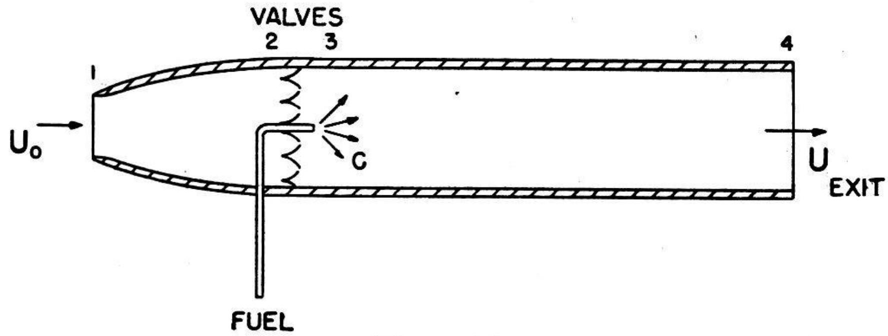


Figure 10.
The Aeropulse.

timed fuel injection and ignition, or a combination of these. The aeropulse was first proposed by the author in 1943 as a result of his morphological investigations on jet engines. The engine has not yet been operated successfully on this principle. The aeropulse operates as shown in Figure 10.

Air enters a diffusor and is rammed into the valve bank section. The valves may be of the flapper reed type. Fuel is injected before or behind the valve bank at given time intervals and is ignited artificially by a spark. The fuel may also be self-igniting because of of the high temperature spots from the previous cycle or the fuel may be self-igniting even with cold air, such as, for instance, aluminium trimethyl or aluminum borohydride. Because of the instantaneous combustion *C* which takes place at 3 the pressure rises at constant specific volume of the gas from p_2 to p_3. The reed valves close under the action of this pressure and the gases are expanded isentropically and exhausted to the rear, thus producing the thrust. The thermodynamic working cycle is represented in Figure 11.

In reality, because of the inertia of the exhausted gases there will be overexpansion, resulting in a minimum pressure $p_4' < p_0$. This underpressure p_4' makes possible the fast scavenging of the duct even at zero forward speed u_0 when no ram is present. The combination of ram and underpressure opens the reed valves and allows fresh air to enter from the diffusor into the duct. With under-

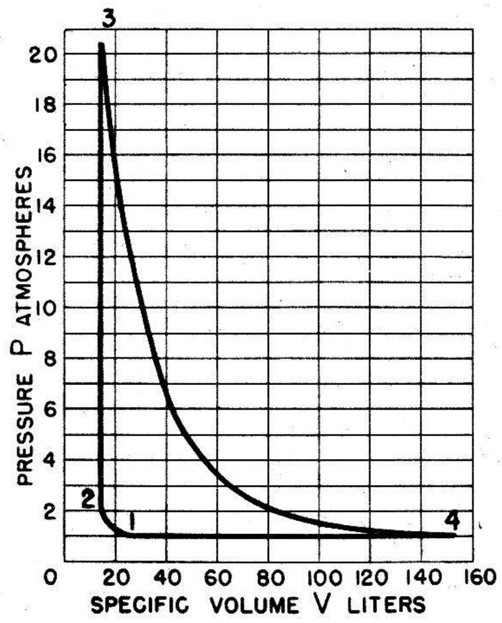


Figure 11.

Thermodynamic Cycle for the Aeropulse.

Pressure vs. Volume for one Mole Air Fuel: Gasoline-Air at Stoichiometric Ratio.

pressure, new air is also sucked into the duct through the rear entrance. For simplicity, we shall assume in our performance calculation that no underpressure is present and we shall put $p_4 = p_0$. An actual engine, at zero forward speed, would of course not operate, were it not for this underpressure or some auxiliary means of recharging the duct with fresh air. The exact thermodynamic diagram of the aeropulse can only be constructed if pressure and temperature or pressure and density are simultaneously recorded as functions of time. With underpressure actually present after the exhaust the performance of the aeropulse is better than without underpressure. The performance derived in the following therefore represents a lower limit for the ideal aeropulse in which all parasite losses such as heat losses and duct friction are neglected.

For the ideal simplified aeropulse, with no underpressure, we have

$$p_2 = p_0 [1 + \varrho_0 u_0^2 (\gamma - 1)/2 \gamma p_0]^{\frac{\gamma}{\gamma-1}} \tag{45}$$

like in (23) for the aeroduct. Any possible difference between u_1 and u_0 or p_1 and p_0, again is neglected. Equation (45) may also be written in this form

$$p_2 = p_0 [1 + u_0^2/2 c_p T_0]^{\frac{\gamma}{\gamma-1}} \tag{46}$$

and

$$T_2 = T_0 + u_0^2 / 2 c_p \tag{47}$$

The thermodynamic efficiency for the ideal aeropulse cycle is

$$\eta_{th} = 1 - \gamma (p_2/p_0)^{\frac{1-\gamma}{\gamma}} \frac{(p_3/p_2)^{1/\gamma} - 1}{(p_3/p_2) - 1} \tag{48}$$

Introducing this expression into the universal thrust formula and neglecting the beneficial effects of the underpressure or suction after the exhaust, we obtain for the effective exhaust velocity of the ideal aeropulse

$$u^* = \frac{u_0}{\beta} [\beta - 1 + \sqrt{1 - \beta + 2 \beta \eta_{th} \Delta h/u_0^2}] \tag{49}$$

where Δh is the heat of combustion available per unit mass of the fuel.

At zero forward velocity it is $p_2/p_0 = 1$ and

$$\eta_{th}(0) = 1 - \gamma [(p_3/p_2)^{1/\gamma} - 1]/(p_3/p_2 - 1) \tag{50}$$

so that $u^* \neq 0$. If we carry out the calculations for octane as the fuel, we obtain

$$\eta_{th}(0) = 0.354$$

The following numerical calculations may serve to illustrate the performance of the aeropulse when operated with octane C_8H_{18}. For stoichiometric combustion we have

$$C_8H_{18} + 12.5\ O_2 = 8\ CO_2 + 9\ H_2O + \text{heat} \tag{51}$$

This reaction therefore requires 400/114.22 parts of O_2 by weight to one part of C_8H_{18}. Since the air contains 23 % of O_2 by weight, the fuel parameter for stoichiometric combustion becomes

$$\beta = 1/16.2 \tag{52}$$

For normal aviation gasoline, however, the stoichiometric fuel parameter is

$$\beta = 1/15.9 \tag{52a}$$

corresponding to an empirical formula C_nH_{2n}. As in the previous examples, we shall make use of (52a) for our actual numerical calculations.

