

## 13.3 Proof of Theorem 13.2

Objektyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **49 (2003)**

Heft 3-4: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **22.09.2024**

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern. Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden. Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

### **Haftungsausschluss**

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Thus  $a_I \omega_1 b_I b_I^{-1} \omega_1^{-1} \omega^{-1} a_I^{-1} \in \text{Im}(\alpha)$ ,  $a'_I \omega_1 b'_I b'_I^{-1} \omega_1^{-1} \omega^{-1} a_I'^{-1} \in \text{Im}(\alpha)$ ,  $a_I \omega_1 b'_I b_I^{-1} \omega_1^{-1} \omega^{-1} a_I'^{-1} \notin \text{Im}(\alpha)$ . Now  $(a_I \omega^{-1} a_I'^{-1})^{-1} a_I \omega^{-1} a_I^{-1} (a_I \omega^{-1} a_I'^{-1}) = a_I' \omega^{-1} a_I'^{-1} \in \text{Im}(\alpha)$ , whereas  $a_I \omega^{-1} a_I'^{-1} \notin \text{Im}(\alpha)$  and  $a_I \omega^{-1} a_I^{-1} \in \text{Im}(\alpha)$ . We thus get a contradiction to the malnormality of  $\text{Im}(\alpha)$  in  $F_n$ . This completes the proof.  $\square$

### 13.3 PROOF OF THEOREM 13.2

From Lemmas 13.6 and 13.7, the Cayley complex  $\mathcal{C}(G_\alpha)$  is the mapping-telescope of a strongly hyperbolic forest-map, equipped with the standard metric. A Cayley complex is connected. Thus, from Theorem 12.4,  $\mathcal{C}(G_\alpha)$  is a Gromov-hyperbolic metric space for any mapping-telescope standard metric. From Lemma 13.5 the group  $G_\alpha$  acts cocompactly, properly discontinuously and isometrically on  $\mathcal{C}(G_\alpha)$  equipped with a mapping-telescope standard metric. A classical lemma of geometric group theory (usually attributed to Effremovich, Svàrc, Milnor – see [19] or [17] for instance), applied to quasi geodesic metric spaces, tells us that  $G_\alpha$  and  $\mathcal{C}(G_\alpha)$  are quasi-isometric so that  $G_\alpha$  is a hyperbolic group.  $\square$

REMARK 13.8. Another way of stating our main theorem about ‘forest-stacks’, using the language of trees of spaces, goes roughly as follows: “An oriented  $\mathbf{R}$ -tree of  $\mathbf{R}$ -trees with the gluing-maps satisfying the conditions of hyperbolicity and strong hyperbolicity with uniform constants is Gromov-hyperbolic.” Here ‘oriented  $\mathbf{R}$ -tree’ means an  $\mathbf{R}$ -tree  $T$  equipped with an orientation going from the domain to the image of each attaching-map, and a surjective continuous map  $f: T \rightarrow \mathbf{R}$  respecting this orientation. As a corollary of our theorem, and in order to illustrate it, we chose to concentrate on mapping-telescopes. We could as well consider spaces similar to mapping-telescopes but where we allow the attaching-maps not to be the same at each step. Our only requirement is to have uniform constants of quasi-isometry, hyperbolicity and so on. Also, with respect to groups, a corollary could have been stated dealing with HNN-extensions rather than just semi-direct products.

Another result which easily follows from our work could be more or less stated as follows. “Let  $T$  be a tree of spaces  $X_i$ ,  $i = 0, 1, \dots$ . Let  $\psi: T \rightarrow T$  be a map of  $T$  such that the mapping-telescope of each  $X_i$  under  $\psi$  is Gromov-hyperbolic. If  $\psi$  induces a hyperbolic map on the tree resulting of the collapsing of each  $X_i$  to a point, then the mapping-telescope of the tree of spaces  $T$  under  $\psi$  is Gromov-hyperbolic.” We leave the precise statement of such corollaries to the reader. Together with [14] where a new proof of the

Bestvina-Feighn theorem is given for mapping-tori of surface groups, the last one gives, thanks to [26], a new proof of the full version of the Combination Theorem for mapping-tori of hyperbolic groups, namely: “If  $G$  is a hyperbolic group and  $\alpha$  is a hyperbolic automorphism of  $G$ , then  $G \rtimes_{\alpha} \mathbf{Z}$  is a hyperbolic group.”

## REFERENCES

- [1] ALONSO, J.M. et al. Notes on word hyperbolic groups. Edited by H. Short. In: *Group Theory from a Geometrical Viewpoint, Trieste (1990)*, 3–63. World Sci. Publishing, 1991.
- [2] BESTVINA, M.  $\mathbf{R}$ -trees in topology, geometry and group theory. In: *Handbook of Geometric Topology*, 55–91. North-Holland, Amsterdam, 2002.
- [3] BESTVINA, M. and M. FEIGN. A combination theorem for negatively curved group. *J. Differential Geom.* 35 (1) (1992), 85–101. With an addendum and correction *J. Differential Geom.* 43 (4) (1996), 783–788.
- [4] BOWDITCH, B. Stacks of hyperbolic spaces and ends of 3-manifolds. Preprint, Southampton.
- [5] BRIDSON, M. and A. HAEFLIGER. *Metric Spaces of Non-positive Curvature*. Fundamental Principles of Mathematical Science 319. Springer-Verlag, 1999.
- [6] BRINKMANN, P. Hyperbolic automorphisms of free groups. *Geom. Funct. Anal.* 10 (5) (2000), 1071–1089.
- [7] COOPER, D. Automorphisms of free groups have finitely generated fixed point sets. *J. Algebra* 111 (1987), 453–456.
- [8] COORNAERT, M., T. DELZANT et A. PAPADOPOULOS. *Géométrie et théorie des groupes*. Lecture Notes in Math. 1441, Springer-Verlag, 1990.
- [9] CULLER, M. and J.W. MORGAN. Group actions on  $\mathbf{R}$ -trees. *Proc. London Math. Soc.* (3) 55 (1987), 571–604.
- [10] DICKS, W. and E. VENTURA. The group fixed by a family of injective endomorphisms of a free group. *Contemporary Mathematics* 195 (1996).
- [11] FARB, B. and L. MOSHER. The geometry of surface-by-free groups. *Geom. Funct. Anal.* 12 (5) (2002), 915–963.
- [12] FATHI, A., F. LAUDENBACH et V. POÉNARU. *Travaux de Thurston sur les surfaces*. *Astérisque* 66/67 (1979).
- [13] GAUTERO, F. Hyperbolicité relative des suspensions de groupes hyperboliques. *C. R. Acad. Sci. Paris* 336 (11) (2003), 883–888.
- [14] GAUTERO, F. and M. LUSTIG. Relative hyperbolicity of (one ended hyperbolic)-by-cyclic groups. To appear in *Math. Proc. Cambridge Philos. Soc.* (2005).
- [15] GERSTEN, S.M. Cohomological lower bounds for isoperimetric functions on groups. *Topology* 37 (5) (1998), 1031–1072.
- [16] GHYS, E. et P. DE LA HARPE. *Sur les groupes hyperboliques d’après Mikhael Gromov*. Progress in Mathematics 83, Birkhäuser, 1990.