Another general question is:

QUESTION 3. *What residually finite groups have a faithful chaotic action on some smooth connected compact manifold?*

Clearly finite groups are residually finite but have no faithful chaotic actions on any connected compact manifold. On the other hand, if a group G acts faithfully and chaotically on a compact manifold, then is G necessarily countable?, finitely generated?, discrete as a subgroup of $\mathrm{Hom}(M)$?, closed as a subgroup of $\mathrm{Hom}(M)$? These properties hold for the known examples of chaotic actions constructed from the action of $SL(n, \mathbf{Z})$ on $\mathbf{T}^n$. The properties would seem unlikely to hold in general, but counterexamples have proved to be elusive. Notice that for a smooth compact manifold M, a discrete subgroup $G \leqslant \mathrm{Hom}(M)$ is necessarily countable, since $\mathrm{Hom}(M)$ is second countable. So on smooth compact manifolds one has the following implications:

$$\begin{tikzcd}
 & \text{finitely generated} \arrow[dl, Rightarrow] \arrow[dr, Rightarrow] & & \text{discrete} \arrow[dl, Rightarrow] \arrow[dr, Rightarrow] & \\
\text{Hopfian} & & \text{countable} & & \text{closed}
\end{tikzcd}$$

Notice that there is a simple partial result: Every topological group acting continuously, faithfully and chaotically on a Hausdorff space is totally pathwise disconnected. To see this, notice that if $G \subseteq \mathrm{Hom}(M)$ acts chaotically, then the only continuous paths in G are the constant paths. Indeed, if γ_t is a path in G and if x has finite orbit under G, then as $\gamma_t(x)$ is a continuous path in M and as $\gamma_t(x)$ belongs to the (finite) orbit of x, so $\gamma_t(x)$ is independent of t. (We remark in passing that it is easy to see that every manifold admits a non-discrete totally pathwise disconnected group of homeomorphisms.)

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Department of Mathematics
La Trobe University
Melbourne
Australia 3083
E-mail address: matgc@lure.latrobe.edu.au

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