4. Ibn Labbn's arithmetic in brief

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are on the syllogism, the foundations of the Torah, and a commentary on the *Physics* of Aristotle [13].

The Arabic text is divided into two major books: the first is concerned with the fundamental operations using the decimal system while the other takes up the pure sexagesimal reckoning.

The Hebrew manuscript comprises the following twelve chapters: 1. numerals, 2. addition, 3. subtraction, 4. multiplication, 5. addition of the multiplication, 6. division, 7. remainder in division, 8. square root, 9. what comes from the root, 10. cube root, 11. what comes from the cube root, 12. checking by casting out nines. The first eight chapters are analogous to the first eight of the first book in the Arabic [14]. The twelfth Hebrew chapter is in the ninth and tenth (the last) sections of the first Arabic book. The subject of the tenth chapter of the Hebrew is found in the sixteenth section of the Arabic, book II. In most of the appropriate Hebrew chapters, an appendix discusses operations with the sexagesimal system. In this way, an attempt was made to cover the two books of ibn Labbān [15]. It is evident, therefore, that the integral sexagesimal system was in use by some people at this time.

The appreciation of the integral decimal system in the history of reckoning encountered quicker acceptance than has generally been supposed. This is seen in the reckoning treatise of al-Nasawī, the pupil of ibn Labbān. From the two extant texts of ibn Labbān's arithmetic, it is obvious that the author had written them in such an abbreviated style that it was difficult to understand when studied alone. Al-Nasawī's [16] text is essentially an elaboration of that of his teacher; it is very clear and practical and may be used without oral teaching.

4. Ibn Labbān's arithmetic in brief.

a) *Addition* (Chap. 2).

$$\text{Ex.}\quad \begin{array}{r} 5627 \\ \underline{482} \\ \hline \end{array}$$

The two amounts are written, like order to like, one above the other. Addition is begun on the left instead of on the right as in the present day method. In this example, the 4 is added to the 6 to give 10; a zero is set down and the 1 is carried to the 5. Next, the 8 is added to the 2 to give 10; a zero is set down and the 1 is carried to the row on the left. The addition is completed by adding the 7 to 2 and the 9 is set down in the proper place.

b) *Subtraction* (Chap. 3).

$$\begin{array}{r} \text{Ex. } 5627 \\ -\ 482 \\ \hline \end{array}$$

First, begin on the left and subtract 4 from 6 to give 2; then subtract 8 from 2 to give 4; and 2 from 7 to give 5. Now correct for the borrowed 1 to give 1 instead of 2 in the third order from the right.

Mediation is considered as part of subtraction just as duplation is one of addition.

c) *Multiplication* (Chap. 4).

Set down the multiplicand and under it the multiplier, the first order of the lower under the last order of the upper.

$$\begin{array}{r} \text{Ex.} \quad 325 \\ \times\ 243 \\ \hline \end{array}$$

First, multiply 3 of the upper line, by 2 of the lower line, then 4, then 3 of the lower line. As each is multiplied, the new subproduct is formed. Each one is erased on the dust board as it is no longer needed and the new digit is substituted for it. The result in each case is placed on each order of the upper number in apposition to these upper orders. Or, it may be done in reverse, for example, 243 is multiplied by 3, then $3 \times 3 = 9$. The 9 is set in the place of the upper 3; then $3 \times 4 = 12$; a 2 is placed above the 4 and a 1 is carried. Then $3 \times 2 = 6$; a 6 plus 1 or 7 is placed above the lower 2. This gives

$$\begin{array}{l} 72925 \\ 243 \end{array}$$

Next, shift the orders left as shown below and multiply 243 by 2. First $2 \times 3 = 6$. This 6 is set in the place of the upper 2.

Then $2 \times 4 = 8$, and $8+9 = 17$. Set down the 7 in place of the 9 and carry the 1. Then $2 \times 2 = 4$; $4+2+1$ (carried) $= 7$

77765
243

Then move the upper line to the left one place. Then $5 \times 3 = 15$. Put the 5 in place of the 5 and carry 1; $5 \times 4 = 20$, $20+1+6 = 27$. Put 7 in place of 6 and carry the 2. Then $5 \times 2 = 10$; $10+2+7 = 19$.

