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THE NORMALISER ACTION AND STRONGLY MODULAR LATTICES

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ABSTRACT. We derive group theoretical methods to test if a lattice is strongly modular. We then apply these methods to the lattices of rational irreducible maximal finite groups.

1. INTRODUCTION

Let $L \subseteq \mathbf{R}^d$ be an even integral lattice in the Euclidean space of dimension d and let $L^\# \subseteq \mathbf{R}^d$ be its dual lattice. Let $\pi(L)$ be the set of all intermediate lattices $L \leq L' \leq L^\#$ that are inverse images of sums of Sylow subgroups of the finite abelian group $L^\#/L$. Then, L is said to be *strongly modular* if L is similar to L' for all $L' \in \pi(L)$ (cf. [Que 96]). Recall that L and L' are called *similar* if there exists $s \in GL(\mathbf{R}L)$ and $a \in \mathbf{R}_{>0}$ such that $Ls = L'$ and $(vs, ws) = a(v, w)$ for all $v, w \in \mathbf{R}L$, where $(\,,)$ denotes the Euclidean scalar product.

The automorphism group

$$G := \mathrm{Aut}(L) = \{g \in O(\mathbf{R}L) \mid Lg \subseteq L\}$$

is conjugate to a finite subgroup of $GL_d(\mathbf{Z})$. Since G acts as group automorphisms on $L^\#/L$ it preserves the lattices $L' \in \pi(L)$.

In Section 3 it is shown that the similarities $L' \to L$ normalise G. So one may use the normaliser

$$N_{GL_d(\mathbf{Q})}(G) := \{n \in GL_d(\mathbf{Q}) \mid n^{-1}gn \in G \quad \text{for all} \quad g \in G\}$$

*) Supported by the DFG.