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INVOLUTIONS IN SURFACE MAPPING CLASS GROUPS

by John McCarthy and Athanase Papadopoulos

1. Introduction

Let F be a compact orientable surface with negative Euler characteristic. A *mapping class* is an isotopy class of orientation-preserving homeomorphisms of F. Mapping classes form a group under composition, the *mapping class group*, $M(F)$. An element of order two of the mapping class group will be called an involution. In this article, we prove two theorems about products of involutions.

The first theorem is group-theoretical. We assume that the surface F is closed and we study the subgroup of $M(F)$ which is generated by involutions. In particular, for closed surfaces of genus greater than or equal to three, we prove that the mapping class group is generated by involutions.

The second theorem is geometric. There is a classification, into 3 types, of mapping classes, which is due to Thurston. This theorem is about the type of the product of two involutions. As is natural in this setting, the theorem and its proof are in terms of the action of the mapping class group on Teichmüller space and its Thurston boundary.

The second theorem is analogous in nature to the following elementary facts about the product of order-two elements of a discrete group of isometries of hyperbolic 2 or 3-space:

(i) the product is an elliptic isometry if and only if the two order-2 isometries have a common fixed-point in hyperbolic space.

(ii) if the two order-2 isometries have no common fixed point in hyperbolic space and have a common fixed point on the boundary at infinity, their product is a parabolic isometry.

(iii) if the two isometries do not have any common fixed point, neither in hyperbolic space nor on the boundary at infinity, their product is of hyperbolic type.

In section 2, we prove the group theoretical result. Then, in section 3, we give an outline of some of the background material on Thurston's classification. For a complete exposition of Thurston's theory, we refer the reader to [4], and for more information on Teichmüller space, to [1]. Finally, in section 4, we prove the theorem on the type of the product of two involutions.

The problem of studying the types of products of involutions in the mapping class group was suggested to the second author by François Laudenbach. The theorem on the subgroup generated by involutions arose out of an attempt to obtain more precise information about the mapping classes which occur as products of two involutions.

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2. The subgroup generated by involutions

Let $M(F_g)$ denote the mapping class group of a closed orientable surface F_g of genus $g \geqslant 2$. Let $I(F_g)$ denote the subgroup of $M(F_g)$ which is generated by involutions. We wish to describe $I(F_g)$ as a subgroup of $M(F_g)$. Clearly $I(F_g)$ is a normal subgroup of $M(F_g)$. Hence, we shall give our description in terms of the quotient, $M(F_g)/I(F_g)$.

We begin by recalling some algebraic facts about $M(F_g)$. For a general introduction to the algebraic structure of $M(F_g)$, we refer the reader to [2].

It is a classical fact that $M(F_g)$ is generated by Dehn twists about nonseparating simple closed curves on M. In fact, $M(F_g)$ is generated by a finite number of such twists. The minimum number of twists which is required to generate $M(F_g)$ was given by Humphries [5].

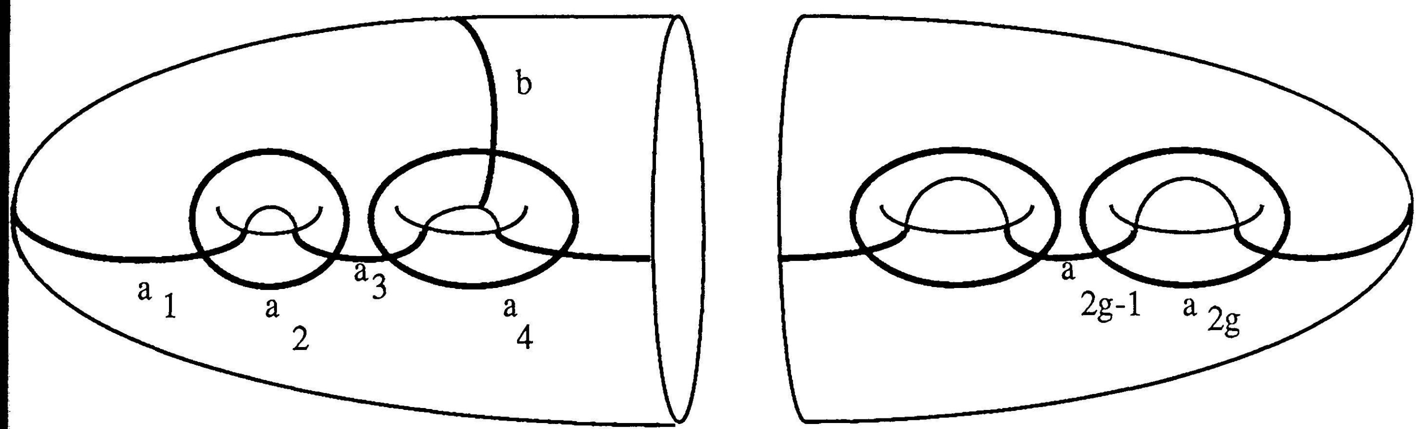

FIGURE 1.

THEOREM (Humphries). *$M(F_g)$ is generated by the Dehn twists about the curves $a_1, \ldots, a_{2g}$ and b of figure 1.*

Let us denote the Dehn twist about a curve, a, by t_a. (We remind the reader that the sense, right or left, of a Dehn twist is not dependent upon an orientation of the curve a, but rather upon the orientation of the surface; t_a denotes a right Dehn twist with respect to the given orientation on F.) Given any element of $M(F_g)$, h, we have the following identity:

$$ht_ah^{-1} = t_{h(a)} . \tag{1}$$

It follows that any two Dehn twists about nonseparating simple closed curves on F are conjugate in $M(F_g)$. As a consequence, $H_1(M(F_g))$ is cyclic. The order of $H_1(M(F_g))$ was computed by Powell [8]:

$$H_1(M(F_2)) = Z_{10} , \tag{2}$$

$$H_1(M(F_g)) = 1 , \quad \text{if} \quad g \geqslant 3 . \tag{3}$$

Due to the peculiar nature of genus 2, we shall need more precise information concerning involutions in $M(F_2)$. It is a consequence of a theorem of Nielsen (cf. [7]) that every involution in $M(F_2)$ is represented by an involution of F_2. Hence, we need only describe involutions of F_2.

