1. Introduction

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AN ASYMPTOTIC FREIHEITSSATZ FOR FINITELY GENERATED GROUPS

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ABSTRACT. Given two fixed integers $k \geq 2$ and $l \geq 3$, let $\Gamma = \langle X \mid R \rangle$ be a presentation of the group Γ with $k = \#X$ generators and $l = \#R$ relations. We show that the following property of presentations of groups is generic in the sense of Gromov: for any $y \in X$, the subgroup of Γ generated by $X - \{y\}$ is free of rank $k-1$. This gives some generic estimates for the spectral radius of the adjacency operator in the Cayley graph of Γ relative to the generating system $S = X \cup X^{-1}$.

1. INTRODUCTION

The existence of free subgroups in some finitely generated group Γ gives some information about the structure of Γ. For example, it implies that Γ is non-amenable, and in particular that Γ has exponential growth. There are several results which ensure that various groups do have non-abelian free subgroups. For example:

THEOREM (Tits's alternative [15]). *Let Γ be a finitely generated linear group. Then either Γ is almost solvable or Γ contains a free subgroup on two generators.*

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THEOREM (Magnus's Freiheitssatz [12]). *Let* $\Gamma = \langle X \mid r \rangle$ *be a one relator group,* $x_0 \in X$ *be a generator of* Γ *that appears in the relation* r *and* r *be a cyclically reduced word in the free group* $\mathbf{F}_X$ *generated by* X*; then* $X - \{x_0\}$ *freely generates a free group in* Γ.

Our purpose in this work is to measure in some sense how frequent it is for a presentation $\Gamma = \langle X \mid R \rangle$ to be such that a proper subset of X is free in Γ. We prove the following result:

THEOREM 1.1. *Let* $\Gamma = \langle X \mid R \rangle$ *be a finite presentation with* k *generators,* l *relations and any fixed* x_0 *in* X*. Then the fact that* $X - \{x_0\}$ *freely generates a free group in* Γ *is generic in the sense of Gromov.*

The key idea is contained in proposition 4.1. Roughly speaking, if you choose at random l long relations and if the presentation satisfies a Dehn algorithm, then every generator x_0 will appear in every sufficiently long subword of every relation and hence it will appear in every product of conjugates of relations. So $X - \{x_0\}$ generates a free group in Γ.

In [6], the first author has shown that "X generates a free semi-group" is generic and that this implies bounds on the spectrum of the adjacency operator associated to the oriented Cayley graph of Γ relative to X. In section 5 below, we consider the adjacency operator h_S of the Cayley graph of Γ relative to $S = X \cup X^{-1}$, and we prove similarly estimates on the norm of h_S.

After finishing this paper we discovered that a result similar to Theorem 1.1 has been proved, using different methods, by G. Arzhantseva and A. Ol′shanskii in [1]. They employed a slightly different definition of the genericity and they proved that the small cancellation condition $C'(\lambda)$ is generic with respect to this new definition.

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