

NOTE ON TABLE I OF "BARKER SEQUENCES AND DIFFERENCE SETS"

Autor(en): **Broughton, Wayne J.**

Objekttyp: **Article**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **40 (1994)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **29.04.2024**

Persistenter Link: <https://doi.org/10.5169/seals-61106>

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

A NOTE ON TABLE I OF "BARKER SEQUENCES AND DIFFERENCE SETS"

by Wayne J. BROUGHTON

In Table I of [EK], S. Eliahou and M. Kervaire show the non-existence of cyclic difference sets with parameters $(2t(t+1)+1, t^2, t(t-1)/2)$, for $3 \leq t \leq 100$, $t \neq 50$, leaving the case $t = 50$ undecided. The purpose of this note is to fill this gap and to generalize the table to non-cyclic difference sets. See any of [EK], [L], or [J] for definitions and notation.

To handle the case $t = 50$ we make use of a multiplier theorem due to McFarland (see [L], Theorem 5.24, p. 218, or [J], Theorem 4.7, p. 254). It refers to a function $M(z)$ which has $M(1) = 1$, and (for $z \geq 5$) is defined recursively to be the product of the distinct prime factors of the numbers

$$z, M\left(\frac{z^2}{p^{2e}}\right), p-1, p^2-1, \dots, p^{u(z)}-1,$$

where p is any prime dividing z with $p^e \parallel z$ and where $u(z) = (z^2 - z)/2$. (Note that the "definition" of M depends on the choice of p made for each z .)

PROPOSITION. *If D is an abelian (v, k, λ) -difference set in G , and m is a divisor of $n := k - \lambda$ such that $M(n/m)$ and v are co-prime, and if d is an integer co-prime with v such that for every prime $p \mid m$ there exists $f \geq 0$ with $p^f \equiv d \pmod{\exp(G)}$, then d is a numerical multiplier of D .*

Now when $t = 50$ we have $v = 5101$, a prime, (so $G = \mathbf{Z}_{5101}$), and $n = 1275 = 3 \cdot 5^2 \cdot 17$. Let $m = 3 \cdot 17$. So $n/m = 5^2$, and $M(5^2)$ has as factors the prime factors of

$$5^2, M(1), 5-1, 5^2-1, \dots, 5^{300}-1,$$

since $u(25) = 300$. But the multiplicative order of 5 modulo 5101 is 425, so $M(25)$ is not divisible by $v = 5101$. Moreover,

$$3^{1088} \equiv 17^1 \pmod{5101},$$

so by the proposition $d = 17$ is a multiplier of any $(5101, 2500, 1225)$ -difference set.

But the non-trivial orbits of multiplication by 17 in $\mathbf{Z}_{5101}$ are all of size 75, so it is impossible for a union of orbits to have size 2500 and hence no such difference set exists.

The primary non-existence theorem used in Table I of [EK] to eliminate difference sets is what they call the Semi-Primitivity Theorem (see Theorem 4.5 of [L] or Theorem 7.1 of [J]). Since this theorem actually applies to abelian difference sets (not just cyclic ones), it can also be used to eliminate almost all of the abelian difference sets in the range $3 \leq t \leq 100$. The only (non-cyclic) abelian case to which the theorem does not apply is $t = 49$, where the parameters are $(4901, 2401, 1176)$ and $n = 1225 = 35^2$. This is easily eliminated by Theorem 4.18 of [L]. Since $4901 = 13^2 \cdot 29$, we can (using Lander's notation) take a subgroup H in G of order $h = 29$, and let $m = 35$. So $m^2 \mid n$, and m is semi-primitive mod $|G/H| = 169$ since $5^{26} \equiv 7^{78} \equiv -1 \pmod{169}$; but by the theorem this implies $h \geq m$ (note the misprint in [L]), which is a contradiction.

Next, the only values of $t \in \{3, \dots, 100\}$ for which there exists a non-abelian group of order $v = 2t(t+1) + 1$ are $t = 26, 28, 36, 41, 48, 51, 52, 66, 73, 76, 86, 88, 96$, and 98 . In every one of these cases we can apply Theorem 4.4 of [L] (Theorem 7.6 in [J]), using the semi-primitivity relations already listed in Table I of [EK].

So we conclude that there do not exist *any* $(2t(t+1) + 1, t^2, t(t-1)/2)$ -difference sets for $3 \leq t \leq 100$.

We now point out a few misprints in Table I:

- (i) At $t = 12$, v should be "313" (a prime), not " $3 \cdot 13$ ".
- (ii) At $t = 17$, the semi-primitivity relation should read " $3^{51} \equiv -1 \pmod{613}$ ".
- (iii) At $t = 28$, the factorization for n should read " $2 \cdot 7 \cdot 29$ ".
- (iv) At $t = 61$, v should be " $5 \cdot 17 \cdot 89$ ".

S. Eliahou and M. Kervaire have also pointed out that on page 375 the polynomial $\theta_0(y)$ should read

$$y^3 + y^6 + y^7 + y^9 + y^{11} + y^{12} + y^{13} + y^{14}.$$

Finally, they also requested mention of the fact that at the time of writing [EK], they were not aware of the paper [C], which contains the complete classification of $(255, 127, 63)$ cyclic difference sets and should have been included in their bibliography.

REFERENCES

- [C] CHENG, U. Exhaustive Construction of $(255, 127, 63)$ -Cyclic Difference Sets. *J. Comb. Theory., Ser. A* 35 (1983), No. 2, 115-125.
- [EK] ELIAHOU, S. and M. KERVAIRE. Barker Sequences and Difference Sets. *L'Ens. Math.* 38 (1992), 345-382.
- [J] JUNGnickel, D. Difference Sets. In *Contemporary Design Theory*, J. H. Dinitz and D. R. Stinson, Ed., Wiley-Interscience (1992), 241-324.
- [L] LANDER, E. S. *Symmetric Designs: An Algebraic Approach*. London Math. Soc. Lecture Note Series 74, Cambridge University Press (1983).

(Reçu le 21 décembre 1993)

Wayne J. Broughton

Department of Mathematics
California Institute of Technology
Sloan 253-37
Pasadena, CA 91125
U.S.A.

Vide-leer-empty