part of the issue. The second part consists of particular question(s) that serve the purpose of pinpointing some crucial aspects of the challenge that deserve to be dealt with in the Study. These are set out in boxes for the respective issues identified in §3.

From the viewpoint of this Study, an issue concerning applications and modelling in mathematics education may be viewed and approached – depending on its nature – from a variety of different *perspectives*, each indicating the category of answers sought.

- *doing:* actual teaching and learning practice (enacted or potential) as carried out in classrooms.
- *development and design:* for example curriculum design, teaching, learning, and assessment materials or activities, and so forth.
- *research:* focus is on the generation of answers to questions as yet unanswered.
- *policy:* focus is on the strategies and policies that exist or ought to be adopted in order to place matters pertaining to applications and modelling on the appropriate agenda.

A given issue may be addressed from any or all of these four perspectives, which are not intended to convey a hierarchy of importance.

3. Examples of important issues

In this section a number of selected *issues* – consisting of *challenges* and *questions* – are raised. Although certain inherent features have influenced their grouping, there is overlap between them and different groupings are certainly possible. They are intended as a guide to the kinds of issues that the present Study intends to address, and readers are invited to identify additional relevant issues.

3.1 Epistemology

Different characterisations of modelling and applications include: posing and solving open-ended questions, creating, refining and validating models, mathematising situations, designing and conducting simulations, solving word problems and engaging in applied problem solving. These present challenges if individuals are to engage successfully in applications and modelling activities.

Issue 1. *Which aspects of applications and modelling invite further epistemological analysis? How is the relationship between applications and modelling and mathematics best described? What is the relationship between applications and modelling and the world we live in?*

Examples of specific questions that could be addressed here are:

- What are the process components of modelling? What is meant by or involved in each?
- How does our knowledge of applications and modelling accumulate, evolve and change over time?
- What are the various meanings of 'authenticity' in modelling?

3.2 Application problems

There exists a plethora of applications and modelling problems and materials for use in mathematics classrooms at various educational levels. These materials range from mere 'dressed up' mathematical problems to authentic problem situations.

Issue 2. *What does research have to tell us about the significance of authenticity to students' acquisition and development of modelling competency?*

Examples of specific questions:

- What authentic applications and modelling materials are available worldwide?
- Taking account of teaching objectives and students' personal situations (experience, competence), how can teachers set up authentic applications and modelling tasks?
- How does the authenticity of problems and materials affect students' ability to transfer acquired knowledge and competencies to other contexts and situations?

3.3 Modelling abilities and competencies

With the teaching and learning of mathematical modelling and applications, many goals and expectations are combined.

Issue 3a. *How can modelling ability and modelling competency be characterised, and how can it be developed over time?*

Examples of specific questions:

- Can specific subskills and subcompetencies of 'modelling competency' be identified?
- How can modelling ability be distinguished from general problem solving abilities?
- Are there identifiable stages in the development of modelling ability?
- What are characteristic differences between expert modellers and novice modellers?
- What is the role of pure mathematics in developing modelling ability?

Issue 3b. *How can modelling in teacher pre-service and in-service education courses be promoted?*

Examples of specific questions:

- What is essential in a teacher education programme to enable prospective teachers to experience real, non-trivial modelling situations, and hence acquire modelling competencies for purposes of teaching applications and modelling in their professional future?
- Which training strategies can help teachers develop security in using applications and modelling in their teaching?

3.4 Beliefs, attitudes, and emotions

Beliefs, attitudes and emotions play important roles in the development of critical and creative senses in mathematics.

ISSUE 4. *To what extent does applications and modelling have the potential to provide an environment to support both students and teachers in their development of appropriate beliefs about and attitudes towards mathematics ?*

Examples of specific questions:

- What are the implications of research on the role of beliefs, attitudes and emotions for changing teaching practice and classroom cultures with respect to applications and modelling ?
- What strategies are feasible for in-service teacher education that will address the fear experienced by some teachers when faced with applications and modelling ?

