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Autor(en): **Arzhantseva, Goulnara N.**

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THE UNIFORM KAZHDAN PROPERTY FOR $\mathrm{SL}_n(\mathbf{Z})$, $n \geq 3$

by Goulnara N. ARZHANTSEVA

Let Γ be a discrete group, and let S be a finite subset of Γ. For a unitary representation π of Γ in a separable Hilbert space $\mathcal{H}$ we define the number

$$K(\pi, \Gamma, S) = \inf_{0 \neq u \in \mathcal{H}} \max_{s \in S} \frac{\|\pi(s)u - u\|}{\|u\|}.$$

Then the *Kazhdan constant* of Γ with respect to S is defined as

$$K(\Gamma, S) = \inf_{\pi} K(\pi, \Gamma, S),$$

where the infimum is taken over unitary representations π having no invariant vectors. We also define the *uniform Kazhdan constant* of Γ as

$$K(\Gamma) = \inf_{S} K(\Gamma, S),$$

where the infimum is taken over all finite generating sets S of Γ.

A group Γ is said to have *Kazhdan property* (T) (or to be a *Kazhdan group*) if there exists a finite subset S of Γ with $K(\Gamma, S) > 0$. A group Γ is *uniform Kazhdan* if $K(\Gamma) > 0$.

Shortly after its introduction by David Kazhdan in the mid 60's, property (T) was used by Gregory Margulis to give a first explicit construction of infinite families of expander graphs of bounded degree. In particular, a major problem of practical application in the design of efficient communication networks was solved.

A classical example of a Kazhdan group is the group $\mathrm{SL}_n(\mathbf{Z})$ for $n \geq 3$ (for more details and a general context of locally compact groups see the recent book [2]). Surprisingly, the following question is still open.

QUESTION 3.1. *Is the group $\mathrm{SL}_n(\mathbf{Z})$, for $n \geq 3$, uniform Kazhdan?*

Infinite finitely generated uniform Kazhdan groups were discovered very recently, see [6] or [1]. However, these groups are neither finitely presented nor residually finite. The latter construction provides an infinite uniform Kazhdan group that weakly (see [4]) contains an infinite family of expanders in its Cayley graph.

An affirmative answer to the above question would give, in particular, the first example of a residually finite (and, in addition, finitely presented) infinite uniform Kazhdan group. It is crucial for applications: infinite families of expanders could be constructed independently of the choice of the group generating set.

A negative answer would be interesting as well. In that case, this classical group would belong to the class of non-uniform Kazhdan groups. First examples of such groups were obtained using Lie groups (see [3]). Then, all word hyperbolic groups were also shown to have zero uniform Kazhdan constant (see [5]).

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Goulnara N. Arzhantseva

Département de Mathématiques
Université de Genève
2-4, rue du Lièvre
CH-1211 Genève 4
Switzerland
e-mail: goulnara.arjantseva@math.unige.ch