interesting observation is, that in general the monomials of φ themselves will have a smaller order in t than φ.

6.7. THE CASE THAT f IS ARBITRARY.

If $f = f_1^{m_1} \cdots f_r^{m_r} g$ with f_i irreducible, $m_i \geqslant 2$ and g reduced, we still have that $f + \varepsilon\varphi$ has the diagram of f with the multiple arrows replaced. We know exactly which replacements are possible (see section 3.8). To find out what is the type of $f + \varepsilon\varphi$, it again suffices to investigate linking behaviour. Some possibilities that only become apparent when f_i and $f + \varepsilon\varphi$ are drawn in one diagram (that is the diagram of their product), have to be opted out by considering linking with cables which are known to be correct, using such valuations as $\upsilon^{(2)}$.

Although the tests become increasingly difficult, this gives a way to generalize theorem 6.5.

6.8. IOMDIN TYPE SERIES.

We end with a remark on series of the form $f + \varepsilon l^k$, where l is a linear form not tangent to any branch of f and $k \geqslant k_0$, the largest polar ratio of f. These series have been studied by Iomdin and Lê, see [Lê], not only in the curve case but for general dimensions. Siersma [Si] has given a formula for the Δ_* of these series. In the curve case this is just a special case of our results. Notice that:

$$\upsilon_i(l) = d_i k \quad \text{where} \quad d_i = e_l(\Sigma_i) = \Sigma_i \cdot l\,,$$
$$\upsilon_i^{(2)}(l) = 2d_i k\,.$$

We would like to stress again that these Iomdin type series are generally much coarser than our topological series: they are single indexed and for example the Milnor number increases with steps of $d = d_1 + \cdots + d_r$ within the series.

APPENDIX

In this appendix the EN-diagrams of the series of plane curve singularities listed in [AGV] are drawn.

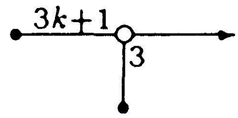
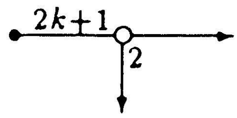
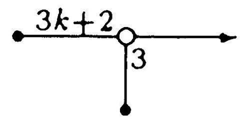
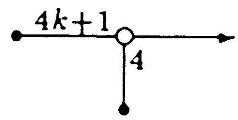
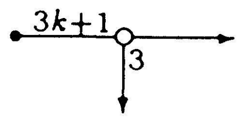
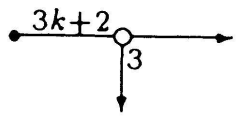
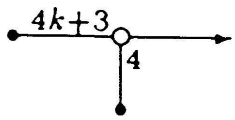
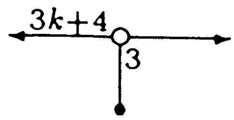
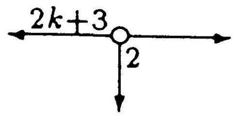
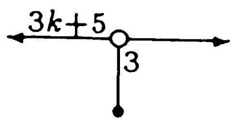
The first part consists of the *exceptional families E, W* and *Z*.

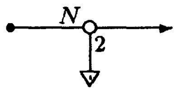
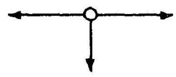
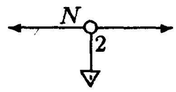
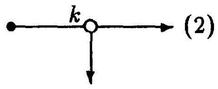
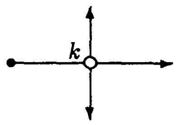
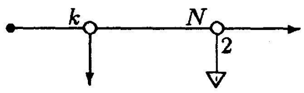
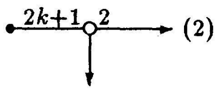
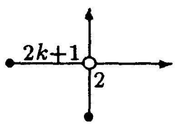
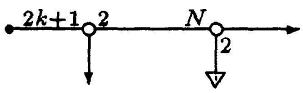
The second part contains the *infinite series A, D, J, W, $W^{\#}$, X, Y* and *Z*. All variants are given. In the tables, we have that:

