3. MISCELLANEOUS COBORDISM THEORIES.

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Example 3. Let X be a space on which O operates trivially. Then an X-structure on V is just a prefered homotopy class of maps $V \to X$. As cases of particular interest X might be an Eilenberg-MacLane space or the classifying space of a group. How does one compute he groups $N_k(X)$?

The above definitions can be modified slightly by admitting only oriented manifolds. Thus one obtains groups $\Omega_k(X)$ where X is any space on which the rotation group SO acts. Again I do not know how to compute these groups. (Added in proof: See Conner and Floyd [21].)

Example 4. Let P denote the infinite real projective space, with the infinite rotation group SO acting in the natural way. The cobordism groups $\Omega_k(P)$ for oriented manifolds with P-structure can be called the *spinor cobordism groups.* This name is appropriate since a P-structure is roughly a " lifting " of the structural group of the tangent bundle to the infinite spinor group. A manifold admits a P-structure if and only if its Stiefel-Whitney class w_2 is zero. The groups $\Omega_k(P)$ have no odd torsion, but otherwise I do not know much about them.

3. Miscellaneous cobordism theories.

So far we have concentrated on differentiable manifolds. However one could equally well define a cobordism group based on the class $\mathscr{T}$ of all compact topological manifolds. (Compare Brown [3, Theorem 3].) The natural correspondence $\mathscr{D} \to \mathscr{T}$ induces a homomorphism from the differentiable cobordism group $N_k = H_k(\mathscr{D})$ to the topological cobordism group $H_k(\mathscr{T})$.

Since Thom [16] has shown that Stiefel-Whitney classes can be defined topologically, we have:

Theorem 3 (Thom). — *The homomorphism* $\mathrm{N}_k \to H_k(\mathscr{T})$ *has kernel zero.*

Problem: Is this homomorphism onto ?

Another possibility would be to consider the class $\mathscr{C}_o$ of all compact, oriented, combinatorial manifolds. Whitehead [20] has shown that each differentiable manifold has a preferred class of triangulations. Hence there is a natural homomorphism from

$\Omega_k = H_k(\mathscr{D}_o)$ to $H_k(\mathscr{C}_o)$. Thom, Rohlin and Švarč have shown that Pontrjagin classes can be defined for combinatorial manifolds. Therefore we have:

THEOREM 3′. — *The homomorphism* $\Omega_k \to H_k(\mathscr{C}_o)$ *has kernel zero.*

However examples show that this homomorphism is not onto. The reader is referred to [13, 18].

Another interesting possibility would be to look at the class of compact homology manifolds.

Returning to the differentiable case, interesting cobordism groups can be obtained by restricting the connectivities of the manifolds involved. As an extreme case we can consider only differentiable manifolds which are either homotopy spheres or homotopy cells. The resulting cobordism groups are closely related to the problem of classifying differentiable structures on spheres. The reader is referred to Milnor [8] and Smale [14].

As a final, quite different, example consider differentiable imbeddings of the circle S^1 in the 3-sphere S^3. Such an object (a knot) is said to *bound* if it can be extended to a differentiable imbedding of the disk D^2 in the disk D^4. The resulting cobordism group has been studied by Fox and Milnor [5]. This group is not finitely generated.

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