78975
243

The answer is in the upper line.

d) *Division* (Chap. 6).

Set down the dividend and under it the divisor, the last order of the lower under the last order of the upper number.

Ex. 5627
243

First, find the first digit of the quotient; it is 2. Multiply the 243 by 2 and then subtract it from 562 to give 76. This is shown:

2
767
243

The lower number is shifted one to the right and this process is repeated to give 3. The final appearance of the problem is

23
38
243

or 23 and 38 as a remainder.

e) *Taking the square root* (Chap. 8) [17].

Ex. The root of the number 65342.

First, from the right, mark off the number by twos; 6 is left over

65342

The largest square to go into 6 is 4; its root is 2. Subtract the 4 from the 6 and set down a 2 in place of the 6. Then since the root is 2, put a 2 above the 6 and below it as follows:

2
25342
2

Double the lower 2, then move it one order to the right to give:

2
25342
 4

Then [18], obtain a number such that when it is multiplied by itself and then by 4, the product can be subtracted from 253. The number is 5. Write it as follows:

25
25342
 45

Now multiply 45 by 5 and subtract it from 253; 28 remains. It is written as follows:

25
2842
45

This process is continued [19].

f) *Taking the cube root* (Chap. 10 [29].

Ex. 2986100

Mark off this number by threes going from right to left; 2 remains on the left. The closest cube root to 2 is 1. Put a 1 over the 2 and under it twice. Subtract 1 from 2 to give:

```
1
2986100
1
1
```

Now, double the lowest 1 to give 2; multiply this 2 by the uppermost 1 (giving 2), and then adding the 1 of the third line. Then the 1 of the uppermost line is added to the 2 of the fourth; this gives:

```
1
1986100
 3
  3
```

Now, a number is desired which when multiplied by the lower 3 and then by itself and both these products are added, then added to the amount in the third line, then this sum is multiplied by the desired number, and the product can be subtracted from the part of the number (2986100) concerned. The number is 4—put the 4 after the lower 3 to get 34; then multipy by 4 to get 136; then add the 3 to get 436; multiply the whole thing (the 436) by 4 to get 1744. Subtract this from 2986

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 1 4
242100
436
 34
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here, $436 = 3a^2+(3a+b)\,b = 3a^2+3ab+b^2$; then $34 = 3a+b$.

Now, double the 4 in the lowest line to give 8; add 8 to the 30 of the lowest line to give 38. Now multiply 38 by 4 of the uppermost line to give 152. Add it to 436 to give 588; add the upper 4 to the 38 to give 42. Move the third line one place to the right and the fourth line two places to the right to get

```
1 4
242100
 588
   42
```

In this figure, $588 = 436+152 = 3a^2+3ab+b^2+(3a+2b)\,b = 3a^2+6ab+3b^2 = 3\,(a+b)^2$; $42 = 3\,(a+b)$ [21]. Now, a third number (of the cube root) is sought as was the second; this gives 4 which is placed in the first line after the 14 and also after the 42 on the lowest line. Multiply the 424 of the lowest line by 4 to give 1696 and add to the third line to give 60496; multiply by 4 and subtract it from the remainder to give 116. This is shown in the diagram:

144 [22]
116
60496
424

Now, double the 4 on the right in the lowest line to give 8 or 428. Multiply this by the 4 on the right in the upper line to give 1712. Add this to the 60496 to give 62208; add 1 to the third line. The diagram is then:

144 = cube root;
116 = the remainder or 116 parts of 62209 according to ibn Labbān.

62209 [24]
428

5. Ibn Labbān's influence.

The fundamental operations of ibn Labbān are to be found reproduced almost exactly, although in much greater detail, in al-Nasawī. In the Arabic manuscript, there is a paragraph in which al-Nasawī remarks on his debt to his great teacher. The origin of ibn Labbān's algorisms is unknown. They are not so radically different from those of Indian sources to claim them as independent inventions. The main difference between the Indian algorisms and those of ibn Labbān seems to be in the shortened process effected by the latter. For example, $\{3a^2+(3a+b)\,b\}\,b$, in the cube root process, is calculated at one time to shorten the work instead of working out $3a^2\,b$, $3ab^2$, and