We may now define $|\xi|_h^2$ for $\xi \in L_x$ with $x \in X$ by

$$|\xi|_h^2 = \begin{cases} e^{-\gamma(x)}|\xi/s(x)|^2 & \text{if } x \in X \setminus Y, \\ e^{-\alpha(x)-\epsilon\beta(x)}|\xi|_k^2 & \text{if } x \in V. \end{cases}$$

Then h is a well-defined C^∞ Hermitian metric in L since, for $x \in V \setminus Y$ and $\xi \in L_x$, we have

$$e^{-\gamma(x)}|\xi/s(x)|^2 = e^{-\left(\alpha(x)-\log|s(x)|_k^2+\epsilon\beta(x)\right)}|\xi|_k^2/|s(x)|_k^2 = e^{-\alpha(x)-\epsilon\beta(x)}|\xi|_k^2.$$

Furthermore, on $X \setminus Y$ we have

$$\mathcal{R}_h = \Delta_g\left(-\log|s|_h^2\right) = \Delta_g\gamma \begin{cases} > 0 & \text{on } X \setminus Y \\ > 1 & \text{on } V \setminus Y \end{cases}$$

By continuity, we also have $\mathcal{R}_h \geq 1 > 0$ at points in Y. Thus $\mathcal{R}_h > 0$ on X. $\square$

For X a Riemann surface, the above proofs become especially simple. For example, the construction of α in the proof of Theorem 2.3 is trivial for $\dim X = 1$ because Y is discrete. For X an open Riemann surface, Theorem 0.1 provides a C^∞ strictly plurisubharmonic exhaustion function and, therefore, by [Gr] and [DG], one gets the theorem of [BS] that an open Riemann surface is Stein. For a compact Riemann surface X, Theorem 2.3 becomes the familiar fact (see, for example, [GriH]) that the holomorphic line bundle associated to a nontrivial effective divisor admits a C^∞ Hermitian metric h with positive curvature Θ_h.

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Terrence Napier

Department of Mathematics
Lehigh University
Bethlehem, PA 18015
U. S. A.
e-mail: tjn2@lehigh.edu

Mohan Ramachandran

Department of Mathematics
SUNY at Buffalo
Buffalo, NY 14260
U. S. A.
e-mail: ramac-m@newton.math.buffalo.edu