There are two obvious such involutions. First, there is the hyperelliptic involution, i, with six fixed points which is depicted in figure 2 as a rotation by 180 degrees about the horizontal axis.

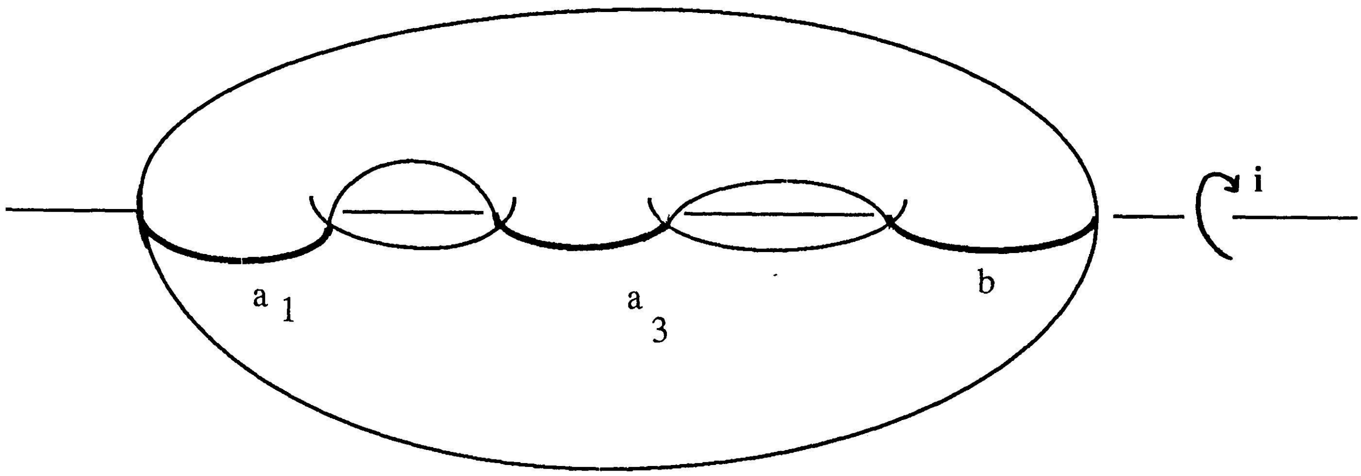

FIGURE 2.

Let us denote the Dehn twist along a_i by t_i. From equation (1), we see that i commutes with t_1, t_2, t_3, t_4 and t_b. Hence, i is a central element of $M(F_2)$.

The following identity was established by Birman [2]:

$$i = t_1 t_2 t_3 t_4 t_b^2 t_4 t_3 t_2 t_1 . \tag{4}$$

Secondly, there is the involution, s, with two fixed points, obtained by a rotation about the vertical axis as in figure 3.

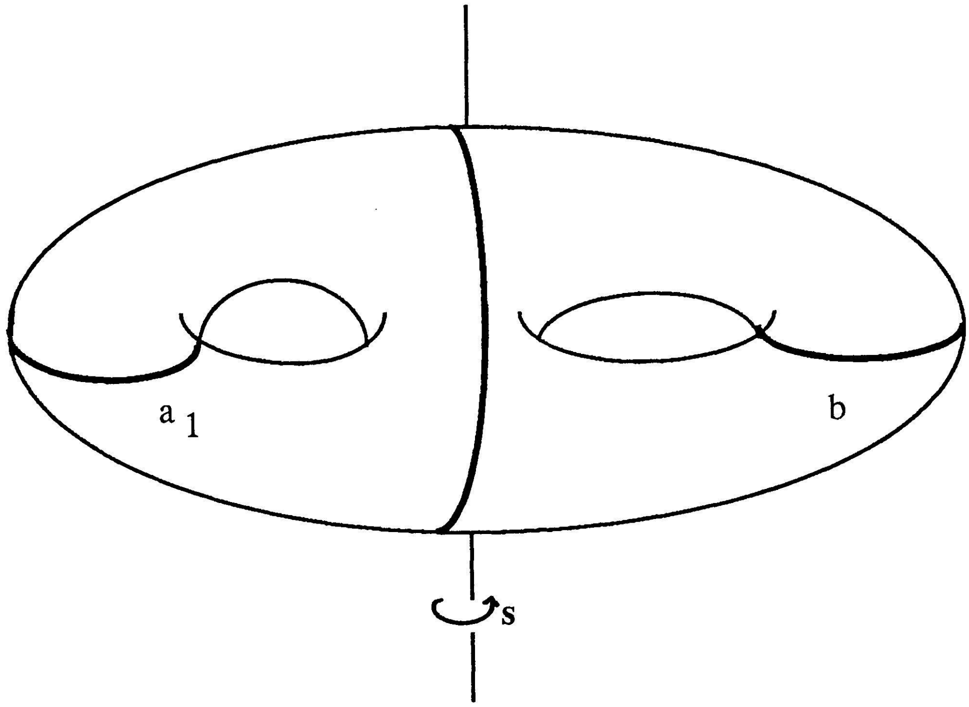

FIGURE 3.

We shall also need to express s as a product of Dehn twists. We thank Roger Tchangang Tambekou for explaining the technique used below for finding such an expression.

LEMMA 1. *We have*

$$s = t_3 t_2 t_4 t_3 t_2 t_4 t_1 t_b t_2 t_4 t_3 t_2 t_4 t_1 t_b .$$

Proof. Since s commutes with i, it induces a map, s', on the quotient orbifold of F_2 by the action of i, N. N is a sphere with six distinguished points of index 2, as in figure 4.

