irreducible representations of a finite group, a theorem of Brauer says this number equals the number of conjugacy classes in the group consisting of elements with order prime to p.

7. Recent results

Character tables do not provide a way to distinguish any two finite groups, in general. For example, for any prime p the two nonisomorphic nonabelian groups of order p^3 have the same character table. Can we find a computational tool extending the character table which will distinguish any two non-isomorphic finite groups? In 1991, Formanek and Sibley [19] showed that if there is a bijection between two groups G and H which converts $\Theta(G)$ to $\Theta(H)$, then G and H are isomorphic. Since the irreducible characters can be read off (in principle) from the factors of $\Theta(G)$, we see $\Theta(G)$ is one answer to the question. However, if $\#G$ is large then $\Theta(G)$ is too hard to compute. Is there something closer to the character table which works? Yes. See the articles of Hoehnke and Johnson [28, 29] and Johnson and Sehgal [31].

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Keith Conrad

Department of Mathematics,
Ohio State University,
Columbus, OH 43210-1174
U. S. A.
e-mail: kconrad@math.ohio-state.edu