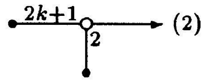
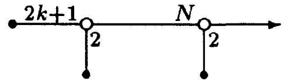
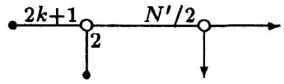
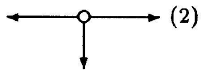
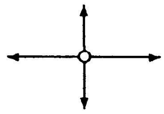
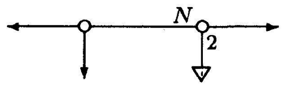
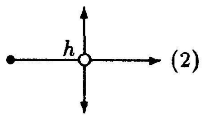
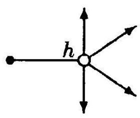
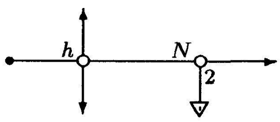
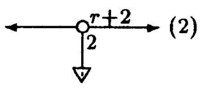
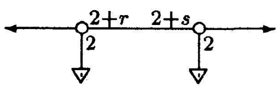
(a) μ = the Milnor number;

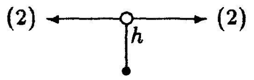
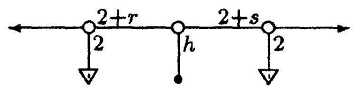
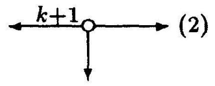
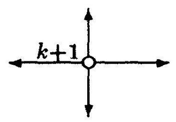
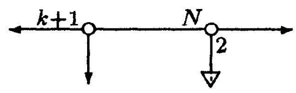
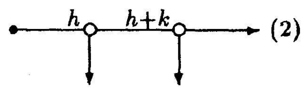
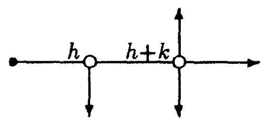
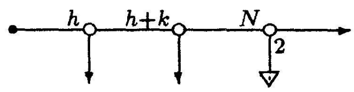
(b) N_0 and the graph constant c are as in theorem 3.4;

(c) Δ_*^∞ is the Δ_* of the non-isolated singularity, the Δ_* of an element of the series can be obtained by multiplying with $t^{N+c} - (-1)^N$.

Name	Formula	μ	EN-diagram
E_{6k}	$y^3 + x^{3k+1}$	$6k$	
E_{6k+1}	$y^3 + x^{2k+1}y$	$6k + 1$	
E_{6k+2}	$y^3 + x^{3k+2}$	$6k + 2$	
W_{12k}	$y^4 + x^{4k+1}$	$12k$	
W_{12k+1}	$y^4 + yx^{3k+2}$	$12k + 1$	
W_{12k+5}	$y^4 + yx^{3k+2}$	$12k + 5$	
W_{12k+6}	$y^4 + yx^{3k+3}$	$12k + 6$	
Z_{6k+11}	$x(y^3 + yx^{2k+3} + x^{3k+4})$	$6k + 11$	
Z_{6k+12}	$x(y^3 + yx^{2k+3} + x^{3k+5})$	$6k + 12$	
Z_{6k+13}	$x(y^3 + yx^{2k+4} + x^{3k+5})$	$6k + 13$	

Name	Formula	μ	EN-diagram	
A_∞	y^2	0		$\Delta_*^\infty = \dfrac{1}{t^2-1}$
A_0	y	0		$p \geq 2,\ N = p+1,$
A_1	y^2+x^2	1		$N_0 = 1,\ c = 0$
A_p	y^2+x^{p+1}	p		
D_∞	xy^2	1		$\Delta_*^\infty = 1$
D_4	xy^2+x^3	4		$p \geq 5, N = p-2,$ $N_0 = 2, c = 1$
D_p	xy^3+x^{p-1}	p		
$J_{k,\infty}$	$y^3+x^ky^2$	$3k-2$		$\Delta_*^\infty = \dfrac{t^{3k}-1}{t^3-1}$
$J_{k,0}$	$y^3+x^ky+x^{3k}$	$6k-2$		$k \geq 2, p \geq 1, c = k$ $N = p+2k, N_0 = 2k$
$J_{k,p}$	$y^3+x^ky^2+x^{3k+p}$	$6k-2+p$		
$W_{k,\infty}$	$y^4+y^2x^{2k+1}$	$8k+1$		$\Delta_*^\infty = \dfrac{t^{8k+4}-1}{t^4-1}$
$W_{k,0}$	$y^4+y^2x^{2k+1}+x^{4k+2}$	$12k+3$		$k \geq 1, p \geq 1,$ $N = p+2k+1$
$W_{k,p}$	$y^4+y^2x^{2k+1}+x^{4k+2+p}$	$12k+3+p$		$N_0 = 2k+1,$ $c = 2k+1$

