Reliability Aalysis of a Highway Bidge
Analyse de la fiabilité d'un pont-route
## Zuverlässigkeitsanalyse für eine Autobahnbrücke

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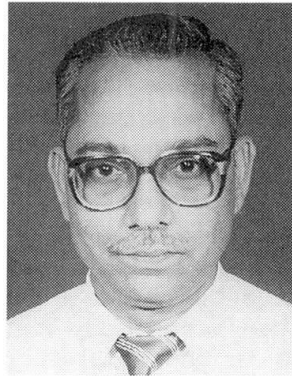
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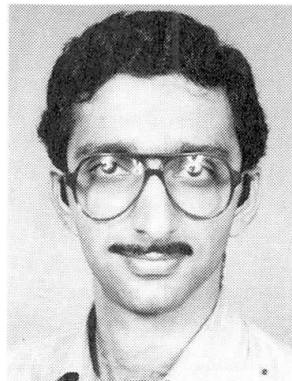
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SUMMARY

The paper deals with the reliability analysis of a continuous prestressed concrete box girder bridge which is under construction across Thana Creek in Bombay, India. The bridge is considered as a system and the reliability of the same is evaluated at limit states of collapse in flexure and shear using Monte Carlo technique. Results of the statical analysis of the field data on material strength and load variables have been used in the Monte Carlo method. It is found that the probability of failure of the bridge at limit states of collabse in flexure and shear are of the order of 10^{-16} and 10^{-10} respectively.

RÉSUMÉ

Cet article traite de l'analyse de la fiabilité d'un pont-route à plusieurs travées, du type à poutre-caisson en béton précontraint, actuellement en construction pour franchir le Thana-Creek à Bombay. La méthode de Monte-Carlo est utilisée pour déterminer la fiabilité de ce système de pont à l'état limite de rupture par flexion et cisaillement. Les données statistiques en travée sur la résistance des matériaux et l'effet des charges ont été déduites pour servir de variables. Le calcul de la probabilité d'une rupture à l'état limite par flexion et cisaillement a donné une valeur variant entre 10^{-16} et 10^{-10}.

ZUSAMMENFASSUNG

Der Beitrag behandelt die Zuverlässigkeitsanalyse einer mehrfeldrigen Spannbetonkastenbrücke, die über den Thana-Creek in Bombay gebaut wird. Die Systemzuverlässigkeit der Brücke auf Biege- und Schubversagen wird mit dem Monte-Carlo-Verfahren ermittelt. Für die Variablen wurden statistische Felddaten der Werkstoffestigkeit und der Einwirkungen herangezogen. Die Wahrscheinlichkeit eines Einsturzes im Grentzzustand des Biege- oder Schubversages wurde zu 10^{-16} bzw. 10^{-10} berechnet.

1. INTRODUCTION

The development of reliability analysis and reliability based design criteria has led to the rational evaluation of structural safety. One area of reliability analysis which is receiving considerable attention from engineers is system reliability. The major source of hidden safety reserve is in the system behaviour of structures. A bridge is a system of interacting members with redundancies and load sharing. However, there is a different degree of redundancy and load sharing in different types of bridges. The difference between member and system reliabilities increases with increase in the level of redundancy. This paper uses a probability based approach, developed by Nowak and Zhou [1], considering a bridge as a system of interconnected elements. System approach has been used to evaluate the reliability of a continuous prestressed concrete bridge which is under construction across Thana Creek in Bombay, India. For the evaluation of the reliability of a bridge, different limit states of flexure, shear, fatigue, deflection, etc. are to be considered. However, only the limit states of collapse in flexure and shear are considered and the evaluation of structural reliability is presented for the same using Monte Carlo technique. Field data on the strengths of materials and traffic load have been collected and statistically analysed. Results of the same have been used in the reliability analysis.

2. DETAILS OF THE BRIDGE

The new Thana Creek Bridge is a prestressed concrete box girder bridge of total length 1.837 km. It consists of six spans viz. 205.46 m, four spans of 321 m each and 347.63 m. It is a Class AA bridge [2] with three lanes on each of the two parallel continuous girders. From the structural analysis point of view, the two girders may be considered to act independently of each other. Each span consists of a variable depth box girder - a square parabola from either support upto the midspan. The girder is supported on steel rocker/roller bearings on each pier. A longitudinal section of a typical intermediate unit and cross section are shown in Fig. 1. Details of critical sections are shown in Table 1.

