6. KEY ISSUES AND CHALLENGES FOR THE FUTURE

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Computers have also made it possible to construct "virtual realities" and to generate interactively animations or marvellous pictures (e.g. fractal images). Moreover, electronic devices can be used to achieve experiences that in everyday life are either inaccessible, or accessible only as a resut of time-consuming and often tedious work.

Of course, in all these activities geometry is deeply involved, both in order to enhance the ability to use technological tools appropriately, and in order to interpret and understand the meaning of the images produced.

Computers can be used also to gain a deeper understanding of geometric structures thanks to software specifically designed for didactical purposes. Examples include the possibility of simulating traditional straightedge and compass constructions, or the possibility of moving basic elements of a configuration on the screen while keeping existing geometric relationships fixed, which may lead to a dynamic presentation of geometric objects and may favour the identification of their invariants.

Until now, school practice has been only marginally influenced by these innovations. But in the near future it is likely that at least some of these new topics will find their way into curricula. This will imply great challenges:

— How will the use of computers affect the teaching of geometry, its aims, its contents and its methods?

— Will the cultural values of classical geometry thereby be preserved, or will they evolve, and how?

— In countries where financial constraints will not allow a massive introduction of computers into schools in the near future, will it nevertheless be possible to restructure geometry curricula in order to cope with the main challenges originated by these technological devices?

6. Key issues and challenges for the future

In this section we list explicitly some of the most relevant questions which arise from the considerations outlined in the preceding sections. We believe that a clarification of these issues would contribute to a significant improvement in the teaching of geometry. Of course we do not claim that all the problems quoted below are solvable, nor that the solutions are unique and have universal validity. On the contrary, the solutions may vary according to different school levels, different school types and different cultural environments.

6.1. Aims

Why is it advisable and/or necessary to teach geometry?

Which of the following may be considered to be the most relevant aims of the teaching of geometry?

— To describe, understand and interpret the real world and its phenomena.

— To supply an example of an axiomatic theory.

— To provide a rich and varied collection of problems and exercises for individual student activity.

— To train learners to make guesses, state conjectures, provide proofs, and find out examples and counterexamples.

— To serve as a tool for other areas of mathematics.

— To enrich the public perception of mathematics.

6.2. Contents

What should be taught?

Is it preferable to emphasize "breadth" or "depth" in the teaching of geometry? And is it possible/advisable to identify a core curriculum?

In the case of an affirmative answer to the second question above, which topics should be included in syllabi at various school levels?

In the case of a negative answer, why is it believed that teachers or local authorities should be left free to choose the geometric contents according to their personal tastes (is this point of view common to other mathematical subjects, or is it peculiar to geometry)?

Should geometry be taught as a specific, separate subject, or should it be merged into general mathematical courses?

There seems to be widespread agreement that the teaching of geometry must reflect the actual and potential needs of society. In particular, geometry of three-dimensional space should be stressed at all school levels, as well as the relationships between three-dimensional and two-dimensional geometry. How could and should the present situation (where only two-dimensional geometry is favoured) therefore be modified and improved?

In which ways can the study of linear algebra enhance the understanding of geometry? At what stage should "abstract" vector space structures be introduced? And what are the goals?

Would it be possible and advisable also to include some elements of non-euclidean geometries into curricula?

6.3. Methods

How should we teach geometry?

Any topic taught in geometry can be located somewhere between the two extremes of an "intuitive" approach and a "formalized" or "axiomatic" approach. Should only one of these two approaches be stressed at each school level, or should there be a dialectic interplay between them, or else should there be a gradual shift from the former to the latter one, as the age of students and the school level progresses?

What is the role of axiomatics within the teaching of geometry? Should a complete set of axioms be stated from the beginning (and, if so, at what age and school level) or is it advisable to introduce axiomatics gradually, e.g. via a "local deductions" method?

Traditionally, geometry is the subject where "one proves theorems". Should "theorem proving" be confined to geometry?

Would we like to expose students to different levels of rigour in proofs (as age and school level progress)? Should proofs be tools for personal understanding, for convincing others, or for explaining, enlightening, verifying?