3.5 CURRICULUM AND GOALS

It is argued that applications and modelling can make fundamental contributions to the development of students' mathematical competencies.

ISSUE 5a. *What would be an appropriate balance – in terms of attention, time and effort – between applications and modelling activities and other mathematical activities in mathematics classrooms at different educational levels ?*

Examples of specific questions:

- Is it possible – or desirable – to identify a core curriculum in applications and modelling within the general mathematical curriculum ?
- Which applications, models and modelling processes should be included in the curriculum ? Do answers depend on each teacher or should there be some minimal indications in national and state curricula ?
- Is it beneficial to generate specific courses or programs on applications and modelling or is it better to integrate applications and modelling into standard mathematical courses ?

The university level represents a particularly problematic case. Although there are differences between places and countries, university graduates in mathematics embark on a large variety of professional careers, many of which will have links involving applications and modelling – including research mathematicians through their research or teaching responsibilities.

ISSUE 5b. *Should all university graduates in mathematics acquire some applications and modelling experiences as part of their studies ? If so, what kinds of experiences should they be ?*

Concerning general education at the school level, some special questions arise. Mathematics accounts for a large proportion of time in school – this is only justified if mathematics can contribute to general education for life after school.

ISSUE 5c. *How and to what extent can applications and modelling contribute to building up fundamental competencies and to enriching a student's general education ?*

Examples of specific questions:

- What meanings can be given to 'general education', and what is the role of mathematical modelling therein ?

- What is a suitable balance within general education between creating one's own models of real situations and problems, and making judgements about models made by others ?

3.6 Modelling pedagogy

The pedagogy of applications and modelling intersects the pedagogy of pure mathematics in a multitude of ways and requires at the same time a variety of practices that are not part of the traditional mathematics classroom. Approaches to teaching applications and modelling vary from the use of traditional methods and course structures, to those that include a variety of innovative teaching practices.

Issue 6. *What are appropriate pedagogical principles and strategies for the development of applications and modelling courses and their teaching ? Are there different principles and strategies for different educational levels ?*

Examples of specific questions:

- What research evidence is available to inform and support the pedagogical design and implementation of teaching strategies for courses with an applications and modelling focus ?
- What criteria are most helpful in selecting methods and approaches suggested by theories of human development and/or learning ?
- What obstacles appear to inhibit changes in classroom culture e.g. the introduction of interactive group work in applications and modelling ?
- What criteria can be used to choose the most desirable option at a particular point within an applications and modelling teaching segment (e.g. whether to use individual and group activity) ?

3.7 Sustained implementation

To sustain change in an educational system is a major challenge as it involves and impacts upon many different parties, including politicians, curriculum developers, teachers, teacher educators, and mathematics faculty members at the post secondary level.

Issue 7. *In spite of a variety of existing materials, textbooks, etc., and of many arguments for the inclusion of modelling in mathematics education, why is it that the actual role of applications and mathematical modelling in everyday teaching practice is still rather marginal, for all levels of education ? How can this situation be reversed to ensure that applications and mathematical modelling is integrated and preserved at all levels of mathematics education ?*

Examples of specific questions:

- What are the major impediments and obstacles that have existed to prevent the introduction of applications and mathematical modelling, and how can these be changed ?
- What are the requirements for developing and sustaining a mathematical modelling environment in traditional courses at school or university ?
- How can it be ensured that the mathematical modelling philosophy in curriculum documents is mirrored in classroom practice ?

3.8 ASSESSMENT AND EVALUATION

The teaching and learning of mathematics at all levels is closely related to assessment of student achievement. There seem to be many indications that the assessment modes traditionally used in mathematics education are not fully appropriate to assess students' modelling competency.