The map, s', is again a rotation. In particular, it fixes the point P which is indicated in figure 4. Hence, we can isotope s' in a neighborhood of the disk D (again indicated in figure 4). This isotopy lifts, in an obvious way, to M_2. Hence, we may assume that s' fixes the disc D pointwise. Therefore, we may consider s' as a map of a six-punctured disc fixing the boundary. In other words, s' is a braid. It is easy to see that the corresponding geometric braid is that depicted in figure 5.

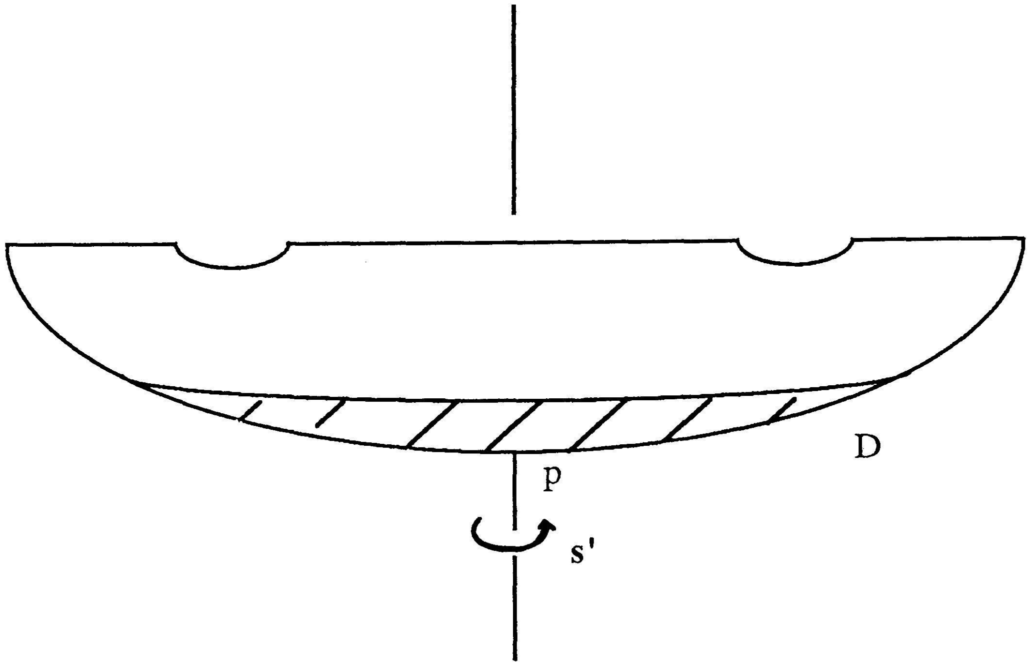

FIGURE 4.

Hence, in terms of the standard braid generators, we have

$$s' = s_3 s_2 s_4 s_3 s_2 s_4 s_1 s_5 s_2 s_4 s_3 s_2 s_4 s_1 s_5 \,. \tag{5}$$

As explained in [3], the braid generators s_1, s_2, s_3, s_4 and s_5 lift to t_1, t_2, t_3, t_4 and t_b respectively. Hence, we see that h is a lift of s'. It

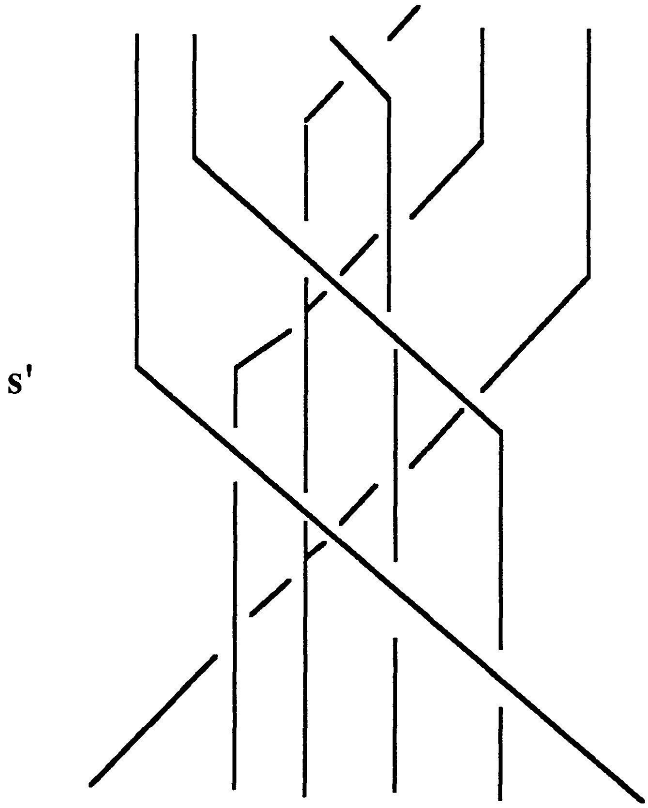

FIGURE 5.

follows that s is equal to h or $h \circ i$. Since i acts on $H_1(F_2, Z)$ as $-Id$, we may decide which of the two inequalities holds by considering the action of s and of h on $H_1(F_2, Z)$. We leave it to the reader to verify that s and h have the same action on $H_1(F_2, Z)$. This completes the proof of lemma 1.

We shall now show that every involution of F_2 is equivalent to either i or s.

LEMMA 2. *There are exactly two conjugacy classes of involutions in $M(F_2)$, the classes of i and of s.*

Proof. By equation (1) above, $st_1s^{-1} = t_b$.

Hence, s is not in the center of $M(F_2)$. Since i is central, i and s represent distinct conjugacy classes in $M(F_2)$.

It remains to see that all involutions in $M(F_2)$ are conjugate to either i or s.

Hence, suppose t is an involution in $M(F_2)$. Let t also denote a representative involution of F_2. Of course, the orbit space of F_2 under the action of $gp(t)$, N, is the base space of a branched covering with total space F_2. Hence, by the Riemann-Hurewicz formula, we have the following identity:

$$2(X(N)-b)) = X(F_2) - b' \tag{6}$$

where X denotes Euler characteristic,

b = number of branch points in N,

and b' = number of branch points in F_2.

