1. WHY A STUDY ON THE TEACHING AND LEARNING OF MATHEMATICS AT UNIVERSITY LEVEL ?

1. Why a study on the teaching and learning of mathematics at university level?

A number of changes have taken place in recent years which have profoundly affected the teaching of mathematics at the university level. Five changes which are still having considerable influence are

(i) the increase in the number of students who are now attending tertiary institutions;

(ii) major pedagogical and curriculum changes that have taken place at pre-university level;

(iii) the increasing differences between secondary and tertiary mathematics education regarding the purposes, goals, teaching approaches and methods;

(iv) the rapid development of technology; and

(v) demands on universities to be publicly accountable.

Of course, all of these changes are general and have had their influence on other disciplines. However, because of its pivotal position in education generally, and its compulsory nature for many students, it could be argued that these changes have had a greater influence on mathematics than perhaps on any other discipline.

There is no doubt that, in many countries, significantly more students are now entering university and taking mathematics courses than was the case ten years or so ago. On the other hand, an increasingly smaller percentage of students appears to be opting for studies which require substantial amounts of mathematics. Thus university departments are faced with a double challenge. On the one hand, they have to cope with the influx of students whose preparation, background knowledge and even attitudes are quite different to those of past students. On the other hand, they have to attract students to pursue studies in mathematics, where employment opportunities and well-paying jobs appear not to be as certain as in some other disciplines.

Some new developments in the teaching and learning of mathematics attempt to come to grips with these issues. For example, alternative approaches to calculus and linear algebra in the United States reflect, in part, attempts to make these subjects more engaging and meaningful for the majority of students. There have been content changes too, with increased emphases in some universities on applications and modelling, history and philosophy of mathematics, and so on. But a general perception remains in some quarters that the teaching of mathematics at the undergraduate level has not to date made sufficient effort to deal with the backgrounds and needs of present day students.

There is also often perceived to be a discontinuity between mathematics education in secondary schools and mathematics education in universities. Certainly the levels of ambition and demand placed on students are increased at the tertiary level. There is not the same attention paid to learning theories in the delivery of university mathematics as there is in the teaching of the subject at lower levels. University teaching methods tend to be more conservative. Often university teachers have joint responsibility for research and teaching. This is clearly beneficial but it can cause more emphasis to be placed on mathematical research in places where that is the main criterion for promotion.

Teachers of university mathematics courses, on the whole, have not been trained to, and do not often consider educational, didactic or pedagogical issues beyond the determination of the syllabus; few have been provided with incentives or encouragement to seek out the results of mathematics education. In days gone by responsibility was placed largely on students' shoulders: it was assumed that faculty's responsibilities

were primarily to present material clearly, and that good students would pass and poor ones fail. The climate today is that academic staff are considered to have greater overall responsibility for students' learning. The role of instruction (specifically, of lectures) and staff accountability are being reconsidered.

Worldwide, increasing use is also being made of computers and calculators in mathematics instruction. Much mathematical software and many teaching packages are available for a range of curriculum topics. This, of course, raises such issues as what such software and packages offer to the teaching and learning of the subject, and what potential problems for understanding and reasoning they might generate. It would be good to collect examples of the use of information technology and software which enrich students' experience of mathematics and result in better understanding and learning.

Many academic mathematicians are aware of changes occurring around them, and of experimentation with different teaching approaches, but they have limited opportunities to embrace change owing to faculty structures and organisation. Further, the relationships between mathematicians in mathematics departments and their colleagues in mathematics education are often strained, with less productive dialogue between them than there might be. The same can be said of relationships between mathematicians and engineers, economists, etc., even though mathematics service teaching to students in other disciplines is an enormous enterprise. These general factors tend to work against, or delay, improvements in the teaching and learning of mathematics, particularly for those students whose main interests are in other disciplines.

As a result of the changing world scene, ICMI feels that there is a need to examine both the current and future states of the teaching and learning of mathematics at university level. The primary aim of this ICMI Study is therefore *to pave the way for improvements in the teaching and learning of mathematics at university level for all students.* To achieve this aim it is important for the professionals involved to

- exchange views and experiences from a wide variety of places and backgrounds;
- report about developments and projects that have taken place;
- consider the contributions from theory and research, and identify areas still to be investigated.

More specifically the Study will cover the following and related points:

◊ to identify, review, encourage, and disseminate research in educational matters at the tertiary level;

◊ to identify and describe major approaches to tertiary mathematics teaching within different cultures and traditions;

◊ to identify obstacles which might prevent the learning of mathematics;

◊ to discuss equity and other issues relating to mathematics education at university level;

◊ to discuss the goals of teaching mathematics to a range of students with different backgrounds and needs, and who should be responsible for that teaching;

◊ to find ways to meet changing needs without compromising the integrity of the subject;

◊ to identify, publicise, and expose to scrutiny, new teaching methods and the positive use of technology;

◊ to discuss the transition and the relations between secondary school and university;

◊ to consider ways to improve the preparation of teachers of mathematics at university level.

Leading up to and during the Conference relating to this Study, it is expected that there will be debate as to why mathematics is taught and what mathematics education is at university level. In addition, consideration will be given as to what is the current teaching and learning situation in universities, what it is believed that the situation should be, and how desired changes can be effected.

2. Themes and issues pertaining to research on the teaching and learning of mathematics at university level

Most academic mathematicians know little about the research that has been undertaken in mathematics education in general, or at the tertiary level in particular. Generally speaking, they are unaware of the methods used by researchers in education. One of the most valuable aspects of the current study is that it could collect together the major findings of mathematics education research, review them, and make them readily accessible to a wide audience. The potential usefulness and limitations of this research should then be considered in the light of the practice of teaching. At the same time, it would be valuable to determine research areas which have not yet been explored and to encourage work in them.

The following questions are of particular interest for the Study.

⁂ *What is mathematical understanding and learning, and how are these achieved? What are the underlying theories behind these and how do they relate to teaching at university level?*

⁂ *What research methods are employed in mathematics education? What are the major research findings of mathematics education? What are the obstacles to having teaching practice become informed/influenced by research findings?*

⁂ *Might insights into the nature of the learning process play out differently at different grade levels? Are the theories that are relevant at school level, relevant at university level as well? Is there a need for theories that are specific to university level?*

⁂ *What research has there been into traditional and alternative methods of teaching and what do the results of such research tell us?*

⁂ *In what ways can teaching change to take into account the different background, abilities and interests of the learner? What methods are effective for teaching large classes?*

⁂ *What do we know about the learning and teaching of specific topics such as calculus and linear algebra? Are there characteristics which are relevant to specific topics? Are there characteristics which are pertinent to a number of topics?*

⁂ *What alternative forms of assessment exist? How can assessment be used to promote better learning and understanding?*