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QUICK LOWER BOUNDS FOR REGULATORS OF NUMBER FIELDS

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ABSTRACT. A short and simple proof for Zimmert's lower bounds for regulators of number fields is presented.

1. Introduction.

Let K be an algebraic number field with r_1 real and $2r_2$ complex embeddings, let R denote its regulator and w its number of roots of unity. The purpose of this note is to present a surprisingly short proof of the following theorem.

THEOREM. *For any real number* $s > 1$ *one has*

$$\frac{R}{w} \geqslant \frac{s(s-1)}{e} \exp\left(-\frac{s}{s-1}\right) \gamma(s) \exp\left(-s\frac{\gamma'}{\gamma}(s)\right),$$

Here $\gamma(s) = 2^{-r_1}\Gamma(s/2)^{r_1}\Gamma(s)^{r_2}$, *where* $\Gamma(s)$ *denotes the gamma function, and* $\gamma'(s)$ *is the derivative.*

If we let $s = 4/3$ then we obtain

$$\frac{R}{w} \geqslant 0.00299 \cdot \exp(0.48 r_1 + 0.06 r_2) .$$

From this one deduces that regulators of number fields are bounded from below by an absolute constant and grow exponentially in the degree of K.

This result and an estimate similar to the one of the above theorem was first stated and proved by Zimmert [Z, Satz 3 and Korollar]. He proved the sharper estimate

$$\frac{R}{w} \geqslant \frac{s(2s-1)}{2e} \exp\left(-\frac{2s}{s-1}\right) \gamma(2s) \exp\left(-2s\frac{\gamma'}{\gamma}(s)\right)$$

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