THEOREM 8. *Let X be a Kählerian threefold and $h \in H^2(X, \mathbf{R})$ be a Kähler class. Then the map*

$$\begin{array}{rccc} \langle .,. \rangle : & H^2(X,\mathbf{R}) \times H^2(X,\mathbf{R}) & \longrightarrow & \mathbf{R} \\ & (a,b) & \longmapsto & a \cup b \cup h \end{array}$$

is a non-degenerate, symmetric bilinear form of signature $(2h^{2,0}+1, h^{1,1}-1)$.

One can restate this theorem in such a form as to obtain – at least in theory – some explicit inequalities in the coefficients of cubic polynomials which are satisfied by the cup forms of Kählerian and hence projective algebraic threefolds. The main result of [Sch2] is

THEOREM 9. *For $n \geq 4$, the polynomial*

$$x_0 \left(\frac{4-n}{4} x_0^2 - 3x_1^2 - \cdots - 3x_n^2 \right)$$

cannot occur as the (real) cup form of a projective algebraic threefold with $b_1 = 0$ and $b_3 = 0$.

As a corollary, one obtains the following generalization of a result of Campana and Peternell [CP]:

THEOREM 10. *For $n \geq 4$, twistor spaces over $\sharp_{i=1}^n \mathbf{P}_2$ are not homeomorphic to projective algebraic threefolds.*

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