Since this is a two-fold branched cover, we know that b' is equal to b. Hence, if we denote the genus of N by g, we obtain the formula:

$$6 = 4g + b \tag{7}$$

From this, we obtain precisely two solutions:

$$(g=0, b=6) \qquad \text{or} \qquad (g=1, b=2)\,. \tag{8}$$

Now, we know that the branched cover is a regular branched cover. Hence, it is determined by a representation of $\pi_1(N\backslash\sigma)$ onto Z_2, where σ is the set of branch points.

By the definition of a branch point, we know that the representation must be nontrivial on loops encircling a branch point.

Suppose that g is zero. Since $\pi_1(N\backslash\sigma)$ is generated by such loops, there is only one such representation. Hence, t must be topologically conjugate to i.

Now suppose that g is one. $N\backslash\sigma$ is a twice-punctured torus. The representation of the group $\pi_1(N\backslash\sigma)$ onto Z_2 factors through $H_1(N\backslash\sigma, Z_2)$ which is a free Z_2-module with basis (x, y, z) given by the loops depicted in figure 6.

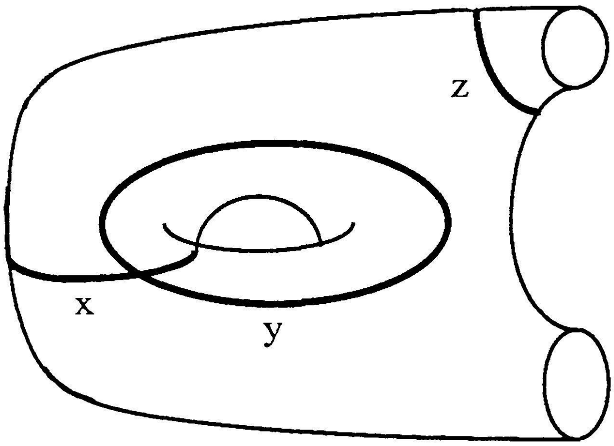

FIGURE 6.

Hence, there are four possible representations, given the previous restrictions:

$$(9) \qquad \begin{array}{llll} \text{(i)} & (x, y, z) & \to & (0, 0, 1) \\ \text{(ii)} & (x, y, z) & \to & (1, 0, 1) \\ \text{(iii)} & (x, y, z) & \to & (0, 1, 1) \\ \text{(iv)} & (x, y, z) & \to & (1, 1, 1)\,. \end{array}$$

It suffices to show that these representations are topologically equivalent.

There is an homeomorphism, f, as depicted in figure 7, which acts as follows:

$$(10) \qquad f:(x, y, z) \;\to\; (x, y+z, z)\,.$$

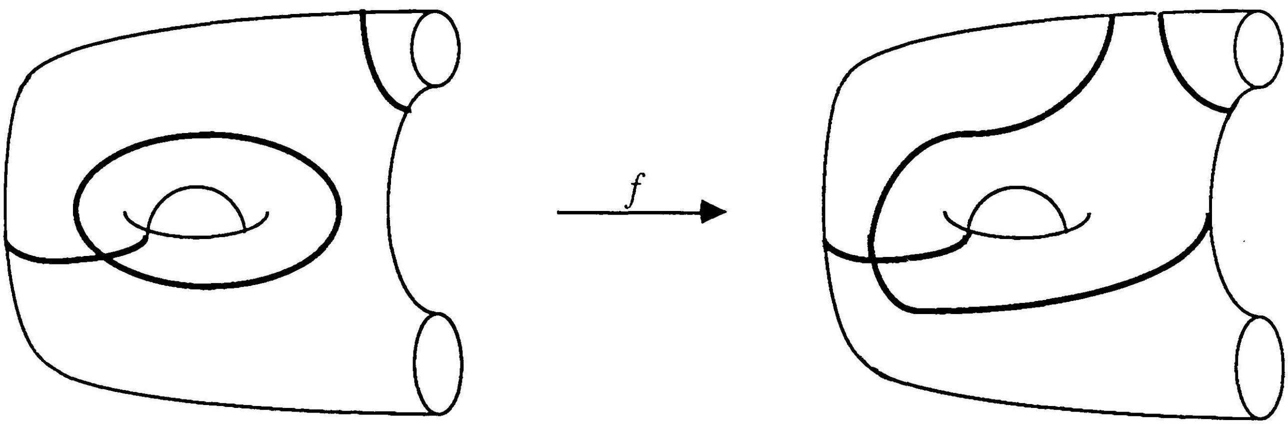

FIGURE 7.

Hence, (i) and (iii) are equivalent representations, and (i) and (iv) are equivalent representations. In a similar manner, by "pulling x over the puncture z", we see that (i) and (ii) are equivalent. Hence, all four representations are topologically equivalent.

It follows, as in the genus zero case, that t must be topologically conjugate to s. This completes the proof of lemma 2.

Let p be the abelianization map given by Powell's result:

$$\begin{aligned} p: M(F_2) &\to Z_{10} \\ t_d &\to 1\,, \qquad d \text{ nonseparating}\,. \end{aligned} \tag{11}$$

From equation (4) and lemme 1, it follows that

$$p(i) = 1\,, \quad p(s) = 5\,. \tag{12}$$

From lemma 2, it follows that $p\big(I(F_2)\big)$ is the subgroup of Z_{10} which is generated by Z_5.

