the Sylow-2-subgroup of $L^{\#}/L$) may be derived from the equality

$$\left[(Sp_4(3) \circ C_3) \underset{\sqrt{-3}}{\overset{2}{\boxtimes}} SL_2(3)\right]_{16} = \left[(Sp_4(3) \circ C_3) \underset{\sqrt{-3}}{\overset{2(2)}{\boxtimes}} SL_2(3)\right]_{16}$$

and

$$\left[SL_2(5) \underset{\infty,3}{\overset{2(3)}{\boxtimes}} (SL_2(3) \overset{2}{\square} C_3)\right]_{16} = \left[(SL_2(5).2 \circ C_3) \underset{\sqrt{-3}}{\overset{2(2)}{\boxtimes}} SL_2(3)\right]_{16}$$

using Proposition 5.

Similarly one uses Proposition 5 to show the 2-modularity of the lattices of the r.i.m.f. group 6 in $GL_{24}(\mathbf{Q})$ using the description

$$\left[6.U_4(3).2 \underset{\sqrt{-3}}{\overset{2}{\boxtimes}} SL_2(3)\right]_{24} = \left[6.U_4(3).2 \overset{2(2)}{\circ} SL_2(3)\right]_{24}.$$

For the groups 44 and 64, which are the only groups which are not $p$-lattice sparse for a relevant prime $p$ (=2), one has to note that the invariant sublattice of index $2^{12}$ in $L$ is unique.

The theorem now follows from the next lemma. □

LEMMA 9. *The lattices (of determinant* $3^8 \cdot 5^8$*) of the r.i.m.f. subgroup* $G := [\pm \mathrm{Alt}_6 .2^2]_{16} \leq GL_{16}(\mathbf{Q})$ *(number 20 of* [NeP 95]*) are not (strongly) modular.*

*Proof.* Let $L$ be such a $G$-invariant lattice and $L' \in \pi(L)$. Assume that there is a similarity $s : L' \to L$. By Proposition 3, this similarity $s$ normalises $G$. Let $U \cong \mathrm{Alt}_6$ be the characteristic subgroup $\cong \mathrm{Alt}_6$ of $G$. Since the full automorphism group of $U$ is already induced by conjugation with elements of $G$, there exists $g \in G$, such that $n := gs \in GL_{16}(\mathbf{Q})$ centralises $U$. Hence $n \in C_{M_{16}(\mathbf{Q})}(U) \cong \mathbf{Q}[\sqrt{5}]$. Since this number field does not contain an element of norm 3, one concludes that $[L' : L] = 5^8$. So the lattice $L$ is neither similar to $L^{\#}$ nor to the lattice $L' \in \pi(L)$ corresponding to the 3-Sylow subgroup of $L^{\#}/L$. Note that if $[L' : L] = 5^8$, an element $x \in C_{M_{16}(\mathbf{Q})}(U)$ with $x^2 = 5$, induces a similarity by Proposition 4. □

## REFERENCES


[CCNPW 85] CONWAY, J. H., R. T. CURTIS, S. P. NORTON, R. A. PARKER and R. A. WILSON. *Atlas of Finite Groups.* Oxford University Press, 1985.

[Neb 95] NEBE, G. Endliche rationale Matrixgruppen vom Grad 24. Dissertation RWTH Aachen. *Aachener Beiträge zur Mathematik 12* (1995).

[Neb 96] NEBE, G. Finite subgroups of $GL_{24}(\mathbf{Q})$. *Exp. Math. 5* (1996), 163–195.

[Neb 96a] —— Finite subgroups of $GL_n(\mathbf{Q})$ for $25 \leq n \leq 31$. *Comm. Alg. 24 (7)* (1996), 2341–2397.

[NeP 95] NEBE, G. and W. PLESKEN. Finite rational matrix groups of degree 16. *AMS Memoirs, vol. 116, No. 556* (1995).

[PlN 95] PLESKEN, W. and G. NEBE. Finite rational matrix groups. *AMS Memoirs, vol. 116, No. 556* (1995).

[Que 95] QUEBBEMANN, H.-G. Modular lattices in Euclidean spaces. *J. Number Theory 54* (1995), 190–202.

[Que 96] —— Atkin-Lehner eigenforms and strongly modular lattices. *L'Ens. Math. 43* (1997), 55–65.

*(Reçu le 25 juin 1996; version révisée reçue le 22 novembre 1996)*

Gabriele Nebe

Lehrstuhl B für Mathematik
RWTH Aachen
Templergraben 64
52062 Aachen
Germany

*E-mail:* gabi@willi.math.rwth-aachen.de