If constant volume combustion takes place, the pressure rises from atmospheric p_0 to the explosion pressure $p_3 = 10.2\ p_0$ or about 10.2 atmospheres, while the temperature jumps from the ambient $T_0 \sim 300^0$ K to about $T_3 \sim 3070^0$ K.

The aeropulse may in principle be run in two different ways. For instance, the fuel, M_f grams per second, may be uniformly dispersed throughout the M_a grams of air which flow through the aeropulse per second. Constant volume combustion then may take place. This will happen for instance if the fuel is spontaneously inflammable with air such as aluminum trimethyl or aluminum borohydride. However, with gasoline a combustion of this type is not possible if there is a large excess of air. Also, the dispersal of the fuel throughout the whole of the available air is not advantageous thermodynamically since the explosion pressure decreases steadily as the excess of air grows and β tends toward zero. As a consequence, the thermal efficiency of the working cycle also decreases.

For high efficiency, one must therefore endeavor to burn a stoichiometric fuel-air mixture and to transfer immediately after explosion a proportionate part of the energy stored in this mixture to the remainder of the air contained in the aeropulse. This second mode of operation of the aeropulse leads to various processes of internal thrust augmentation, the theory of which is as yet little known. The performance values listed in the following have been derived on the assumption of ideal constant volume combustion $(p_2 \rightarrow p_3)$ with subsequent transmission of the liberated energy to the whole gas contained in the duct of the aeropulse motor.

A set of characteristic performance values of the aeropulse is given in Figures 12, 13, 14 and 15.

In all calculations it was assumed that the initial temperature is $T_0 = T_1 = 300^0$ K. The average specific heat at constant volume between the stagnation temperature T_2 and the explosion temperature T_3 was taken as $c_v = 0.2590$ cal/gram $\times$ degrees Centigrade. The partial dissociation of the gaseous products of the combustion was not taken into account.

The following comments must be made. The aeropulse is the only engine among the four discussed in this paper whose performance characteristics cannot be entirely deduced on the basis of first principles. Indeed, strict integration of the differential equations of the aeropulse will be necessary to arrive at a correct description

Aeropulse.

The Ratio of Explosion Pressure P_3 to Atmospheric Pressure P_0 versus forward Velocity.

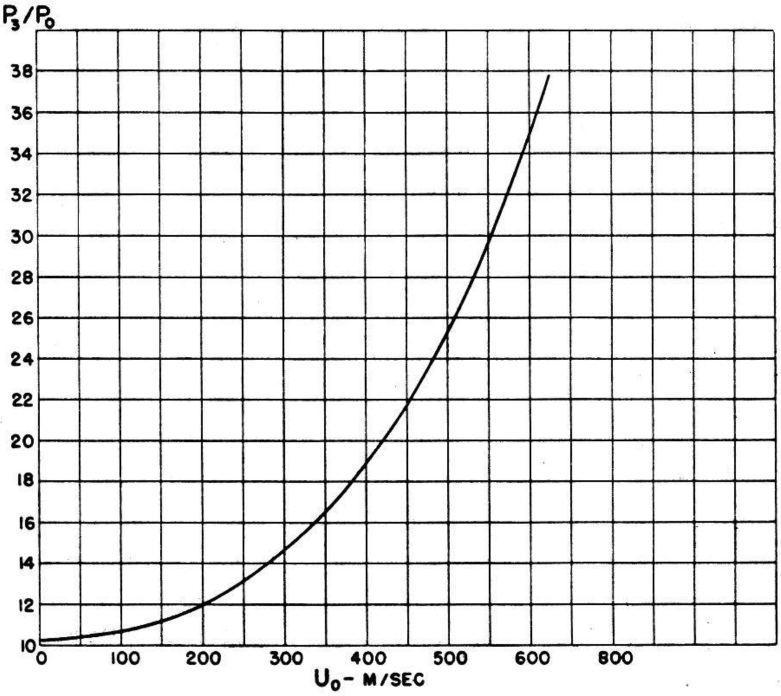


Figure 12.

Aeropulse.

Explosion Temperature as a Function of Forward Speed.

Fuel-Gasoline Fuel Parameter $\beta = 1/15.9$

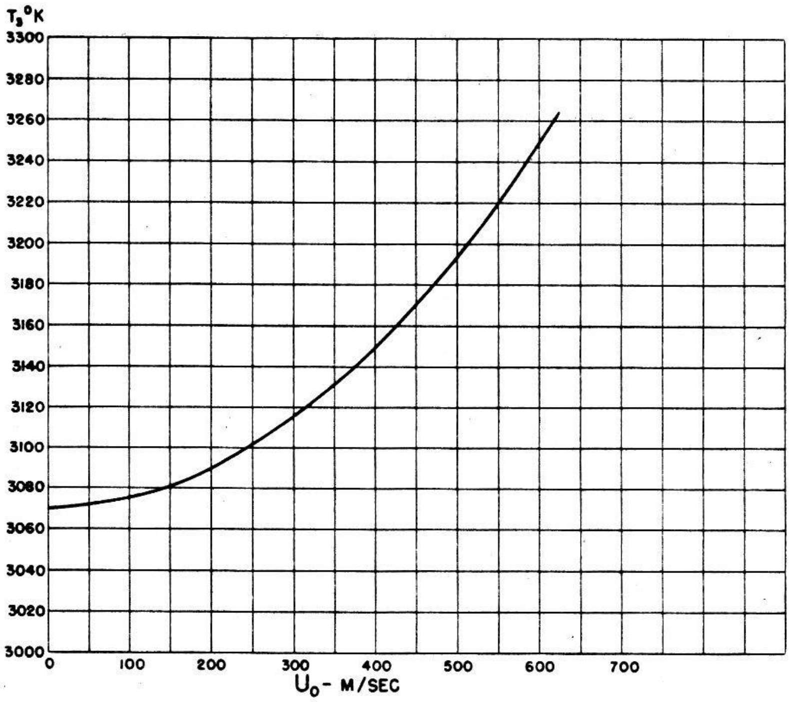


Figure 13.