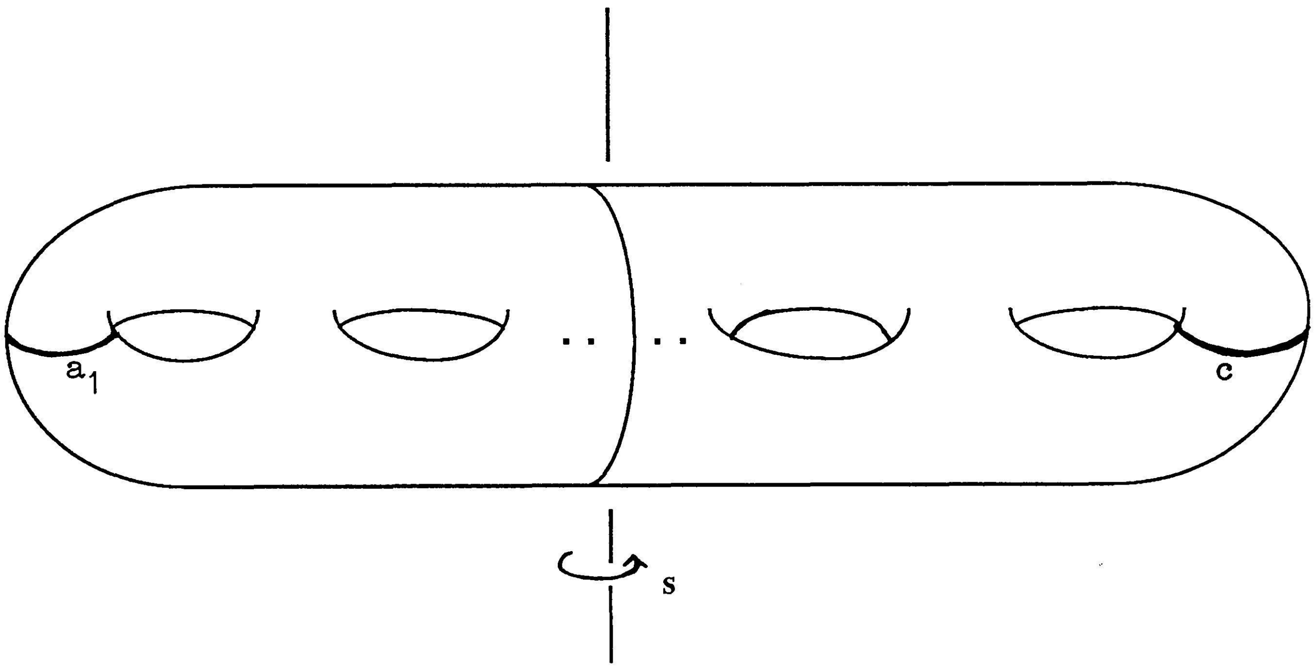

FIGURE 8.

For surfaces of even genus, s will continue to denote an involution with two fixed points as in figure 8. For surfaces of odd genus, s will denote an involution with four fixed points as in figure 9.

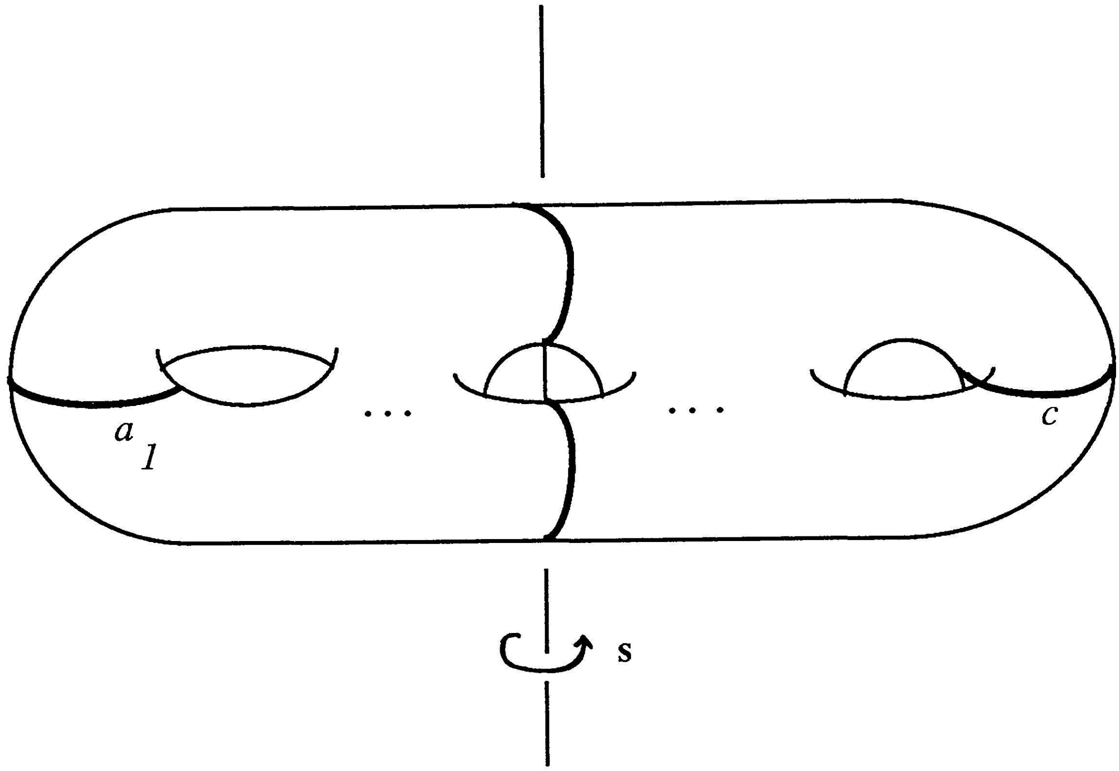

FIGURE 9.

We are now prepared to give the promised description of $I(F_g)$.

THEOREM 1.

(a) $I(F_g)$ *is normally generated by* s *for all* $g \geqslant 2$.

(b) $M(F_2)/I(F_2) = Z_5$.

(c) $M(F_g)/I(F_g) = 1$, for all $g \geqslant 3$.

Proof. Let c denote the curve depicted in figure 8 or in figure 9, depending upon the genus, g. Let Σ be the normal closure of s. We begin by showing that $M(F_g)/\Sigma$ is cyclic.

Let t_i denote as before the Dehn twist along the curve a_i.

By equation (1), we conclude that:

$$t_c t_1^{-1} = (st_1 s^{-1})t_1^{-1} = s(t_1 s t_1^{-1}), \tag{13}$$

and this is an element of Σ.

