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LOOKING BACK TO THE FUTURE OF THE TEACHING AND LEARNING OF ALGEBRA

by Kaye STACEY

The 12th ICMI Study, which ran from 2000 to 2004, was on *The Future of the Teaching and Learning of Algebra,* and its Study Conference was held at the University of Melbourne, Australia from December 9th to 14th, 2001. There are several reasons why the future of the teaching and learning of algebra was a timely focus at the beginning of the twenty-first century. The strong research base developed in the previous decades enabled us to take stock of what has been achieved and also to look forward to what should be done and what might be achieved in the future. In addition, trends evident in the previous decades were intensifying, and continue to do so today. Those trends particularly affecting school mathematics are the "massification" of education — continuing in some countries whilst beginning in others — and the advance of technology. Algebra is centrally affected by both of these trends. For *mass education,* algebra teaching highlights questions of equity and relevance. For progression to higher mathematics, students need algebra but its abstraction makes it hard to learn and hard for beginners to see a reason for learning it. To be equitable to students from all backgrounds and with a wide range of abilities, it is important to provide access to algebra because it is a gateway to advanced mathematics, and hence to the technological education that depends on it, and consequent life chances. However, the reality is that many students struggle to learn algebra and do not see that it contributes positively to their lives. Addressing this demand for more meaningful and productive ways of teaching algebra to all was a key purpose of the ICMI Study.

The second trend of *advancing technology* also affects the teaching of algebra. New technology provides rich prospects for improving teaching, and many of these were discussed at the ICMI Study. However, advancing technology also provides a challenge to the existing curriculum because so many of the routines that have been the standard diet of school algebra to date are now available "at the press of a button".

The consequence is that an algebra curriculum that serves its students well in the twenty-first century may well look very different from an ideal curriculum from some years ago. At the classroom level, the consequence is that a high priority is put on ways to make the objects and processes of algebra meaningful to students and to help teachers have a clearer idea of what algebra might be beyond symbolic manipulation.

The International Program Committee met in January 2000 to draft the Discussion Document [2] that was then disseminated throughout the international mathematics education community and is reprinted in the conference proceedings [1], available from the University of Melbourne. In response to the call for contributions, over 150 papers were submitted from nearly 200 authors. This strong response indicated the wide concern with the teaching of algebra, the strong research base upon which we can go forward, and an optimism that attention to this issue can result in real gains for students around the world. Finally, there were 110 participants from North and South America, Eastern and Western Europe, China, Indonesia, Israel, Japan, Malaysia, New Zealand, Singapore, and the Pacific, as well as a strong contingent of Australians. Since the individual contributions are available in the published Proceedings, the final study volume [6] takes a summative and reflective point of view. Each chapter contains a substantial literature review and also puts forward a scholarly perspective on the issues involved, which was shaped by and tested through the group discussions and prepared by leading researchers.

The main themes of the ICMI Study

Looking back at the ICMI algebra study, one is immediately struck by the comprehensiveness of the issues and styles of enquiry that it encompassed. Even though it was focussed on a single area of mathematics teaching, the study brought in perspectives from across mathematics education and drew on the epistemology and history of mathematics.

What is algebra?

At the level of policy and curriculum, the ICMI Study has assisted in understanding how what is regarded as “algebra” varies around the world, and how its teaching is structured in different school systems. Lack of awareness often operates as a barrier to learning from each other, whereas awareness of difference provides a basis for countries to examine other practices and explore possibilities for change. In different countries, educators emphasise that algebra is: (i) a way of expressing generality; (ii) a study of symbol manipulation and equation solving; (iii) a study of functions; (iv) a way to solve certain classes of problems; (v) a way to model real situations; or (vi) a formal system involving set theory, logic, and operations on entities other than real numbers. At the study conference, educators from many countries (including developing countries in the Asia-Pacific region) had the opportunity to examine in detail the changes that were being made elsewhere and to evaluate their potential for local contexts. Several participants commented on the unexpectedly great variety of what constitutes ‘algebra’ between and even within countries. Kieran [5], for example, comments that the divide between traditional algebra courses that emphasise symbol manipulation and ‘reform courses’ that emphasise functions and technology-supported problem solving methods is still strong within the United States. As educators in many countries work to extend ideas of algebra from rule-based symbol manipulation, their search for a livelier subject is assisted by understanding possibilities tried elsewhere. We do not expect that the algebra curricula of different countries will converge, but that decisions will be made based on informed choices and awareness of wider possibilities.

