3. Open problems

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In all the other cases, the moves of disc $n+1$ are determined by Lemma 7, the moves of the other n discs are governed by Lemmas 5 and 6, and their uniqueness follows by induction hypothesis, keeping in mind that the paths of Lemma 5 are actually paths from $\bar{\sigma}^d$ to $\hat{j}^{n-h_\sigma(d)}$. $\square$

Using the methods of this chapter, one finds the shortest path from σ to $\hat{0}$ in Figure 1 with length 102.

3. Open problems

Much of the discussion of the **TH** in computer science literature has been a controversy between recursion and iteration. It has turned out here that problems involving just regular states, can be solved by iteration very elegantly (Chapter 1). On the other hand, as soon as irregular states are considered, only recursive solutions are available (Chapter 2). While for $\mathfrak{P}3$ the solution is essentially unique and the recursion will work efficiently, the situation for $\mathfrak{P}4$ is less straightforward. Although the number of cases to be considered can be further limited by methods as in Section 2.3 (e.g. the shortest path (of length 108) from σ to r in Figure 1 is unique), and one can show that no three moves of the largest disc $n+1$ occur if $r(n+1) = t(n+1)$ and $h(n+1) > 1$, it is not clear whether there are shortest path problems with even three different solutions. Also it seems that the minimal length in $\mathfrak{P}s$ 3 and 4 can only be determined recursively.

The only existing solution to the **TH** with more than three pegs is also recursive, and the preceeding chapters should have demonstrated that things are not as easy as many authors might hope (see the remarks in the Introduction). To move the largest disc $n+1$ in the solution of $\mathfrak{P}0$ with four pegs, the n other discs have to be transferred to two different pegs; after the last move of disc $n+1$, discs 1 to n have to be sent from some two pegs to the top of disc $n+1$. Again it follows by symmetry that disc $n+1$ will only be moved once in a shortest path. But this time, this does not reduce the problem for $n+1$ discs to a similar one with only n discs, but to the different setting of how to transfer n discs from a perfect state to two different pegs in the shortest possible way. Here is where the hitherto unjustified assumption made in literature enters, namely that this will be achieved by dividing the perfect state in a suitable way into two parts, then first solving $\mathfrak{P}0$ for the smaller discs using four pegs, leaving them untouched thereafter and solving the old problem for the larger discs using three pegs only.

The validity of this hypothesis is the most interesting open problem about the **TH**. It might be found by checking a suitable guess about the minimal length for $\mathfrak{P}2$ with four pegs against the recursive solution which can easily be constructed using the fact, proved as Lemma 1, that the largest disc will not move more than three times.

In contrast to this recursive solution, the use of the hypothesis leads to a very elegant iterative solution to $\mathfrak{P}0$ with four (or more) pegs (see Hinz [26]), resembling algorithm i in 1.2.1, with the astonishing result that the transfer of 64 discs can be carried out in less than 6 hours (compare the time needed with three pegs, indicated in the Introduction!).

To conclude, it can be said that the invention of Edouard Lucas, besides its appeal as a puzzle for human beings as well as for computer performance, has been endowed with enough structure to be treated mathematically (the problem $\mathfrak{P}5 :=$ irregular $\rightarrow$ irregular without the "devine rule" (0) seems to have almost no mathematical structure), but not with so much to be trivial and uncapable of meaningful generalizations. As long as there are still open problems, a mathematical subject is not dead. The brahmins are alive and as long as they are still moving golden discs, the world will, according to legend, not fall to dust. Let us hope so!

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