For any j with $3 \leqslant j \leqslant 2g$, we may construct a homeomorphism, h, which takes (a_1, c) to (a_1, a_j). By applying equation (1), we conclude that:

$$t_j t_1^{-1} \in \Sigma \quad \text{for all} \quad 3 \leqslant j \leqslant 2g. \tag{14}$$

By the same reasoning, we deduce that:

(15) $$t_b t_1^{-1} \in \Sigma\,,$$

(16) $$t_b t_2^{-1} \in \Sigma\,.$$

Hence, Humphries' generators are all conjugate modulo Σ. This implies that $M(F_g)/\Sigma$ is cyclic.

If the genus is greater than 2, equation (3) implies that $\Sigma = M(F_g)$. Since Σ is contained in $I(F_g)$, we see that $\Sigma = I(F_g) = M(F_g)$. Hence, theorem 1 is true for genus greater than two.

Now suppose that the genus is two. By equation (12), we conclude that i belongs to Σ. By lemma 2, it follows that $\Sigma = I(F_2)$. On the other hand, by equation (12), we conclude that

$$M(F_2)/\Sigma = Z_5\,.$$

This establishes theorem 1 for genus two.

This completes the proof of theorem 1.

3. Thurston's classification of mapping classes

The Teichmüller space of F, denoted by **T**, is the space of hyperbolic metrics on F up to isometry. It has a natural topology and is homeomorphic to an open ball of dimension $6g-6+2b$, where g is the genus of F and b the number of its boundary components.

Thurston's boundary of **T** is the space of projective classes of measured foliations on F.

A measured foliation is a foliation with isolated singularities of a special type (p-prong singularities, where p is any integer > 2, see figure 10), with a measure on transverse segments which is a Lebesgue-measure, and which is invariant by isotopy of the segment keeping each point on the same leaf.

There's an equivalence relation between measured foliations, generated by isotopy and the operation of collapsing a leaf connecting two singular points. **MF** denotes the space of equivalence classes.

There's a natural action on **MF** by the positive reals; **PMF** is the quotient projective space. **PMF** is homeomorphic to a sphere of dimension $6g-7+2b$ which constitutes, by Thurston's work, a natural boundary for Teichmüller space. $M(F)$, The mapping class group of F, acts continuously

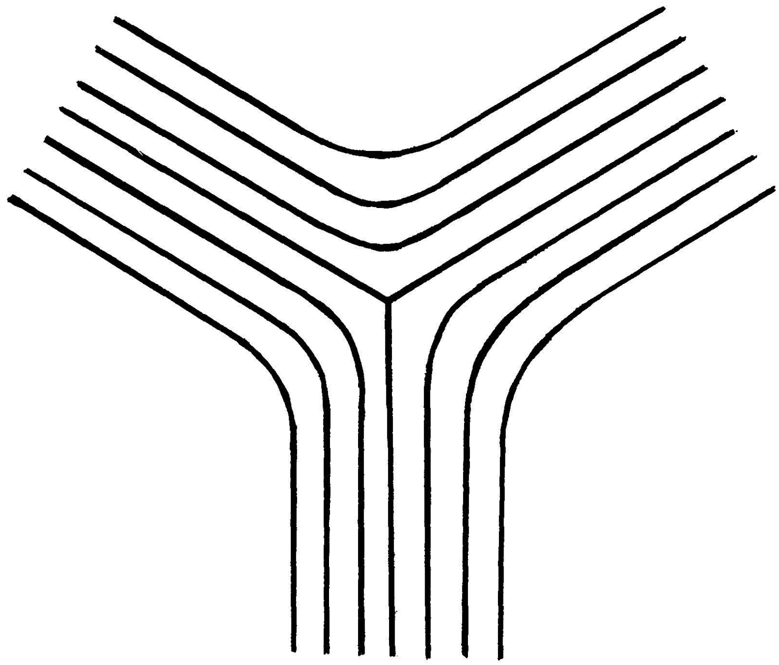

FIGURE 10.

on the closed ball $\mathbf{T} \cup \mathbf{PMF}$, and Thurston's classification of the elements of $M(F)$ can be formulated in terms of this action.

If an element of $M(F)$ has a fixed point in $\mathbf{T}$, then it is of finite order, i.e. there is an integer n such that the n-th iterate of that element is the class of the identity. In fact, there is a representative of this element which is globally periodic of order n, and which is an isometry of the hyperbolic metric corresponding to that fixed point in $\mathbf{T}$.

If an element of $M(F)$ does not have a fixed point in $\mathbf{T}$, then by the Brouwer fixed-point theorem it has a fixed point in $\mathbf{PMF}$.

There are two cases: either this point is the equivalence class of a foliation which has no closed cycles of leaves, and then this element is of *pseudo-Anosov* type, and can be represented by a homeomorphism of the surface which preserves a pair of measured foliations, acting as an expansion with respect to the transverse measure of one of them, and a contraction with respect to the other, or the fixed point in $\mathbf{PMF}$ is the class of a foliation which has a cycle of leaves; in this case the map is said to be *reducible*. There's an isotopy class of a (nonnecessarily connected) simple closed curve on M which is preserved by this mapping class, and the mapping class naturally splits into *components*.

We refer to [1] or [4] for the details of this classification.

4. Products of involutions

Let s_1 and s_2 be two involutions. We are interested in the type of the element $s_1 \circ s_2$. This type will be seen to depend upon the intersection of the two sets $\mathrm{Fix}(s_1)$ and $\mathrm{Fix}(s_2)$, where $\mathrm{Fix}(s_i)$ denotes the fixed point set of s_i in the closed ball $\mathbf{T} \cup \mathbf{PMF}$.

THEOREM 2.