Section No.	Area of section <m²>	Moment of inertia <m⁴>	Number of prestressing cable ducts in		Slope of duct <rads>	Area of stirrups <mm²/m>	Centroid distance from top <m>
			deck	soffit			
1	7.4660	13.511	2	6	0.0000	4400.83	1234.21
2	9.3258	36.725	36	6	0.1000	4997.73	2040.96
3	14.470	113.11	58	0	0.1315	6544.93	3832.94
4	9.3258	36.725	40	4	0.1000	4581.47	2040.96
5	7.2300	13.262	2	38	0.0000	4400.83	1215.74
6	9.3258	36.725	40	4	0.1000	4581.47	2040.96
7	14.470	113.11	64	0	0.1315	6544.93	3832.94

Table 1 Section properties for the Thana creek bridge

3. STATISTICS OF STRENGTH AND LOAD VARIABLES

Concrete mix M 40 (cube strength 40 N/mm²) and 12.7 mm diameter, 1/7 ungalvanised stress relieved strands for prestressing the bridge and Torsteel (yield strength 415 N/mm²) for vertical stirrups have been used in the bridge. Field data have been collected on cube strength of concrete, f_{cu}, ultimate tensile strength of strands, f_p, and yield strength of Torsteel, f_{sy}, and statistically analysed. It is found that f_{cu}, f_p and f_{sy} follow normal distributions with mean value and standard deviation equal to 47.28 and 4.28, 1925.8 and 24.5, and 468.9 and 34.2 N/mm² respectively
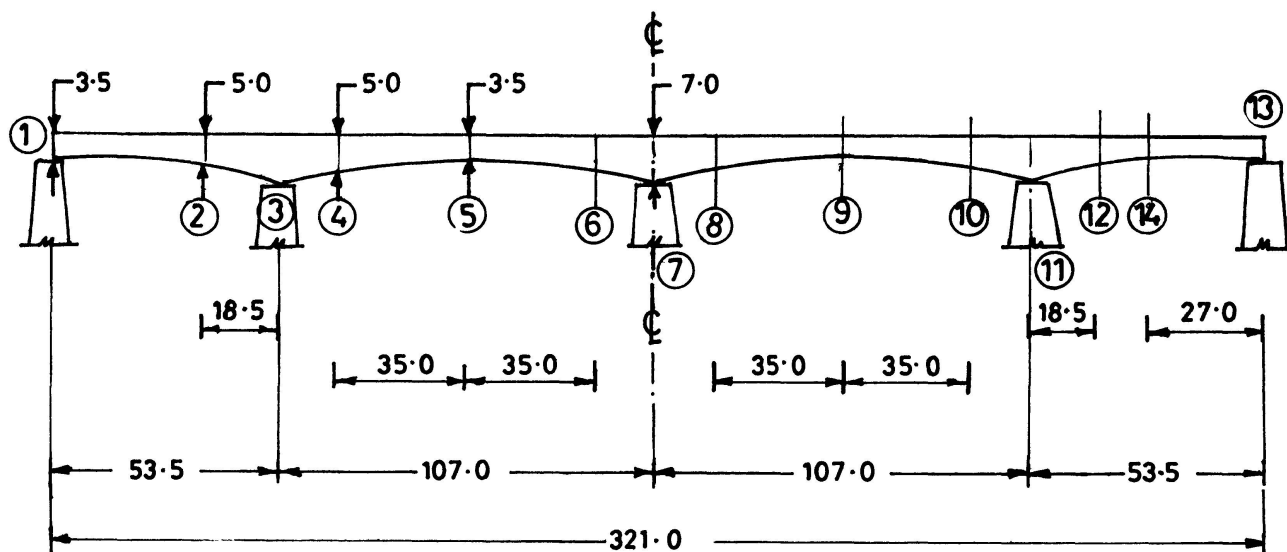
All dimensions in m

(a) Typical intermediate unit
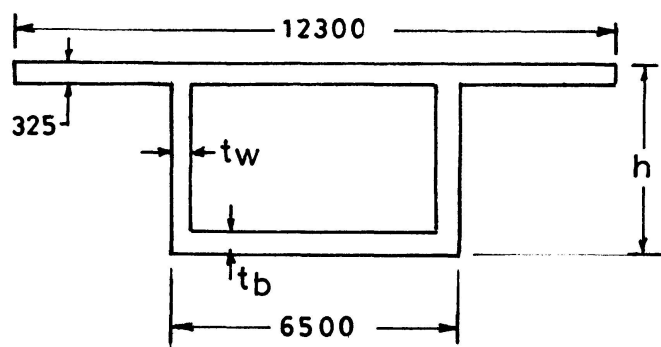
Sections	h	t_w	t_b
1,5,9,13	3500	300	225
3,7,11	7000	350	1000
2,4,6,8 10,12	5000	363	335

All dimensions in mm

(b) Details of cross sections

Fig.1 Details of new Thana Creek Bridge

[3].

