2. Framework for the Study

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **49 (2003)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **28.04.2024**

2. Framework for the Study

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a *structuring framework*. This is particularly important in an area as complex and difficult to survey as the teaching and learning of mathematical modelling and applications, for this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, but it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model.

2.1 Concepts and notions

It is recognised that within the field variations exist with respect to the meaning and interpretation of some technical terms. Hence we give some working definitions that will indicate meanings intended within this document.

By *real world* we mean everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics. Starting with a certain problematic *situation* in the real world, simplification and structuring leads to the formulation of a *problem* and thence to a *mathematical model* of the problem. Here we use the term problem to encompass both basic practical problems, and problems of an intellectual nature aimed at describing, explaining, understanding or even designing parts of the world. The situation – still a part of the real world in our sense – is then *mathematised* – that is the relevant objects, data, relations and conditions involved in it are translated into mathematics, resulting in a *mathematical model* of the original situation.

Now mathematical methods are used to derive *mathematical results*, which when re-translated into the real world, can be interpreted in relation to the original situation. At this point the problem solver *evaluates* the model by checking whether the problem solution is appropriate and reasonable for his or her purposes. If need be, as often occurs with 'genuine' real-world problems, the whole process has to be repeated with a modified or a totally different model. Finally the ultimate solution of the original real world problem is stated and communicated. It has become common practice to use the term *mathematical modelling* for the entire process consisting of structuring, mathematising, working mathematically and interpreting, validating, revisiting and reporting the model.

Sometimes the given problem situation is already pre-structured or is essentially a 'dressing up' of a purely mathematical problem in the words of a segment of the real world. This is often the case with classical school *word problems*. In this case mathematising means merely 'undressing' the problem, and the 'modelling process' extends no further than some use of mathematics and a simple interpretation.

An *application* of mathematics is sometimes used to describe any kind of linking of the real world and mathematics, for example beginning with a standard piece of analysis and showing how it can be applied to address a problem in another discipline. This is sometimes described as the *use of standard models*, as distinct from *mathematical modelling* that represents a complete process in the sense described above. During the last decade the term '*applications and modelling*' has been increasingly used to encompass all kinds of relationships between the real world and mathematics.

2.2 Structure of the topic applications and modelling in mathematics education

We can conveniently represent the 'reality' of applications and modelling in mathematics education as being constituted essentially by two dimensions.

The *first* dimension contains three different domains, each forming some sort of a continuum. The *first* domain consists of the essential *notions of applications and modelling*, i.e. what we mean by an application of mathematics, and by mathematical modelling; their most important components in terms of concepts and processes; epistemological characteristics of applications and modelling vis-à-vis mathematics as a discipline and vis-à-vis other disciplines and areas of practice; who uses mathematics, and for what purposes; etc. The *second* domain is that of the *classroom*, used generically to indicate the location of teaching and learning activities pertaining to applications and modelling. This includes the classroom in a literal sense, but also includes the student doing his or her homework, individual or group activity, the teacher's planning of teaching activities or looking at students' products, and so forth. The *third* and final domain is the *system* domain. The word system, here, refers to the whole institutional, political, structural, organisational, administrative, financial, social, and physical environment that exerts an influence on the teaching and learning of applications and modelling.

The *second* dimension is constituted by the different educational *levels* within which the teaching and learning of applications and modelling are addressed. For the sake of generality we have stayed with the widespread designations of *primary*, *secondary*, and *tertiary* levels, together with the level of *teacher education.* If we then think of the dimensions as being depicted by orthogonal axes, this choice of structure enables us to locate issues in terms of the 4×3 cell matrix defined by the respective domains and levels. To clarify we provide an illustrative example.

Issue 0. An underlying reason for giving prominence to applications and modelling is the goal of students being able to apply mathematical knowledge in situations that are new to them. That is, it is argued that applications and modelling competency if well taught can be transferred to solve unfamiliar problems. However, several research studies suggest that for some students at least this transfer is rather limited in scope and range. A significant question is therefore:

To what extent is applications and modelling competency transferable across areas and contexts? What teaching/learning experiences are needed or suitable to foster such transferability?

As it stands, this issue concerns the applications and modelling domain, the classroom domain, and at least one of the educational levels. The issue is therefore identified within a rectangle whose base extends across two domains, and whose height is determined by the number of educational levels selected. When each issue is viewed in this way the total distribution of issues can be readily identified in terms of the matrix components, and priorities and gaps identified. In this Discussion Document, a number of issues have been identified as particularly significant to the present Study, and readers are invited to comment on these issues or to suggest further issues to be considered in the Study.

The formulation of the issue given above as an example consists of two parts. Firstly, a background part outlining a challenge, i.e. a dilemma or a problem, which may be of a political, practical, or intellectual nature – we call this part the *challenge* part of the issue. The second part consists of particular question(s) that serve the purpose of pinpointing some crucial aspects of the challenge that deserve to be dealt with in the Study. These are set out in boxes for the respective issues identified in §3.

From the viewpoint of this Study, an issue concerning applications and modelling in mathematics education may be viewed and approached – depending on its nature – from a variety of different *perspectives*, each indicating the category of answers sought.

- *doing:* actual teaching and learning practice (enacted or potential) as carried out in classrooms.
- *development and design:* for example curriculum design, teaching, learning, and assessment materials or activities, and so forth.
- *research:* focus is on the generation of answers to questions as yet unanswered.
- *policy:* focus is on the strategies and policies that exist or ought to be adopted in order to place matters pertaining to applications and modelling on the appropriate agenda.

A given issue may be addressed from any or all of these four perspectives, which are not intended to convey a hierarchy of importance.

3. Examples of important issues

In this section a number of selected *issues* – consisting of *challenges* and *questions* – are raised. Although certain inherent features have influenced their grouping, there is overlap between them and different groupings are certainly possible. They are intended as a guide to the kinds of issues that the present Study intends to address, and readers are invited to identify additional relevant issues.

3.1 Epistemology

Different characterisations of modelling and applications include: posing and solving open-ended questions, creating, refining and validating models, mathematising situations, designing and conducting simulations, solving word problems and engaging in applied problem solving. These present challenges if individuals are to engage successfully in applications and modelling activities.

Issue 1. *Which aspects of applications and modelling invite further epistemological analysis ? How is the relationship between applications and modelling and mathematics best described ? What is the relationship between applications and modelling and the world we live in ?*

Examples of specific questions that could be addressed here are:

- What are the process components of modelling ? What is meant by or involved in each ?
- How does our knowledge of applications and modelling accumulate, evolve and change over time ?
- What are the various meanings of 'authenticity' in modelling ?