(i) *$s_1 \circ s_2$ is of finite order if and only if $\mathrm{Fix}(s_1)$ and $\mathrm{Fix}(s_2)$ have a common point in $\mathbf{T}$.*

(ii) *Suppose that $s_1 \circ s_2$ is not of finite order. If $\mathrm{Fix}(s_1) \cap \mathrm{Fix}(s_2) \neq \emptyset$, then $s_1 \circ s_2$ is reducible.*

(iii) *$s_1 \circ s_2$ is pseudo-Anosov if and only if $\mathrm{Fix}(s_1)$ and $\mathrm{Fix}(s_2)$ have empty intersection.*

Proof. (i) If s_1 and s_2 have a common fixed point in $\mathbf{T}$, then $s_1 \circ s_2$ also fixes this point and is therefore of finite order (cf. [4]).

For the converse, suppose that $s_1 \circ s_2$ is of finite order. Then by ([2], remarque p. 67), there is a point m in Teichmüller space such that m is fixed by $s_1 \circ s_2$.

The mapping classes s_1 and s_2 being involutions, we have $s_1(m) = s_2(m)$.

Now Teichmüller space has a metric, the Teichmüller metric (cf. [1]), for which the mapping class group acts by isometries. By Teichmüller's theorem, any two points in $\mathbf{T}$ can be joined by a unique geodesic. Each of the mapping classes s_1 and s_2 interchanges the points m and $s_1(m)$. Therefore, s_1 and s_2 fix the point which is at equal distance from m and $s_1(m)$, on the Teichmüller geodesic joining these points.

(ii) Let $\mathbf{F}$ be a common fixed point of s_1 and s_2 in $\mathbf{PMF}$. There exist two positive real numbers x_1 and x_2 such that if f is an element of $\mathbf{MF}$ in the class $\mathbf{F}$, then $s_1(f) = x_1 . f$ and $s_2(f) = x_2 . f$.

As s_1 and s_2 are of finite order, we have x_1 and $x_2 = 1$, so $s_1 \circ s_2(f) = f$. By ([2], exposé 9, §.III et IV), either $s_1 \circ s_2$ is of finite order or it is reducible.

(iii) Suppose that $\mathrm{Fix}(s_1) \cap \mathrm{Fix}(s_2)$ is empty. By (i), $s_1 \circ s_2$ is not of finite order. Suppose that it is reducible, and let $\mathbf{C}$ be the element of $\mathbf{MF}$ corresponding to the class of the reducing curve. We have $s_1(\mathbf{C}) = s_2(\mathbf{C})$. Let $\mathbf{C}_1$ denote the equivalence class $s_1(\mathbf{C})$.

Let C and C_1 be two simple closed curves on F representing respectively the classes $\mathbf{C}$ and $\mathbf{C}_1$, in such a way that C and C_1 are in a position of minimum-intersection number.

Consider a neighborhood of the union of C and C_1 obtained by taking the union of a thin tubular neighborhood of each of these curves, and let C_2 denote the collection of those boundary curves of this neighborhood which are not null-homotopic.

Suppose first of all that C_2 is not empty. Then we have $s_1(C_2) = C_2$ and $s_2(C_2) = C_2$. (To see this, one can represent s_1 (respectively s_2) by an isometry of some hyperbolic metric, and then consider the geodesics g and g_1 in the classes of C and C_1. The isometry preserves the geodesics union $g \cup g_1$ and therefore it preserves an imbedded ε-neighborhood of that subset, and the boundary of the neighborhood). In this case, s_1 and s_2 have a common fixed point in **PMF**.

Suppose now that C_2 is empty. We have $s_1 \circ s_2(\mathbf{C}) = \mathbf{C}$ and $s_1 \circ s_2(\mathbf{C}_1) = \mathbf{C}_1$, and $\mathbf{C}$ and $\mathbf{C}_1$ have the property that for any element $\mathbf{F}$ in **MF**, we have either $i(\mathbf{F}, \mathbf{C}) \neq 0$ or $i(\mathbf{F}, \mathbf{C}_1) \neq 0$.

By assumption, $s_1 \circ s_2$ is reducible. Let n be an integer s.t. the map $(s_1 \circ s_2)^n$ preserves each component of the surface F cut along the reducing curve.

The mapping class $(s_1 \circ s_2)^n$ cannot have any pseudo-Anosov component, since if it had one, and if $\mathbf{F}^u$ denotes the class of the unstable foliation of that component, we have either $i(\mathbf{F}^u, \mathbf{C}) \neq 0$ or $i(\mathbf{F}^u, \mathbf{C}_1) \neq 0$. By the dynamics of a pseudo-Anosov (component) map on measured foliations space, the two classes of curves cannot be fixed by $s_1 \circ s_2$. Therefore, $s_1 \circ s_2$ cannot have pseudo-Anosov components.

So $(s_1 \circ s_2)^n$ has only finite order components.

By the same argument, $(s_1 \circ s_2)^n$ cannot have a non-trivial Dehn twist along a component of its reducing curve.

Therefore, $s_1 \circ s_2$ has only periodic components with no non-trivial Dehn twists along the reducing curve, so it is globally periodic, i.e. of finite order, a contradiction.

We conclude that $s_1 \circ s_2$ is pseudo-Anosov. This proves theorem 2.

5. Remarks and examples

1. We can easily classify now the structure of the group generated by two involutions:

Given the two involutions s_1 and s_2 of $M(F)$, the subgroup G they generate is an order-2 extension of the cyclic subgroup generated by $s_1 \circ s_2$. The elements of G that are not in that subgroup are all conjugate to s_1 or s_2. If s_1 and s_2 have a common fixed point in $\mathbf{T}$, the subgroup that they generate is finite. Otherwise, it is isomorphic to the infinite dihedral group $Z_2 * Z_2$.

2. In closing, we wish to point out that all three cases of Theorem 2 do in fact occur in every genus: To see that $s_1 \circ s_2$ can be of finite order we can take s_1 to be an horizontal rotation as in figure 2 and s_2 to be a vertical rotation as in figure 3. Since these rotations commute, $s_1 \circ s_2$ is an involution. (This example obviously generalizes to genus greater than two.)