The curriculum and policy issues above were reflected in the attention given in the study conference to the questions of 'what is algebra', 'what is algebraic thinking' and how researchers can describe and categorise the main activities that are engaged in when people do algebra. These questions were prompted by the differences in the content of algebra courses noted above, but also in the intention (which has strengthened since the study conference) to engage students in 'algebraic thinking' even when they are not doing conventional algebra with letters. The core activities of algebra are seen as *generational* activities (where situations, properties, patterns, and relationships are represented algebraically or interpreted), *transformational* activities (including algebraic manipulation), and *global/meta-level* activities which are not algebra-specific but relate to the purpose and context for using algebra (e.g., to prove, or solve problems, or to notice structure etc.). This is Kieran's GTG framework, which since has been widely used to organise research and thinking (cf. [5]). A second framework outcome of the study conference for researchers was a toolkit for analysing teaching approaches to algebra, developed by a large and diverse working group. This calls for consideration of the problem domain, the teaching approach, the theoretical perspective, and the community of students. Many socio-cultural studies published since the conference take this comprehensive view.

The early development of algebraic reasoning

At the study conference, it was clear that a main theme was broadening the range of teaching of algebra, in its content, in the teaching approaches used and also in the age groups of students who could benefit from algebra. One particular strand of this broadening, which was embryonic at the time of the conference but which has now reached some maturity, was the 'early algebra' movement. This attracted a large and active working group. The recent book edited by Kaput, Carraher and Blanton [4], which has contributions from many members of the ICMI Study working group, represents a major milestone in the development of their agenda. The basic tenet of the early algebra movement is that it is possible to introduce young students to the culture of algebra by encouraging them to think with a particular kind of generality throughout schooling, including in the first years. In 2001, there was speculation on 'how young and how early', but the recent volume presents much more extensive empirical evidence about what can be profitably undertaken and how. As noted at the study volume, early algebra is not algebra early, which would be a recipe for disaster. In the United States, an important impetus for 'early algebra' has been to disturb the traditional layer-cake curriculum, where algebra follows arithmetic, and is taught separately from other parts of mathematics. However, in the United States as well as in other countries such as Australia and Japan, where mathematics is taught in an integrated curriculum, early algebra has primarily been of interest as a way of enhancing the total mathematical experience of children of all ages. Early algebra aims to emphasise generalisation and connections between topics. It aims to build a sense of mathematical structure and to develop symbolisation (often not traditional), representation and age-appropriate appreciation of functions. The developing interest in early algebra has further provoked discussion of what algebraic thinking is, what the essence of algebra is, its relation to generalisation and abstraction, and how these more general ideas can be well developed in students.

Technology for algebra

The use of new technology in teaching and learning algebra, and also in changing the algebra curriculum, was a major theme at the study conference with two working groups devoted to it. Predictably, technology has been a major theme of research work in algebra since then. One working group examined a wide set of technological environments, both commercial and experimental, that can contribute to learning algebra. Major themes, such as creating meaning for variables and using multiple representations, which were present then are still present in today's research, although the technology has moved on. Sharing possibilities developed a strong sense of optimism that well-designed, imaginative technological environments can improve the experiences of many students in the future, and provided an introduction for many participants to new capabilities. For example, dynamic control of numerical variables and the consequential possibility of a kinetic sense entering algebraic exploration were new at that time, and have since flowered in both software applications and in research on learning. Ferrara, Pratt and Robutti [3] provide examples in their overview of recent research. A different type of outcome was a heightened awareness that doing mathematics in a technological environment will not exactly mirror doing mathematics in a pen-and-paper environment. This growing understanding of the gap between doing mathematics by hand and doing mathematics with a machine was also a major theme that emerged in the other technology working group, which focussed on the use of computer algebra systems for secondary school and early tertiary mathematics learning. At the time of the study conference, use of computer algebra systems in computer laboratories had been pioneered in a considerable number of early tertiary settings, and a small amount had taken place in secondary schools. The main focus had been to extend the range of problems that students could deal with. The working group, however, focussed on the possibilities of students making pedagogical use of computer algebra systems to learn concepts more deeply. The main dimensions of these possibilities were outlined. This has been a main theme of research since that time, as use at secondary level has become a major topic of research. At the level of theory, the study conference provided an extended opportunity for researchers to learn in depth about the theories used by others. As an example, the conference provided a early opportunity for English-speaking researchers in technology to understand the theories used by Artigue's French team in some depth. These theories have since been used extensively by researchers from many countries.