Name	Formula	μ	EN-diagram	
$W^{\#}_{k,\infty}$	$(y^2+x^{2k+1})^2$	$4k$		$\Delta^{\infty}_{*}=\dfrac{t^{4k+2}+1}{t^4-1}$
$W^{\#}_{k,2q-1}$	$(y^2+x^{2k+1})^2+yx^{3k+1+q}$	$12k+2q+2$		$k\geq 1, q\geq 1$, $c=0$
$W^{\#}_{k,2q}$	$(y^2+x^{2k+1})^2+y^2x^{2k+1+q}$	$12k+2q+3$		$N\ =8k+2q+3$ $N'\ =8k+2q+4$
X_∞	$y^4+x^2y^2$	5		$\Delta^{\infty}_{*}=t^4-1$
X_9	$y^4+x^2y^2+x^4$	9		$p\geq 10, N=p-7$, $N_0=2, c=2$
X_p	$y^4+x^2y^2+x^{4+p-9}$	p		
$X_{h,\infty}$	$y^4+x^hy^3+x^{2h}y^2$	$8h-3$		$\Delta^{\infty}_{*}=\dfrac{(t^{4h}-1)^2}{t^4-1}$
$X_{h,0}$	$y^4+x^hy^3+x^{2h}y^2+x^{3h}y$	$12h-3$		$h\geq 2, p\geq 1$, $N=p+2h$, $N_0=2h, c=2h$
$X_{h,p}$	$y^4+x^hy^3+x^{2h}y^2+x^{4h+p}$	$12h-3+p$		
$Y_{\infty,\infty}$	x^2y^2	4		$\Delta^{\infty,\infty}_{*}=1$
$Y_{r,\infty}$	$y^{4+r}+x^2y^2$	$r+5$		$r,s\geq 1$, $c_1=c_2=2$
$Y_{r,s}$	$y^{4+r}+x^2y^2+x^{4+s}$	$9+r+s$		

Name	Formula	μ	EN-diagram	
$Y^h_{\infty,\infty}$		$4h-2$		$h \geq 2,\ r,s \geq 1$
$Y^h_{r,s}$	See [AGV], p. 248	$12h+r+s-3$		
$Z_{k,\infty}$	$xy^3 + x^{k+2}y^2$	$3k+5$		$\Delta^{\infty}_{*} = t^{3k+4} - 1$
$Z_{k,0}$	$xy^3 + x^{k+2}y^2 + x^{3k+4}$	$6k+9$		$k,p \geq 1, c = k+2$ $N = p + 2k + 2,$ $N_0 = 2k + 2$
$Z_{k,p}$	$xy^3 + x^{k+2}y^2 + x^{3k+4+p}$	$6k+9+p$		
$Z^h_{k,\infty}$		$8h+3k-3$		$\Delta^{\infty}_{*} = \dfrac{(t^{4h}-1)(t^{4h+3k}-1)}{t^4 - 1}$
$Z^h_{k,0}$	See [AGV], p. 249	$12h+6k-3$		$h \geq 2,\ k,p \geq 1,$ $N = p + 2h + 2k,$ $N_0 = 2h + 2k,$ $c = 2h + k$
$Z^h_{k,p}$	See [AGV], p. 249	$12h+6k-3+p$		

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