Starting from a certain school level, should every statement be proved, or should only a few theorems be selected for proof? In the latter case, should one choose these theorems because of their importance within a specific theoretical framework, or because of the degree of difficulty of their proof? And should intuitive or counterintuitive statements be privileged?

It seems that there is an international trend towards the teaching of analytic methods in increasingly earlier grades, at the expense of other (synthetic) aspects of geometry. Analytic geometry is supposed to present algebraic models for geometric situations. But, as soon as students are introduced to these new methods, they are suddenly projected into a new world of symbols and calculations in which the link between geometric situations and their algebraic models breaks down and geometric interpretations of numerical calculations are often neglected. Hence, at what age and school level should teaching of analytic geometry start? Which activities, methods and theoretical frameworks can be used in order to restore the link between the algebraic representation of space and the geometric situation it symbolizes?

How can we best improve the ability of pupils to choose the appropriate tools for solving specific geometric problems (conceptual, manipulative, technological)?

6.4. Books, computers, and other teaching aids

Are traditional textbooks as appropriate for teaching and learning geometry as we would like them to be?

How do teachers and pupils actually use geometry textbooks and other teaching aids? How would we like pupils to use them?

What changes could and should be made in teaching and learning geometry in the perspective of increased access to software, videos, concrete materials and other technological devices?

What are the advantages, from the educational and geometrical point of view, that can follow from the use of such tools?

Which problems and limitations may arise from the use of such tools, and how can they be overcome?

To what extent is knowledge acquired in a computer environment transferable to other environments?

6.5. Assessment

The ways of assessment and evaluation of pupils strongly influence teaching and learning strategies. How should we set out objectives and aims, and how should we construct assessment techniques that are consistent with these objectives and aims? Are there issues of assessment which are peculiar to the teaching and learning of geometry?

How does the use of calculators, computers and specific geometric software influence examinations as regards content, organization and criteria for the evaluation of the answers of the students?

Should assessment procedures be based mainly upon written examination papers (as it seems to be customary in many countries) or else what should be the role of oral communication, of technical drawing and of work with the computer?

What is it exactly that should be evaluated and considered for assessment: The solution outcome? The solution process? The method of thinking? Geometric constructions?

6.6. Teacher preparation

One essential component of an efficient teaching/learning process is good teacher preparation, as regards both disciplinary competence and educational, epistemological, technological and social aspects. Hence, what specific preparation in geometry is needed (and realistically achievable) for prospective and practicing teachers?

It is well known that teachers tend to reproduce in their profession the same models they experienced when they were students, regardless of subsequent exposure to different points of view. How is it then possible to motivate the need for changes in the perspective of teaching geometry (both from the content and from the methodological point of view)?

Which teaching supplies (books, videos, software, ...) should be made available for in-service training of teachers, in order to favour a flexible and open-minded approach to the teaching of geometry?

6.7. Evaluation of long-term effects

All too often the success (or failure) of a curricular and/or methodological reform or innovation for a certain school system is evaluated on the basis only of a short period of observation of its outcomes. Moreover usually there are no comparative studies on the possible side effects of a change of content or methods. Conversely, it would be necessary to look also at what happens in the long term. For instance:

— Does a visual education from a very young age have an impact on geometric thinking at a later stage?

— How does an early introduction of analytic methods in the teaching of geometry influence the visual intuition of pupils? When these pupils become professionals, do they rely more on the intuitive or on the rational parts of the geometry teaching to which they have been exposed?

— What is the impact of an extensive use of technological tools on geometry learning?

6.8. Implementation

At ICME 5 (Adelaide, 1984) J. Kilpatrick asked a provocative question: What do we know about mathematics education in 1984 that we did not know in 1980? Recently the same question has been picked up again in the ongoing ICMI study: "What is research in mathematics education, and what are its results". As for geometry, the possibility of relying on research results would be extremely useful in order to avoid reproposing in the future paths already proved unsuccessful, and conversely in order to benefit from successful solutions. And, as for still unsettled and relevant questions, we would like research to give us useful information in order to clarify the advantages and drawbacks of possible alternatives.

Hence, a key question might be:

What do we already know from research about the teaching and learning of geometry and what would we want future research to tell us?