Other specialist areas looking at algebra

The study conference provided an opportunity for researchers and educators to hear specialists in diverse areas of mathematics education discuss how their special interest related to algebra. The working group on the history of algebra, for example, were able to put into historical perspective the pedagogical need to move students from an arithmetic to an algebraic understanding of equality: from = as a sign meaning "work it out now" to = as a sign to indicate the equality of two expressions. Twists and turns in the history of algebra can highlight aspects of mathematics that have presented challenges in the past and hence are likely to be challenging for students. On the other hand, knowledge of points of difficulty for present-day students can pinpoint areas of history that are worth studying in depth. It also demonstrates the historical roots of many of the difficulties that students are likely to have when learning present-day algebra. Curricula in the future can benefit from these insights.

It is often said that algebra is a language, but does treating algebra as a language, or at least a system of signs, lead to insights into teaching and learning? Are there useful parallels to be made with how languages are used and how they develop analogous processes in algebra? The working group on *Symbols and language* brought a linguistic and a semiotic perspective to studying the learning of algebra. Since the time of the study conference, Drouhard (personal communication) notes that this work has developed considerably as a research field, but that it has not yet been able to influence practice.

Researchers interested in teachers' knowledge and its impact on classrooms and students' learning were able to merge perspectives from the general research area with knowledge of specific problems for the teaching of algebra. They reviewed a series of studies which identified areas where teachers' knowledge was likely to need development and areas where teachers' beliefs about algebra were likely to conflict with new curriculum goals. Since the study conference, research work on teachers' knowledge has flourished, and many of the gaps in research knowledge that were identified have received attention.

The study conference was very comprehensive, but there was one under-subscribed working group. This was on modelling real world problems, and in the end it did not run. This reflected an orientation at the study conference towards learning algebra as a goal in itself, rather than learning algebra as a tool for solving problems from the real world. In most of the real world examples presented in the many working groups and study volume chapters, the purpose of the modelling was for students to learn algebra (for example to give meaning to variables). What was missing was overt presentation of research work where a major purpose of learning algebra is to learn to model the real world. An agenda for future work in this field is to bring these two perspectives into stronger synergy.

REFLECTION

The 12th ICMI Study provided a special opportunity for mathematics educators, researchers and policy leaders to better understand how other countries approach the teaching of algebra, to exchange ideas and to learn at first hand about research findings and theories in the field. It drew together participants who focussed on the learning of algebra at all levels of schooling, including in the university sector, and participants from developed and developing countries. The definition of school algebra as encompassing much more than techniques for symbol manipulation was well established before the study conference in some parts of the world, but at the conference the new way of conceptualising algebraic thinking, algebraic activity and the goals of school algebra reached greater consensus. The study conference was held at a time when there was already a substantial understanding of the cognitive obstacles which students face when they learn algebra. This research underpinned the work of the study conference. In the years that have passed since then, it is interesting to see that new technology has been able to support students through some of these difficulties and that this work is well advanced for basic concepts. However, whilst changes in technology continue to provide new possibilities for learning algebra better, they also continue to provide new challenges as epistemological differences between pen-and-paper and machine mathematics emerge. The pressure on algebra teaching from massification of secondary schooling, the other major trend identified in the Introduction, is also now providing new challenges. There is an on-going movement towards mass education in some countries. In other countries, current mass education approaches are seen to have resulted in disengagement of young adolescents in schooling. Increasingly, the voices of reform are calling for a move away from standard curriculum with recognised subjects and a move towards more flexible structures of schooling that are devised around the student's immediate concerns. There are new challenges for the teaching of algebra and its part in mathematical growth in such an environment.

The on-going demand for copies of the conference proceedings after more than seven years, and the impact of the very carefully prepared overview chapters in the study volume are two further indications of the lasting benefits of the 12th ICMI Study.

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