To see that $s_1 \circ s_2$ can be reducible of infinite order, we can take s_1 to be a vertical rotation as in figure 3 and let $s_2 = s_1 \circ t_1 \circ t_b^{-1}$. Now s_2 is an involution by equation (1):

$$(s_2)^2 = s_1 \circ t_1 \circ t_b^{-1} \circ s_1 \circ t_1 \circ t_b^{-1} = t_b \circ t_1^{-1} \circ t_1 \circ t_b^{-1} = 1\,. \tag{17}$$

Moreover, $s_1 \circ s_2 = t_1 \circ t_b^{-1} - 1$ which is a reducible map of infinite order. (Again, this example obviously generalizes to higher genera.)

To see that $s_1 \circ s_2$ can be pseudo-Anosov we can make a similar construction. Let s_1 be an involution. Suppose that A is a family of disjoint nontrivial simple closed curves. Let $B = s_1(A)$. Now suppose that A and B fill up F. Let t_A be the product of the Dehn twists about the components of A and t_B be the corresponding product associated to B. Let $s_2 = s_1 \circ t_A \circ t_B^{-1}$. As in the reducible case just described, s_2 is an involution. Furthermore, $s_1 \circ s_2 = t_A \circ t_B^{-1}$, which is a pseudo-Anosov map by an algorithm of Long's [6] generalizing Thurston's algorithm described in [4]. An example of this construction of case (iii) of Theorem 2 is depicted in figure 11, where s_1 is again the vertical rotation. (Again, this example easily generalizes.)

Alternatively, one can give a nonconstructive argument as follows. Let s_1 be a vertical rotation as in figure 3. Since $s_1(a_1) = b$, we know that s_1

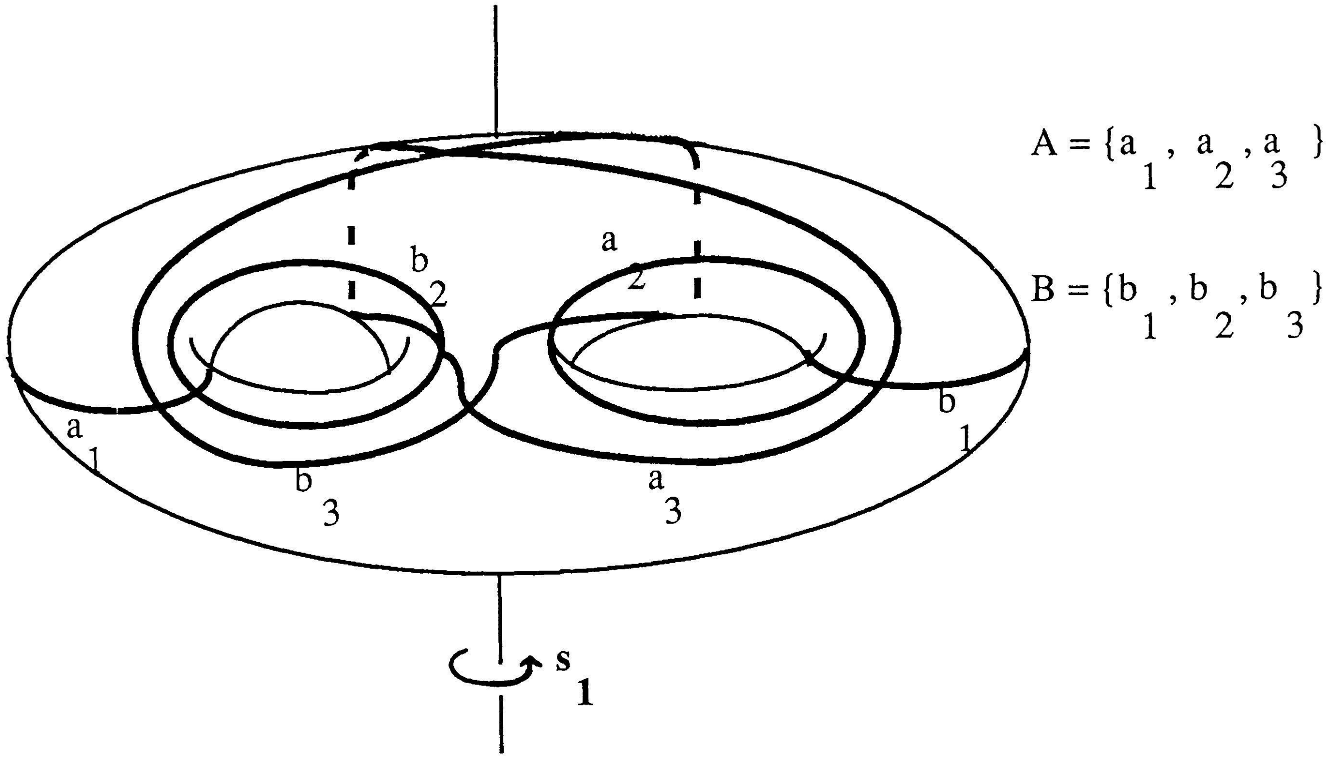

FIGURE 11.

is not in Fix(s_1). On the other hand, Fix(s_1) is clearly a closed set. Hence, we may find an open neighborhood of a_1 in $\mathbf{T} \cup \mathbf{PMF}$, U, such that U avoids Fix(s_1). Now, we may find a pseudo-Anosov, f, both of whose fixed points lie in U. (For example, this can be acheived by conjugating any given pseudo-Anosov by a sufficiently high power of t_1 .) Since Fix(s_1) is a compact set which avoids the repelling fixed point of f, it follows from the well known behavior of pseudo-Anosov maps on $\mathbf{T} \cup \mathbf{PMF}$ that $f^n(\mathrm{Fix}(s_1))$ is contained in U for sufficiently large n. Choose n subject to this condition and let $s_2 = f^n \circ s_1 \circ f^{-n}$. Finally, since Fix($s_2$) is equal to $f^n(\mathrm{Fix}(s_1))$, it follows from Theorem 2 that $s_1 \circ s_2$ is pseudo-Anosov.

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