

# VIII. Applications and questions

Objektyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **36 (1990)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **19.09.2024**

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A particularly nice model occurs for index set in the form

$$\{-n, -n+2, \dots, n-2, n\} \quad \text{with} \quad \lambda = w.$$

This gives a series of one-variable specializations of the Homfly polynomial. (See [40], [58], [93].) *Is there a Yang-Baxter model for the full Homfly polynomial?* This is an open question.

A similar approach works for the Dubrovnik form of the Kauffman polynomial. See [58], [93]. The expansion formula has the appearance.

$$[\times] = z [\text{I}] - z [\text{II}] + w [\text{III}] + w^{-1} [\text{IV}] + [\times]$$

(It is understood that reversing the orientation of a line is accompanied by the negation of its spin.) Once again, the dot on a line means that it has smaller spin.

### VIII. APPLICATIONS AND QUESTIONS

This section is devoted to a few applications of the skein and state models and related questions.

1. Let  $\nabla_K$  denote the Conway polynomial. The skein model is embodied in the formula of section 6:

$$\nabla_K = \sum_{L, |L|=1} (-1)^{t(L)} z^{t(L)}$$

from which we see easily that

$$\max \deg \nabla_K \leq V - S + 1 = \rho(K)$$

where  $V$  is the number of crossings in the diagram  $K$ ,  $S$  is the number of Seifert circuits (the set of circuits obtained by splicing all crossings of  $K$ ). One knows that  $\rho(K) = \text{rank}(H_1(F))$  where  $F$  is the Seifert spanning surface [42] corresponding to the diagram  $K$ . If  $K$  is an alternating link then  $\max \deg \nabla_K = \rho(K)$  [76]. This is generalized to the class of alternative links in [42], using the FKT model. Is there a proof using the skein model?<sup>1)</sup>

In the case where all the crossings are of positive type, we see from the skein model that all terms of  $\nabla_K$  are positive, and it is then easy to see that the highest degree term is of degree  $\rho(K)$ .

<sup>1)</sup> Note added in proof: A proof using the skein model for the theorem on alternative links has been found by John Mathias — University of Maryland, May 1989.

2. Similar remarks apply to the Homfly model of section 3. In the case of the Yang-Baxter model for the Homfly polynomial given in section 7, it is easy to see that the highest  $z$ -degree is  $\rho(K)$  when  $K$  is positive — this time by constructing an appropriate spin state.
3. Thistlethwaite [89] proves that the writhe  $w(K)$  is an ambient isotopy invariant for  $K$  alternating and reduced. It would be useful to see a proof of this result using the skein model for  $D_K$  (section 4).
4. The Alexander polynomial  $\Delta_K$  is given by the formula

$$\begin{aligned}\Delta_K(t) &\doteq \nabla_K(\sqrt{t}-1/\sqrt{t}) \\ &= \sum_{|L|=1} (-1)^{t-(L)}(\sqrt{t}-1/\sqrt{t})^{t(L)}\end{aligned}$$

where  $\doteq$  denotes equality up to sign and powers of  $t$ . One knows ([23]) that if  $K$  bounds a smooth disk in the upper 4-space  $((x, y, z, t)$  with  $t > \theta$ ) then

$$\Delta_K(t) \doteq f(t)f(t^{-1})$$

for some polynomial  $f(t)$ . Can this fact be deduced directly from the skein model or from the FKT model? A solution should generalize to give new information about the full skein polynomial behaviours on slice links.

## IX. RELATIONS WITH MATHEMATICAL PHYSICS

I have deliberately included a description of the Yang-Baxter models in this paper in order to raise the question of the relation of the skein models to mathematical physics. The Yang-Baxter models can be regarded as averages of scattering amplitudes over all possible spin states — hence as discrete Feynman integrals, or as partition functions for two-dimensional statistical mechanics models. The FKT model for the Conway polynomial can be seen [57] as the low temperature limit of a partition function of a generalized Potts model.

### META-TIME

If we interpret the FKT model or the skein models in a particle interaction framework, then a curious and interesting issue arises:

Think of a particle moving forward and backward in “time” on a given universe. The “same” particle may traverse a given site (crossing) twice.