The live load data has been collected for the old Thana Creek Bridge as the bridge under reliability study is still under construction. The last traffic survey was conducted in 1989. The collected data has been statistically analysed. Based on the analysis of the actual loads, a new standard truck has been fixed with a mean axle load of 92 kN per axle (total load 184 kN) with a wheel base of 1.2 m. Considering the uncertainties due to several factors, the coefficient of variation of the load is found to be 30 per cent [3]. The standard truck obtained above represents the arbitrary point-in-time varying load, Q_{apt}, as obtained from the results of the traffic survey. For checking the reliability of the bridge at ultimate limit states, statistics of the lifetime maximum live load have to be used. Considering the life of the bridge as 50 years, and assuming that (i) Q_{apt} follows Type I extremal (largest) distribution [4], (ii) the live load data represents a constant traffic distribution throughout the year and (iii) all annual maximum loads are identically distributed and independent, the statistics of the lifetime maximum total load, Q_m, for the standard truck have been obtained. It is found that Q_m follows Type I extremal (largest) distribution with parameters u= 327.5 kN and α

= 0.232. The mean value of Q_m is 352.4 kN [3].

The dead load, D, is assumed to be normally distributed with mean to nominal ratio equal to 1.05 and coefficient of variation five per cent.

4. RELIABILITY ANALYSIS

Having determined the statistics of the basic random variables, both load and strength, the reliability analysis of the bridge is now considered. The criteria selected are (i) the limit state of collapse in flexure and (ii) the limit state of collapse in shear.

4.1 Limit state of collapse in flexure

The given bridge is a redundant structure. In the analysis of such structures, the failure of a section is assumed to take place when the plastic moment capacity of the section is reached. The failure is called the formation of a plastic hinge at the section. In the case of a redundant structure, a collapse mechanism forms only when a sufficient number of plastic hinges have developed.

4.1.1 Plastic rotation capacity and plastic moment

The failure of a structure through the formation of a collapse mechanism requires the plastic hinges to have a large rotational capacity. In the case of prestressed concrete structures, any critical region, hinged earlier, may fail due to insufficient rotation capacity before a collapse mechanism is formed. This failure is called a rotation failure mode. The plastic rotation capacity, θ_p, of a section is given by

$$\theta_p = d(e_u - e_{cy})/x_u \qquad \text{----(1)}$$

where d is the effective depth, e_u is the ultimate strain in concrete and e_{cy} is the strain in concrete at the extreme fibre at the start of yield in steel. The depth of the neutral axis at failure, x_u, can be determined from the equilibrium condition at failure. As per IS:456-1978 [5], the limiting values e_u= 0.0035 and e_{cy}= 0.002 are used. Thus, knowing the geometry of the section and the depth of the neutral axis at failure, the ultimate rotation capacity of the section can be found.

The ultimate resisting moment of a section is taken as the plastic moment capacity, M_p, of the section. This is given by [5]

$$M_p = B \, f_{pu} \, A_p \, (d - 0.42x_u) \qquad \text{----(2)}$$

where f_{pu} is the ultimate tensile stress in the tendons at failure and A_p is the area of prestressing tendons. Using equilibrium equations, strain compatibility conditions and the stress-strain relationships for steel and concrete, a given section can be analysed and f_{pu} and x_u can be determined. Knowing f_{pu} and x_u, M_p can be calculated. In Eq. 2, the model parameter, B, introduced to take care of uncertainty in the prediction equation, is assumed to be normally distributed with mean value 1.01 and standard deviation equal to 0.0465 [6].

4.1.2 Determination of reliability

The reliability analysis of the bridge as a system starts with the determination of critical sections in the bridge. The standard truck is placed at a critical section. Using the Monte Carlo technique, a set of values is generated for the basic random variables viz. load and strength variables (f_{cu}, f_p, Q_m, B and D). All other parameters are considered to be deterministic. Using the generated values,

plastic moments and plastic rotation capacities of critical sections are calculated as explained in the previous section.

