we see

$$J(\omega_{\mathfrak{p}}^{-k_1}, \ldots, \omega_{\mathfrak{p}}^{-k_r}) \cdot \frac{q!}{(-p)^{\frac{q-1}{p-1}}} \equiv \frac{(k_1 + \cdots + k_r)!}{k_1! \cdot \ldots \cdot k_r!} \bmod \mathfrak{P} \,,$$

so the congruence

$$\frac{q!}{(-p)^{\frac{q-1}{p-1}}} = \frac{q!}{(-p)^{\mathrm{ord}_p(q!)}} \equiv H_p(q) = 1 \bmod p$$

settles Case 3.

Finally, in Case 4, Stickelberger's congruence and Lemma 3 imply that

$$\frac{J(\omega_{\mathfrak{p}}^{-k_1}, \ldots, \omega_{\mathfrak{p}}^{-k_r})}{(\zeta_p - 1)^{s_q(k_1) + \cdots + s_q(k_r) - s_q(k_1 + \cdots + k_r)}} \equiv \frac{h_q(k_1 + \cdots + k_r)}{h_q(k_1) \cdot \ldots \cdot h_q(k_r)} \bmod \mathfrak{P} \,,$$

so

$$\frac{J(\omega_{\mathfrak{p}}^{-k_1}, \ldots, \omega_{\mathfrak{p}}^{-k_r})}{(\zeta_p - 1)^{(p-1)\mathrm{ord}_p\left(\frac{(k_1 + \cdots + k_r)!}{k_1! \cdot \ldots \cdot k_r!}\right)}} \equiv \frac{(k_1 + \cdots + k_r)!}{k_1! \cdot \ldots \cdot k_r!} \cdot \frac{1}{(-p)^{\mathrm{ord}_p\left(\frac{(k_1 + \cdots + k_r)!}{k_1! \cdot \ldots \cdot k_r!}\right)}} \bmod \mathfrak{P},$$

since $s_q(k_i) = S_p(k_i)$ and $s_q(k_1 + \cdots + k_r) = S_p(k_1 + \cdots + k_r)$. Thus

$$J(\omega_{\mathfrak{p}}^{-k_1}, \ldots, \omega_{\mathfrak{p}}^{-k_r}) \equiv \frac{(k_1 + \cdots + k_r)!}{k_1! \cdot \ldots \cdot k_r!} \bmod \mathfrak{P} \,. \qquad \square$$

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