of the thermodynamic cycle of this engine. Especially the over-expansion and the resulting underpressure can only be deduced by going back to the differential equations which describe the combustion and the resulting aerodynamic processes. The performance calculation given in the preceding therefore is a very crude appro-

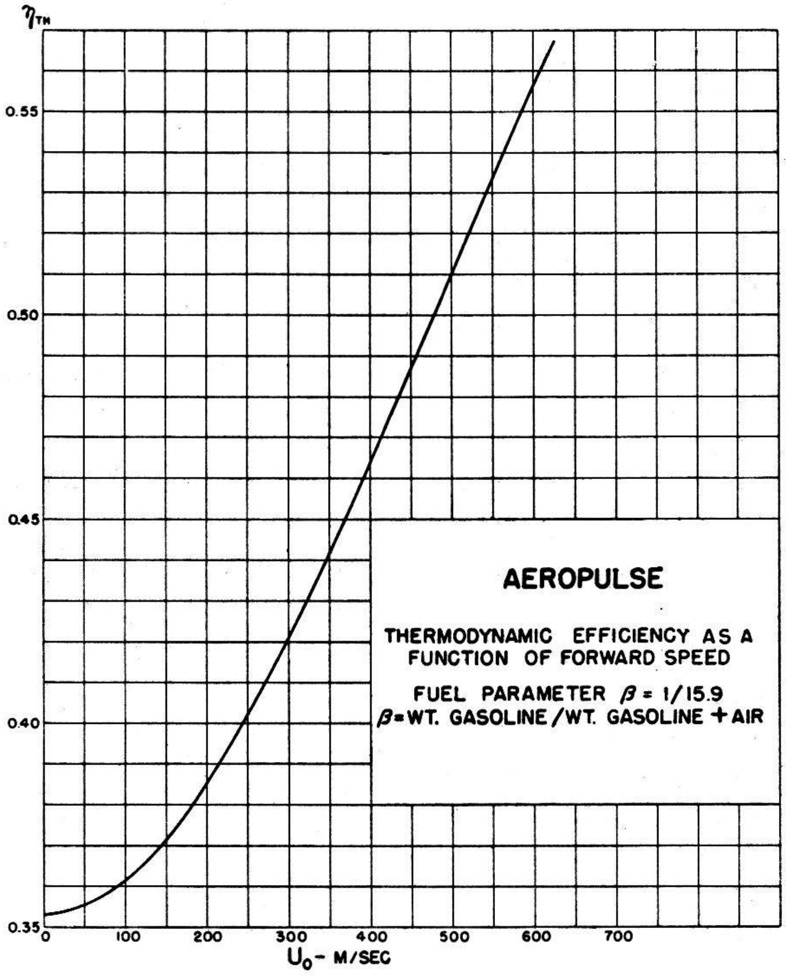


Figure 14.

Thermodynamic efficiency.

ximation which essentially disregards the complications originating in the intermittent operation.

A distinguishing feature of the aeropulse is the fact that the thermodynamic efficiency, in contradistinction to the other engines discussed, depends on the value of the heat of combustion of the fuel used. This is due to the fact that the explosion pressure and therefore η_{th} increase as the heat of combustion per unit mass of the fuel increases.

The aeropulse is also that aero-engine which in practice is capable of the greatest improvements. These must be sought for in the direction of adequate fuels reacting very fast with the oxygen of the air. With such fuels and systematic timing of the combustion process involved, very high pressures can be automatically obtained in the aeropulse. The problem therefore is to achieve precompression of the air by using "phased" combustion rather than mechanical means such as the compressor in the aeroturbojet. Once we master

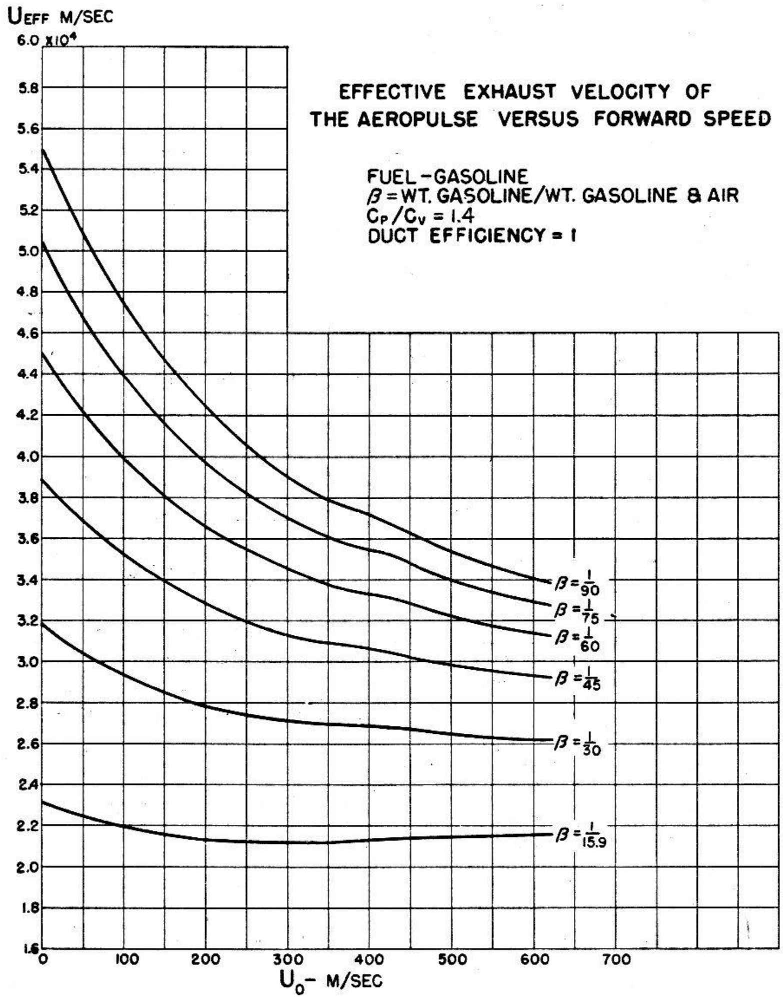


Figure 15.
$u^*(u_0, \beta)$.