For a given position of the wheel load, the structure is analysed using the stiffness method of linear elastic and piecewise linear elastic-plastic analysis [7]. The steps involved are as follows:

i) Develop the load vector , {L}, and the stiffness matrix, [K], for the structure.
ii) Invert the stiffness matrix and obtain the displacement vector {D} as

$$\{D\} = [K]^{-1} \{L\} \qquad\qquad \text{----(3)}$$

iii) Determine the member end forces and check for excessive rotation at all locations where plastic hinges have formed.
iv) Find the load factor, λ, the ratio of the plastic moment capacity to the actual moment, at each critical location. Select λ_{min} and increase the load by multiplying the initial load by λ_{min}.
v) Introduce a plastic hinge at the node for which $\lambda = \lambda_{min}$ and modify [K]. To do this, add a row and a column to [K], the elements of which are the stiffnesses corresponding to the rotational degree of freedom at the hinge point. Add the plastic moment capacity of the section and the hinge rotation to the load and displacement vectors respectively. Check for the formation of a mechanism.
vi) Repeat steps (ii) to (v) until the check in step (iii) fails or a mechanism is formed. A mechanism is said to have formed when the determinant of the matrix, $|[K]| \leq 0$.
vii) The ultimate load factor, λ_{ult} , will be the sum of the minimum values of λ in successive cycles. The above procedure results in a piecewise linear moment-curvature relationship. Hence, during the analysis, if at any stage it is found that the rotation capacity of a section is exceeded, then a direct interpolation will suffice to obtain λ and hence λ_{ult}.

The safety margin, Z, is calculated as

$$Z = \frac{resistance}{action} = \frac{ultimate\ load}{actual\ load} = \lambda_{ult} \qquad\qquad \text{----(4)}$$

Hence a value for Z is obtained. The Monte Carlo technique is repeated and a number of values are generated for Z. As the elastic-plastic analysis is to be carried out each time, the number of samples to be generated is restricted to only 200 to get estimates of mean and standard deviation of Z.

For the generated samples, mean value, $\bar{Z}$, and coefficient of variation,δ_Z, of Z are calculated. For calculating reliability, the lognormal format has been adopted. Hence for the lognormal variate Z, the parameters are calculated as follows:

$$\sigma_{\ln z} = \sqrt{\ln(\delta_Z^2 + 1)} \qquad\qquad \text{----(5)}$$

$$\tilde{Z} = \bar{Z} \exp(-\tfrac{1}{2} \sigma_{\ln z}^2) \qquad\qquad \text{----(6)}$$

The probability of failure, p_f, and the reliability index, β, are given by

$$p_f = \Phi[\ln(\tfrac{1}{\tilde{Z}})/\sigma_{\ln z}] \qquad\qquad \text{----(7)}$$

$$\beta = -\Phi^{-1}(p_f) \qquad\qquad \text{----(8)}$$

This gives the probability of failure of the bridge for a given load position. There are four critical load positions (Table 2). For each critical load position, the above procedure is repeated and p_f and β are calculated. The results are given in Table 2. Since the values of $p_f \ll 1$, the probability of failure of the system, p_{fs}, has been taken as

$$p_{fs} = \sum_{i=1}^{N} p_{fi} \qquad\qquad ----(9)$$

assuming the failure modes to be independent. p_{fi} is the value of p_f for i^{th} load position and N is the number of critical load positions. The value of p_{fs} is also given in Table 2.

Load position Section No.	$\bar{z}$	σ_z	p_f	β
14	4.6643	0.8716	1.018×10^{-16}	8.2191
4	4.6418	0.8581	6.159×10^{-17}	8.2827
5	4.3353	0.8174	4.538×10^{-15}	7.7548
6	4.8220	0.9152	6.537×10^{-17}	8.2691
For system			4.77×10^{-15}	7.75

Table 2 Results of analysis for limit state of collapse in flexure

4.2 Limit state of collapse in shear

4.2.1 Ultimate Shear Strength

The analysis for the limit state of collapse in shear differs from the elastic-plastic analysis for flexure in one major respect. There is no successive formation of hinges as was considered in the previous limit state (Sec. 4.1). This is because once a section fails in shear, the entire unit is considered to have failed. The ultimate shear strength of a prestressed concrete girder is considered to be the lesser of its strengths in the cracked and uncracked states. It is calculated using the Indian Roads Congress code [8]. The equations in the code are used after removing partial safety factors. While estimating the ultimate shear strength of a section, a model error parameter, B, is attached to the prediction equation and this is assumed to be normally distributed with a mean value of 1.09 and a standard deviation of 0.14 [6]. The analysis of a girder at the limit state of collapse in shear is simpler than that at the limit state of collapse in flexure.

