Appendix 2: Restriction to the Temperley-Lieb algebra

Objekttyp: **Appendix**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **40 (1994)**

Heft 3-4: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **19.09.2024**

APPENDIX 2: RESTRICTION TO THE TEMPERLEY-LIEB ALGEBRA

The Temperley-Lieb algebra $P(n, \delta)$ (see §2) is contained (unitally) in $A(n, \delta)$ (indeed in $\overrightarrow{A(n, \delta)}$) by simply connecting the inside $*$ to the outside $*$, which reduces the rest of the annulus to a disc. The structure of $P(n, \delta)$ is very well known, particularly when it is semisimple (see [GHJ], and [GW] in the non-semisimple case). This structure is very easily re-obtained by the method of this paper. We have that there is one irreducible representation of $P(n, \delta)$ for each t, $0 \leqslant t \leqslant n$, $t + n$ even, of dimension $\binom{n}{\frac{n-t}{2}} - \binom{n}{\frac{n-t-2}{2}}$.

Call these representations ψ_t.

THEOREM. *For* $t > 0$,

$$\pi_{t,\omega \mid P(n,\delta)} = \bigoplus_{\substack{t \leqslant k \leqslant n \\ k + t \text{ even}}} \psi_k$$

and when $t = 0$,

$$\pi_{0 \mid P(n,\delta)} = \psi_0$$

(when both algebras are semisimple).

This is easily proved by induction using Theorem 2.8 and Lemma 4.6. It is reassuring to note that the dimensions add up in an obvious way:

$$\dim(\pi_{t,\omega}) = \binom{n}{\frac{n-t}{2}} = \left\{\binom{n}{\frac{n-t}{2}} - \binom{n}{\frac{n-t}{2} - 1}\right\} + \left\{\binom{n}{\frac{n-t}{2} - 1} - \binom{n}{\frac{n-t}{2} - 2}\right\}$$
$$+ \cdots + \left\{\binom{n}{0} - \binom{n}{-1}\right\}.$$

Similarly one may check that the formulas for the traces of minimal idempotents add up.

Our first attempt to derive the structure of $A(n, \delta)$ was using the unital inclusion of the Temperley-Lieb algebra. The only stumbling block was in trying to show that the "trivial" representation ψ_n (of dimension 1) is actually contained in $\pi_{t,\omega}$.