ISSUE 8a. *What alternative assessment modes are available to teachers, institutions and educational systems that can capture the essential components of modelling competency, and what are the obstacles to their implementation ?*

Examples of specific questions:

- In assessing mathematical modelling as a process (instead of a product) what can be learnt from assessment in the arts, music, etc. ?
- When mathematical modelling is introduced into traditional courses at school or university, how should assessment procedures be adapted ?
- When state or national centralised testing of students is implemented, how do we ensure that mathematical modelling is assessed validly ?
- How does one reliably assess individual contributions and achievement within group activities and projects ?

There is a need to develop specific means of evaluating programmes with an applications and modelling content.

ISSUE 8b. *What evaluation modes are available that can capture the essential features of applications and modelling, especially of integrated courses, programmes and curricula, and what are the obstacles to their implementation ?*

Examples of specific questions:

- In what way do usual evaluation procedures for mathematical programmes carry over to programmes that combine mathematics with applications and modelling ?
- What counts as success when evaluating outcomes from a modelling programme ? For example, what do biologists, economists, industrial and financial planners, medical practitioners, etc., look for in a student's mathematical modelling abilities ? How does one establish whether a student has achieved these capabilities ?

3.9 TECHNOLOGICAL IMPACTS

Many technological devices are available today and many of them are highly relevant for applications and modelling. In a broad sense these technologies include calculators, computers, the Internet and computational or graphical software, as well as various kinds of instruments for measuring, experimenting, etc. These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment.

ISSUE 9. *How should technology be used at different educational levels to effectively develop students' modelling abilities and to enrich the students' experience and capability with open-ended mathematical situations in applications and modelling ?*

Examples of specific questions:

- What implications does technology have for the range of applications and modelling problems that can be introduced?
- How is the culture of the classroom influenced by the presence of technological devices? Does technology compromise thinking and reflection or can these be enhanced by technology? In what ways?
- What evidence of successful or failed practice in teaching and learning applications and modelling has been documented as a direct consequence of the introduction of technology?
- With respect to non-affluent countries: can applications and modelling be successfully undertaken without the availability of technology?
- What are the implications of the availability of technology for the design of assessment items and practices?

4. Call for contributions to the Study

The Study Conference will be held in Dortmund (Germany) on February 13–17, 2004. Participation in the Study Conference is by invitation only, given on the basis of a submitted contribution, and is limited to approximately 75 people. The Study Volume, to be published in the ICMI Study Series, will contain selected contributions and reports prepared for the conference, as well as on the outcomes of the conference.

The International Programme Committee (IPC) for the Study invites submission of contributions on specific questions, problems or issues related to this Discussion Document. Contributions, in the form of synopses of research papers, discussion papers or shorter responses, may address questions raised above, or questions that arise in response, or further issues relating to the theme of the Study. Submissions should not exceed 6 pages in length and should reach the Programme Chair at the address below (preferably by e-mail) no later than June 15, 2003, but earlier if possible. All submissions must be in English, the language of the conference.

The members of the International Programme Committee for this Study are:

Werner BLUM (University of Kassel, Germany), *Chair of the IPC*, Claudi ALSINA (University of Technology, Barcelona, Spain), Maria Salett BIEMBENGUT (University of Blumenau, Brazil), Nicolas BOULEAU (École Nationale des Ponts et Chaussées, Marne-la-Vallée, France), Jere CONFREY (University of Texas-Austin, USA), Peter GALBRAITH (University of Queensland, Brisbane, Australia), Toshikazu IKEDA (Yokohama National University, Japan), Thomas LINGEFJÄRD (Gothenburg University, Sweden), Eric MULLER (Brock University, St. Catharines, Canada), Mogens NISS (Roskilde University, Denmark), Lieven VERSCHAFFEL (University of Leuven, Belgium), Shangzhi WANG (Capital Normal University, Beijing, China), Bernard R. HODGSON (Université Laval, Québec, Canada), *ex officio, representing the ICMI Executive Committee*, Hans-Wolfgang HENN (University of Dortmund, Germany), *Chair of the Local Organising Committee*.

For further information and submission of contributions, please contact the Chair of the IPC:

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