the combustion processes, so that we can accurately control them in time and space, the aeropulse bids fair to become one of the most efficient aero-engines. This is particularly true since there seems to be no limitation to the amount of excess air used, a fact which will ultimately enable one to make use of very small fuel parameters β. This in turn means high thrust augmentation and high effective exhaust velocity, as we shall demonstrate in the next section.

A great advantage of the aeropulse lies in its simplicity of construction and in its light weight. The elimination of the strong vibrations characteristic of the present models would not seem to present any insuperable difficulties. Once these problems are solved, the aeropulse should become an efficient propulsive power plant in all ranges of forward speed, a feature which is not easily attained with any of the other aero-engines discussed.

The performance curves in Figures 12, 13, 14 and 15 are self-explanatory. We shall come back later on to an intercomparison of the various aero-engines. Attention is called only to one small detail. It is seen that the small secondary minimum and maximum in the curve u^* versus u_0 for the aeroturbojet operating near stoichiometric operation is much less pronounced in the analogous curve for the aeropulse. This is due to the fact that the aeropulse operates at a higher explosion pressure than the aeroturbojet. The low efficiency of the aeroduct at low forward speeds, which intrinsically also reflects itself in the performance of the aeroturbojet and the aeropulse thus becomes less evident as the operating pressures increase.

D. The Internal Combustion Engine-Propeller-Combination.

The three propulsive power plants which we have just discussed all make use of internal thrust augmentation. The air which is not chemically used in the combustion process nevertheless flows through the combustion chamber of these engines. The engine propeller combination is the typical example of a propulsive power plant with external generation of thrust, F_2. The air which flows through the engine of course also produces some thrust F_1 if the exhausts are properly constructed and directed. Since this effect is only an auxiliary improvement and not typical of the engine-propeller we shall not consider it further and simply write $F_2 = F$.

For simplicity, we assume that the shaft power ΔE_t is constant and is equal to

$$\Delta E_t = \eta_{th} M_f \Delta h \tag{53}$$

The combustion engine used to produce the propeller shaft power may be a reciprocating engine or a gas turbine. Neglecting all losses in the propeller, this power is transformed in its entirety into propulsive power $F u_0$ of the vehicle and kinetic energy $M \Delta u^2/_2$ left in the surrounding air, where M is the mass of air per second flowing through the propeller. The propeller thrust is given by (15).

$$F = \sqrt{2 M \Delta E_t + M^2 u_0^2} - M u_0 \tag{54}$$

In practice, the amount of air flowing through the propeller per second is very large compared with the fuel consumed per second in the engine. It is therefore

$$\beta = M_f/(M + M_f) \cong M_f/M \tag{55}$$

We have assumed $\varDelta E_t$ or M_f independent of forward speed. However, β, in contradistinction to the engines previously discussed, is not constant in function of u_0. In order to be able to calculate the thrust from (54) we must therefore first determine M or $\varDelta u$. We proceed as follows. Equation (54) may be compared with this expression for the thrust

$$F = M \varDelta u \tag{56}$$

where $\varDelta u$ is the total increment of velocity which the propeller imparts to the air rushing through it. The relative velocity of the air to the propeller far in front of it is u_0 and sufficiently far back of it $u_0 + \varDelta u$. At the propeller disc itself the slip stream velocity is

$$u_s = u_0 + \varDelta u/2 \tag{57}$$

as follows from simple theory[4]). The mass of air passing through the propeller therefore is

$$M = \varrho A (u_0 + \varDelta u/2) \tag{58}$$

where A is the area of the propeller disc and ϱ the density of the air. Eliminating M and F from equations (54), (56) and (58) we arrive at the following determining equation for $\varDelta u$

$$\varDelta u \left[u_0 + \frac{\varDelta u}{2}\right]^2 = \varDelta E_t / \varrho A \tag{59}$$

After $\varDelta u$ has been calculated from this relation, M and F follow immediately from (58) and (56).

Some limiting cases are easily derived. For instance, at zero forward speed of the propeller we have

$$\varDelta_0 u = (4 \varDelta E_t/\varrho A)^{1/3} \tag{60}$$

$$M_0 = (\tfrac{1}{2} \varrho^2 A^2 \varDelta E_t)^{1/3} \tag{61}$$

and

$$F_0 = (2 \varrho A \varDelta E_t^2)^{1/3} \tag{62}$$

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4) H. Glauert, Elements of Aerofoil and Airscrew Theory, Cambridge Univ. Press 1943, p. 202.

Furthermore as a first approximation, in the neighborhood of $u_0 = 0$, where $u_0/\Delta u \ll 1$, it is

$$\Delta u = \Delta_0 u - 4\, u_0/3 \tag{63}$$

$$M = M_0 + \varrho A\, u_0/3 \tag{64}$$

$$F = M_0[\Delta_0 u - 2\, u_0/3] \tag{65}$$

$$u^* = \frac{M_0}{M_f}\left[\Delta_0 u - \frac{2}{3}\, u_0\right] = \frac{1}{\beta_0}\left[\Delta_0 u - \frac{2}{3}\, u_0\right] \tag{66}$$

The "effective exhaust" velocity u^* is that velocity with which the fuel (gasoline) fed into the internal combustion engine would have to be exhausted in a hypothetical process to produce by its jet action the same thrust as the propeller achieves by external thrust generation. Attention should be called to the fact that as the value of β decreases because of increase of propeller diameter, $\Delta_0 u$ tends towards zero and the range of velocities u_0 for which (66) can be used also shrinks to zero.

