Introduction

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **29 (1983)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **28.04.2024**

L'Enseignement Mathématique, t. 29 (1983), p. 263-280

MILNOR LATTICES AND GEOMETRIC BASES OF SOME SPECIAL SINGULARITIES [1])

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Introduction

The subject of this note are certain invariants associated to the topology of a complex hypersurface singularity $f : (\mathbf{C}^n, 0) \to (\mathbf{C}, 0), n \equiv 3(4)$, which are defined via the Milnor fibration and deformation theory. These invariants are the homology group of the Milnor fiber of middle dimension $n - 1$ as an abelian group (determined by the Milnor number μ), the signature (μ_0, μ_+, μ_-) of the intersection form, the homology group of dimension $n - 2$ of the link of the singularity as an abelian group, the linking form, the intersection form, weakly distinguished bases, distinguished bases. The order reflects the relative strength of these invariants, but it is not always a strict order: the knowledge of the intersection form turns out to be equivalent to the knowledge of the invariants listed before.

The aim of this note is to give a survey of some recent results on these invariants for special classes of singularities. In [7] we studied some of these invariants for the singularities of Arnold's lists. Here we summarize these results and give additional information. We also consider another class of singularities, namely the minimally elliptic hypersurface singularities as studied by Laufer [14]. We state some general features about the above invariants for these two classes of singularities. One of the problems is to find a normal form of the Dynkin diagrams with respect to bases of a certain type for a whole class of singularities. We indicate such a form with respect to weakly distinguished bases for the minimally elliptic hypersurface singularities. But it turns out that all the above invariants except the class of distinguished bases are to weak to distinguish between singularities of different topological type. We also discuss a canonical form of Dynkin diagrams with respect to distinguished bases for the uni- and bimodular singularities, which are contained in both classes of singularities mentioned above.

[1]) This article has already been published in *Nœuds, tresses et singularités*, Monographie de l'Enseignement Mathématique N° 31, Genève 1983, p. 129-146.

The results were obtained by the following method:

a) Find a distinguished basis for the given singularity. This is done using methods of Gabrielov, especially [11]. But the Dynkin diagrams of these bases are very complicated and contain many cycles.

b) Transform this diagram into a "nicer" form, where the information one is looking for is more transparent.

c) Analyse this diagram.

The paper is organized as follows. In section 1 + 2 we recall the definitions of the invariants and the basic relations among them and discuss the admissible transformations for b) above. Section 3 is devoted to a study of the weaker invariants including weakly distinguished bases of the above mentioned singularities. In section 4 we consider distinguished bases of the uni- and bimodular singularities.

There are also other invariants associated to singularities such as e.g. the monodromy groups. For a discussion of the relations among the various invariants and of their relative strength with respect to geometrical problems we refer to the expository article of E. Brieskorn [3]. For a description of the monodromy groups we refer to [8, 9]. There are also other interesting phenomena related to the above invariants in the class of bimodular singularities such as an extension of Arnold's strange duality. This is the subject of a joint paper with C. T. C. Wall, which is in preparation.

This paper is an extended version of the talk given by the author at the conference on the "Topology of complex singularities" at Les Plans-sur-Bex/Switzerland, March 27-April 2, 1982. The author thanks the organizers of this meeting, especially C. Weber, for their invitation. He is also grateful to E. Brieskorn for helpful discussions, which influenced the presentation in section 2. The final work on this paper was carried out at the State University of Utrecht and was supported by the Netherlands Foundation for Mathematics S.M.C. with financial aid from the Netherlands Organization for the Advancement of Pure Research (Z.W.O.). The author thanks these institutions for their hospitality.

1. The Milnor Lattice of a Singularity

Let $f : (\mathbf{C}^n, 0) \to (\mathbf{C}, 0)$ be the germ of an analytic function with an isolated singularity at 0. Let B_ε denote an open ball of radius ε in $\mathbf{C}^n$ around 0. Then for sufficiently small $\delta > 0$ and $\varepsilon > 0$