4.2.2 Determination of reliability

The standard truck wheel load is fixed at a critical section. Using the Monte Carlo technique, a set of values is generated for the basic random variables viz. load and strength variables (f_{cu}, Q_m, B, f_{sy} and D). Using the generated values, the ultimate shear strength of each critical section is calculated. For a given load position, elastic analysis of the bridge is carried out using stiffness matrix method and values of actual shear forces at critical sections are determined. The load factor, λ, which is the ratio of ultimate shear strength to actual shear, is calculated at each critical section. The lowest value, λ_{min}, is selected and this gives the load factor for the ultimate shear capacity of the weakest section of the girder to be reached. The safety margin is calculated as

$$Z = \frac{resistance}{action} = \lambda_{min} \qquad\qquad ----(10)$$

This λ_{min} is nothing but a value for the safety margin Z. Similarly, using the Monte Carlo technique, the process is repeated and a number of values of Z are generated. Mean and standard deviation of Z are calculated. Values of p_f and β are calculated using Eqs. 7 and 8.

There are seven critical load positions (Table 3). For each critical load position, the above procedure is repeated and probability of failure calculated. The probability of failure of the system is calculated using Eq. 9. The results of the

reliability analysis are given in Table 3.

Load position Section No.	$\bar{z}$	σ_z	p_f	β	Failure at section
1	2.9256	0.4420	7.60×10^{-11}	7.0707	1, 6 or 8
2	2.9203	0.4410	8.33×10^{-11}	7.0618	2 or 6
3	2.9257	0.4419	7.59×10^{-11}	7.0728	6 or 8
4	2.9237	0.4424	9.00×10^{-11}	7.0553	4
5	2.9296	0.4437	8.33×10^{-11}	7.0621	6 or 8
6	2.9095	0.4393	9.61×10^{-11}	7.0383	6 or 10
7	2.9256	0.4418	7.57×10^{-11}	7.0739	6 or 8
For system			5.81×10^{-10}	6.14	

Table 3 Results of analysis for limit state of collapse in shear

5. DISCUSSION AND CONCLUSION

With the available data, a fairly accurate standard truck has been developed. The mean value of the lifetime maximum live (truck) load has been found to be 352.4 kN. It is observed that this value is very close to the standard wheeled vehicle of 400 kN (200 kN per axle) specified by Indian Roads Congress [2] for Class AA bridges.

During the reliability analysis at limit state of collapse in flexure, it has been observed that the failure in each case is in the rotation failure mode. Failures take place before the last hinge is formed to cause a complete failure mechanism.

A study of the results of the reliability analysis at limit state of collapse in flexure given in Table 2, reveals a slight decrease in the reliability index when the load is over the critical section 5 (Fig. 1). This is due to the special method of analysis and design used for the bridge. The bridge is being constructed by the cantilever method. As a result, during construction, the dead load due to self-weight is carried by cantilever action. Even after the fixation of the continuity cables at midspan, only the live load and the super-imposed dead load are carried by continuous beam action. Only after a period of 10 to 12 years, due to the relaxation of steel and the creep of concrete, the dead load will be carried by continuous beam action. As a result, there is an excess of strength in flexure at the supports. When the load is over the critical section 5, the moment at midspan, the weakest section of the girder, is more than that for the other load positions. Hence the reliability index for this position is lower than that for the rest. The reliability index for the limit state of collapse in flexure, for the girder unit taken as a system, is seen to be 7.75 (corresponding to $p_f = 4.77 \times 10^{-15}$) as shown in Table 2. This is much higher than the acceptable value of 2.5 to 3, normally used abroad. Thus, as expected, there appears to be an excess reserve of strength in the girder unit.

A study of Table 3, giving values of reliability index at limit state of collapse in shear, does not reveal any sharp variations in the reliability index as was observed in the case of flexure. This is because the failure in each case is predominantly at one of the 5 m deep sections, close to the interior supports, viz. 2, 4, 6, 8 or 10 (Fig. 1). The reliability index for the limit state of collapse in shear, for the girder unit taken as a system, is found to be 6.137 (corresponding to $p_f = 5.81 \times 10^{-10}$). This is higher than the generally accepted value of 2.5 to 3.0. The reason for this is the same as that for the limit state of collapse in flexure, viz. the use of the cantilever method of construction for the bridge.

Within the limitations of the present study, it can be concluded that the bridge has high reliability (reliability index greater than 6; probability of failure less than 10^{-9}) at limit states of collapse in flexure and shear. Reliability of the bridge in fatigue is an important aspect. Currently the state of knowledge in

fatigue evaluation of concrete structures is not adequate to carry out reliability studies.

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