The other limiting case which is easily treated concerns great forward velocities for which $u_0 \gg \Delta u$. Disregarding efficiency losses of the propellers, which in practice become very great, we should have from (59)

$$\Delta u \cong \Delta E_t/\varrho A\, u_0^2 \tag{67}$$

$$M \cong \varrho A\, u_0 \tag{68}$$

$$F \cong \Delta E_t/u_0 \tag{69}$$

that is, since $F u_0 \cong \Delta E_t$, the propulsive efficiency in this ideal case would become equal to unity, or the kinetic energy $M \Delta u^2/_2$ lost to the surrounding air tends toward the value zero, for $u_0 = \infty$.

Intermediate values for u_0 are best handled by numerical evaluation of the equation (59). We submit values for some typical cases.

We assume the following conditions for purposes of illustration. An internal combustion engine be available with $\Delta E_t = 1000$ HP shaft power. If the engine uses octane gasoline with a heat of combustion $\Delta h = 11.2$ Kcal/gram and a thermodynamic efficiency $\eta_{th} = 0.25$ we have

$$M_f = 4\, \Delta E_t/\Delta h \tag{70}$$

To generate 1000 HP on the shaft of the propeller, this requires a flow of octane into the engine of

$$M_f = 31.3 \text{ grams/sec} \tag{71}$$

or 112.7 grams octane per shaft horsepower per hour. With an air density $\varrho = 0.001205$ grams/cc at $t = 20^0$ C and an air pressure

$p_0 = 760$ mm Hg we obtain the following table I of typical ideal performance characteristics if the propeller has a diameter $D = 2r = 3$ meters.

TABLE I.

Performance Characteristics of an Ideal Engine Propeller Combination. Shaft power = 1000 HP. Propeller diameter = 3 m.

u_0 m/sec	Δu m/sec	M kg/sec	F kp	u^* m/sec	I_{sp} sec	$1/\beta$	Power HP
0	69.7	297	2070	648,000	66,100	9,490	0
10	57	328	1870	586,000	59,700	10,500	249
20	46.3	367	1700	533,000	54,300	11,700	453
30	36.8	412	1516	475,000	48,400	13,200	606
50	22.7	522	1185	371,000	37,800	16,700	790
70	14.4	657	946	294,000	30,000	21,000	883
100	8.0	885	708	222,000	22,600	28,600	944
200	2.13	1712	364.6	114,000	11,600	54,700	972
300	0.955	2558	244	76,400	7,790	83,600	976
400	0.538	3408	183.3	57,000	5,800	111,000	978

In addition we plot in Figures 16, 17, 18, and 19 some of the important performance parameters of the engine propeller combination.

The outstanding feature of the combustion-engine propeller combination is the fact that the mass of air passing through the propeller per second is very large compared with the mass of the propellant or fuel consumed per second. As shown in Table I and in Figure 17 the values of β for the engine-propeller are therefore much smaller than in any of the other propulsive power plants now in common use. The overall efficiency thus becomes very great. Unfortunately, this advantage cannot be maintained at high speeds for which the propeller becomes more and more inefficient because of turbulent and shock losses. The true jet engines with internal thrust augmentation and relatively large values of β become superior to the engine propeller at supersonic and at high subsonic forward velocities.

E. Some General Characteristics of the Universal Thrust Formula.

From the universal thrust formula (10) we obtain for the limiting value of the thrust F at zero forward speed $u_0 = 0$ the expression

$$F_0 = M\sqrt{2\,\beta\,\Delta\varepsilon} \tag{72}$$

This thrust at standstill is different from zero for all of the engines discussed except for the aeroduct. The effective exhaust velocity at $u_0 = 0$ is

$$u^*(0) = \sqrt{2\,\Delta\,\varepsilon/\beta} \tag{73}$$

which formula clearly demonstrates the thrust augmenting action

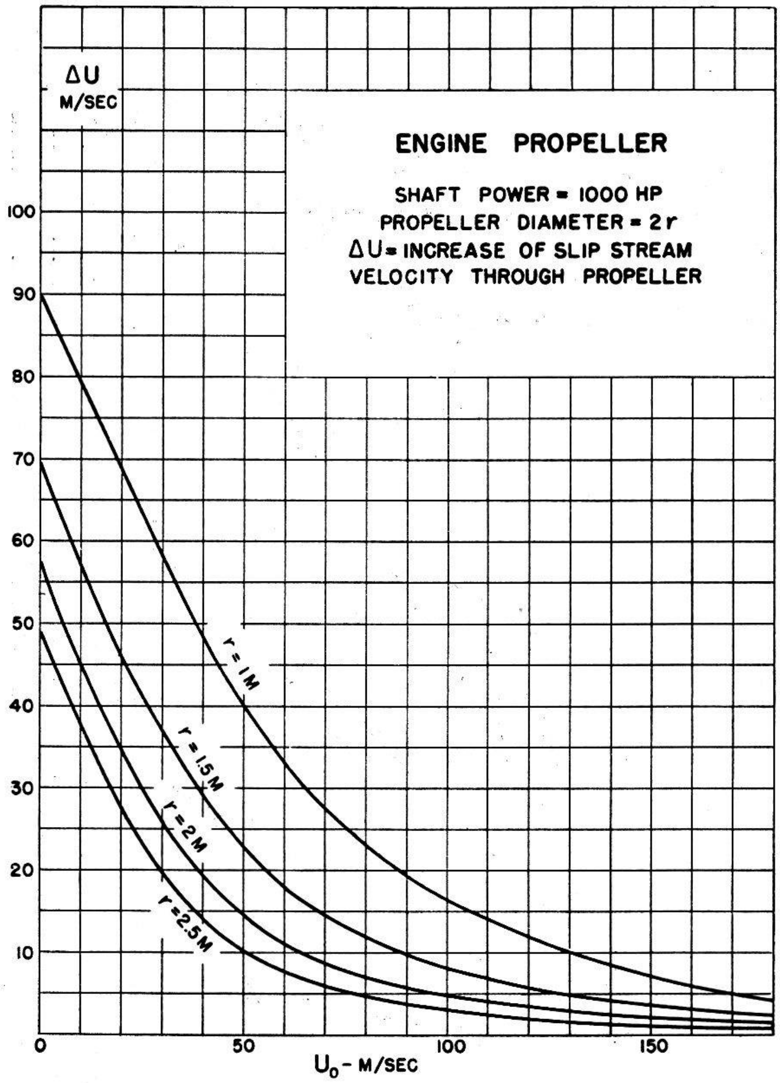

Figure 16.

of large excesses of air whose oxygen does not partake in the chemical combustion. It can be shown generally that the thrust and u^* increase with decreasing β for all engines whose thermodynamic efficiency η_{th} and consequently $\Delta\varepsilon$ does not depend on the propellant parameter β. For intermittently operating engines it may be $\frac{d\,\eta_{th}}{d\,\beta} \neq 0$ and $\frac{d u^*}{d\,\beta} > 0$ in certain ranges. If $\Delta\,\varepsilon$ does not depend on β we have from (11)

$$\frac{d u^*}{d\,\beta} = \frac{u_0}{\beta^2}\left[1 - \frac{1}{2}\left(\frac{1}{\Delta} + \Delta\right)\right] \tag{74}$$

where

$$\Lambda = \sqrt{1 - \beta + 2\,\beta\,\Delta\varepsilon/u_0^2} \tag{75}$$

Analyzing the function

$$f = 1/\Lambda + \Lambda \tag{76}$$

we find that

$$f_{\min} = 2 \text{ for } \Lambda = 1 \tag{77}$$

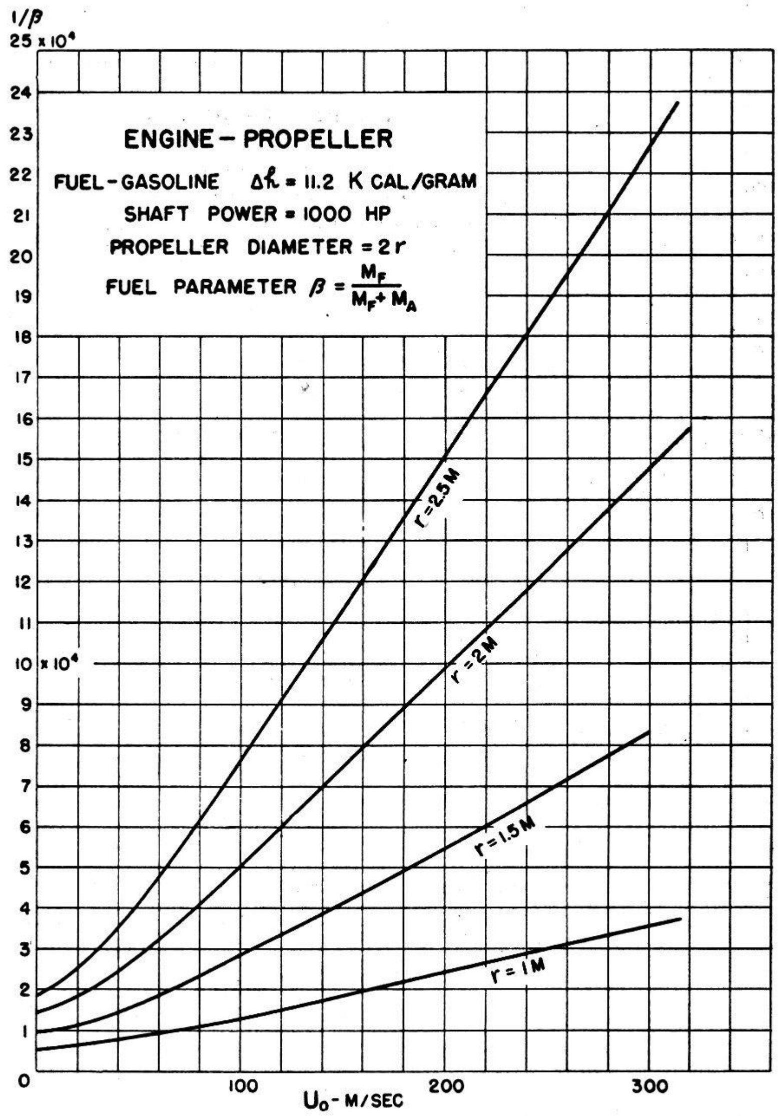

Figure 17.

In this case we have

$$u_0 = \sqrt{2\,\Delta\varepsilon} \tag{78}$$

and

$$\frac{du^*}{d\beta} = 0 \tag{79}$$

In all other cases it is obviously, from (74) and (77)

$$\frac{du^*}{d\beta} < 0 \tag{80}$$

This inequality demonstrates our theorem that the thrust and the overall efficiency of a propulsive power plant for which $\Delta\varepsilon$ does not depend on β increase with decreasing β. With β tending toward zero we get the greatest effective exhaust velocity and specific impulse. If both

$$\beta \ll 1 \tag{81}$$

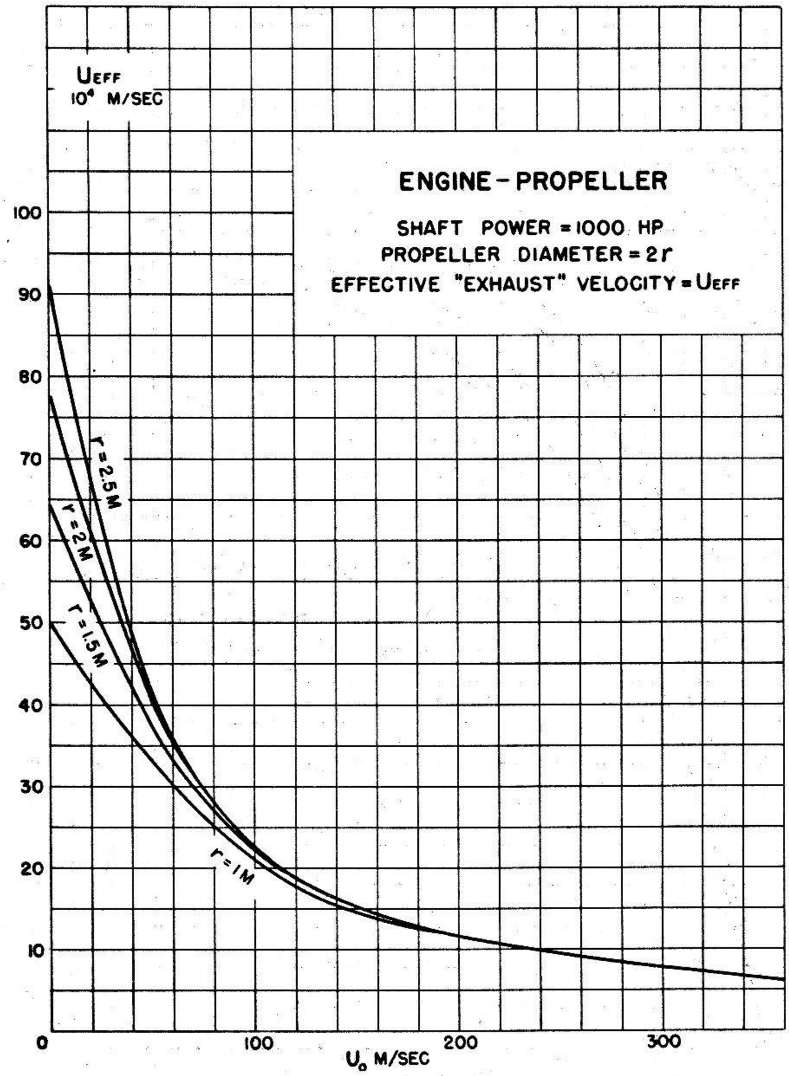

Figure 18.

and

$$\beta\Delta\varepsilon/u_0^2 \ll 1 \tag{82}$$

it follows from the universal thrust formula (10) that

$$u^* = \Delta\varepsilon_t/u_0 = u_0/2 + \Delta\varepsilon/u_0 \tag{83}$$

In the range of speed usually considered u_0 is negligible compared with $\Delta\varepsilon/u_0$. Indeed for gasoline, for instance, it is $\Delta\varepsilon \sim 10^{12}$ ergs/gram. Therefore, if u_0 is equal to the speed of sound in air, that is

347 m/sec, we have $\Delta\varepsilon/u_0 \cong 3\times 10^5$ m/sec, which is large compared with u_0. Neglecting $u_0/2$ it thus is

$$F u_0 = M\Delta\varepsilon \tag{84}$$

In any case we have from (83) that accurately

$$F u_0 = \Delta E_t \tag{85}$$

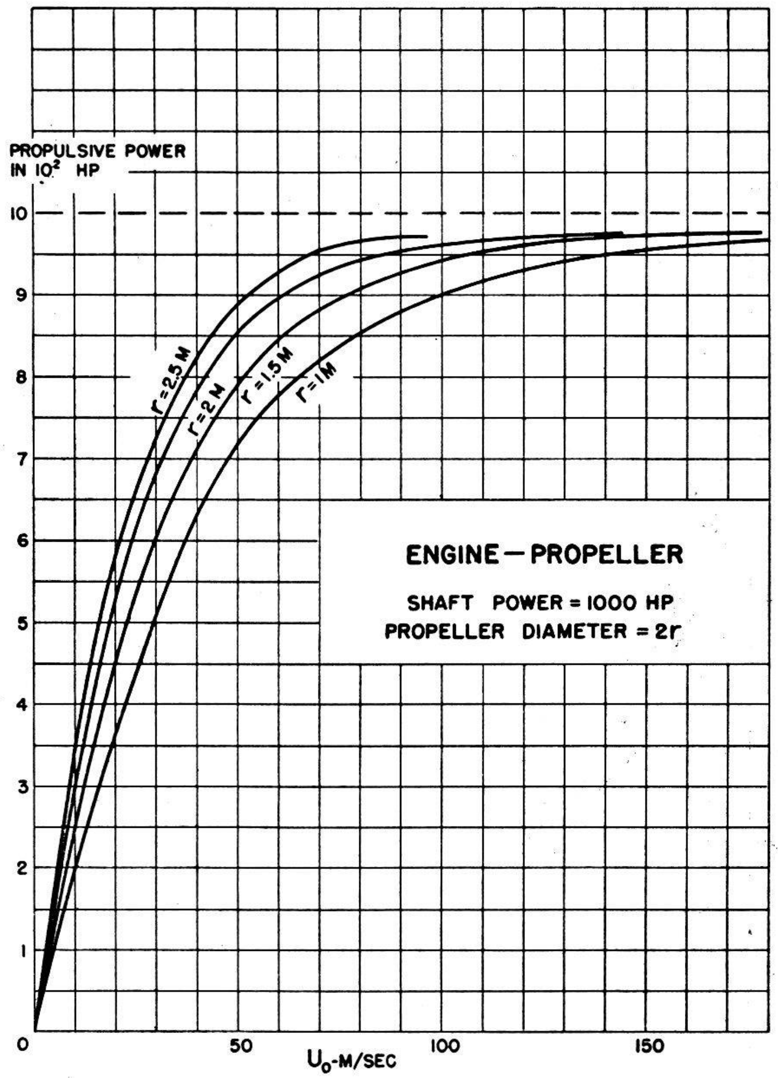

Figure 19.

which means that the propulsive efficiency (equation 17)

$$\eta_p = F u_0/\Delta E_t \tag{86}$$

is equal to unity for $\beta = 0$, that is, for infinite relative amounts of the excess air.

In the limit $u_0 \gg \sqrt{\Delta\varepsilon}$ the universal thrust formula is reduced to the relation

$$u^* = u_0/2 \tag{87}$$

which represents the effect of the energy invested in the propellant (or fuel) traveling on a vehicle at high speed. Formula (87) must of course be corrected for relativistic effects if the ratio of u_0 to the velocity of light becomes appreciable. This correction may be found in an article by ACKERET[1]).

In the limit $\beta = 1$, that is, self-contained propellants, we obtain

$$u^* = \sqrt{2\,\Delta\varepsilon} \tag{88}$$

This is the performance equation of ordinary rockets for which the effective exhaust velocity becomes independent of forward speed.

5. *Intercomparison of Various Aero-Engines, Topological Performance Diagrams.*

For purposes of intercomparison of the four engines discussed we have (logarithmically) plotted in Figure 20 the ideal performances of the aeroduct, the aeroturbojet and the aeropulse operating with a fuel parameter $\beta = 0.01$ and octane as the fuel. For the engine-propeller combination we show the curve for an engine operating with the thermodynamic efficiency $\eta_{th} = 0.25$, a propeller of radius $r = 2$ m and a shaft power of 1000 HP. As the maximum possible performance of any aerial propulsive power plant, which transforms all of heat of combustion of octane into propulsive power the "ideal" curve is shown, the equation of which obviously is given by (83). Otherwise the Figure 20 is self-explanatory.

Obviously for practical engines the parasite losses previously mentioned is of decisive importance. The introduction of these parasite losses into our universal mathematical formalism presents no difficulties if quantitative experimental values for these losses are available. The discussion of the practical performance of aero-engines on the basis established in this paper is, however, somewhat lengthy and must be reserved for another article. Assuming that the necessary analysis has been carried out we wish to point to a method of quick intercomparison of aero-engines with a view to different requirements. These may of course be very varied, depending on whether the engines are used for commercial or for military purposes. Both of these categories may again be discussed in the light of numerous special requirements, including straight performance, economy and all sorts of logistic characteristics.

As an example of a method of intercomparison we mention only the analysis of the performance of aero-engines in the light of load

[1]) J. ACKERET, Helvetica Physica Acta **19**, 103 (1946).

carrying capacity as a function of average cruising speed u_0, time of flight τ and level of flight H. In this analysis the so-called "invested mass μ" is of universal importance. We define

$$\mu = \beta_0 + M_f \tau \tag{89}$$

as the invested mass, where μ_0 is the mass of the empty vehicle and M_f the fuel consumption per second.

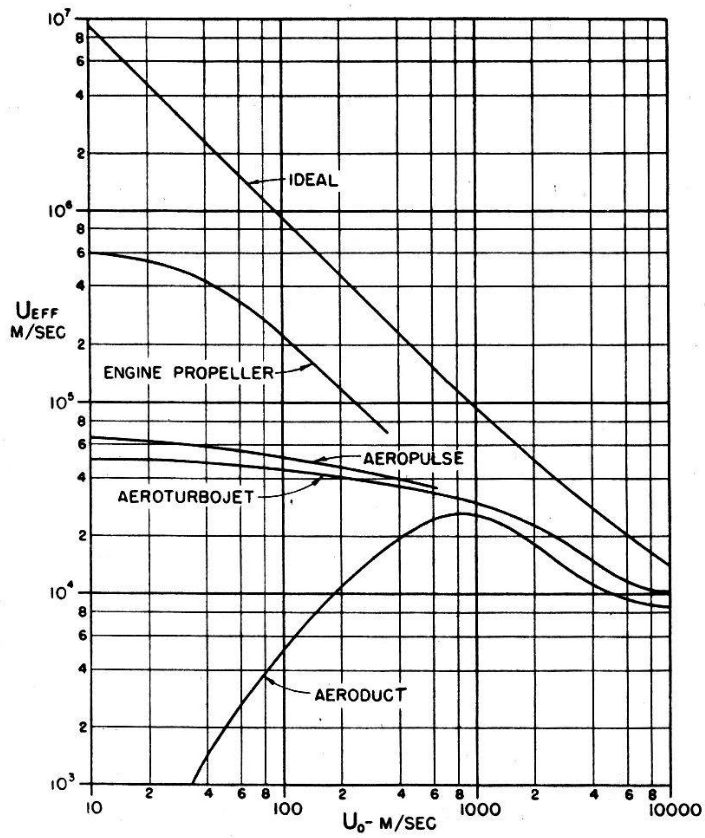

Figure 20.
Comparative Performance of the Principal Power Plants.
$\beta = 0.01$ in all cases except Engine Propeller. Engine Propeller: $r = 2$ meters Shaft Power $= 1000$ HP Fuel-Gasoline.

Every horizontal plane $H =$ const in the three dimensional topological performance space (H, t, u_0) is then subdivided into many regions bounded by curves defined by

$$\mu_i = \mu_k \tag{90}$$

where the indices i and k designate two different aero-engines. Inside of each area bounded by such curves the invested mass μ_σ of some engine σ is then smaller than the invested masses of all other engines. This means that at the heights H, forward speeds u_0 and times of flight t lying within this region the engine σ is the best from the standpoint of load carrying capacity.

	Power Plant (jet motor)	Propellants for the activation of the power plants	BODY OF VEHICLE (Aero-dynamics)	BODY OF VEHICLE (Structure) Materials	BODY OF VEHICLE (Controls)	Investigation of the medium through which the missile will move	Necessary test stands	Instrumentation	Field tests such as tracking of missile
Basic theoretical research									
Basic experimental research									
Construction and testing of model devices									
Construction and testing of full scale component parts of final vehicle									

CONSTRUCTION AND TESTING OF FINAL VEHICLE

Production of Vehicle. (Procurement and Manufacturing Problems).

A topological performance diagram of the type just described is obviously nothing absolute. With the advent of better fuels, better structural materials, design and construction the diagram must be brought up to date again and again. Topological spatial diagrams were simultaneously conceived by Professors von Karman, Tsien and the author in this country as well as by O. Neugebauer and H. Reindorf in Munich, Germany.

Many auxiliary problems must be solved, after a choice of a particular engine has been made, to construct this engine and put it into operation. The following diagram indicates in rough outlines what the nature of these problems is and how they are to be solved in stages through research, fundamental experimentation, model construction and final production.

Acknowledgment.

I wish to thank Dr. J. M. Carter and Mr. S. W. Coffman of the Aerojet Engineering Corporation for their assistance in carrying out the calculations for the aeroduct and the aeroturbojet.

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