Retrieval of system properties of existing structures

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Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **67 (1993)**

PDF erstellt am: **21.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-51369

Retrieval of System Properties of Existing Structures

Détermination des caractéristiques de systèmes de structures existantes

Ermittlung der Systemeigenschaften bestehender Tragwerke

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SUMMARY

Existing older structures are sometimes hard to evaluate, due to certain limitations in identifying their physical properties. It is shown, however, that the application of known forces at the nodes, together with the measurement of the associated displacements, leads to the retrieval of the physical characteristics of the structure, namely the stiffness matrix.

RÉSUMÉ

Il est parfois difficile d'évaluer les anciennes structures, en raison de certaines restrictions à identifier leurs caractéristiques physiques. Il apparait toutefois que l'utilisation de forces connues aux noeuds, combinée à la mesure des déplacements correspondants, permet de de déterminer les caractéristiques physiques de la structure, notamment la matrice de raidissement.

ZUSAMMENFASSUNG

Die Möglichkeiten zur Eruierung der physikalischen Eigenschaften bestehender alter Tragwerke sind naturgemäss beschränkt. Wie jedoch gezeigt wird, kann aus der Applikation bekannter Kräfte an den Knoten und Messung der zugehörigen Verschiebungen die Steifigkeitsmatrix des Tragwerks gewonnen werden.

1. INTRODUCTION

Environmental attacks, corrosion and prolonged use of existing structures make their structural evaluation rather limited because their members' properties may not conform to the design values. Hence, classical methods of structural analysis become inadequate to tackle and overcome the difficulty involved. Therefore, it is both necessary and prudent to improve such methods. In this study, system identification techniques are introduced. In such techniques, the structural stiffness is recovered from known forces and known associated displacements. Once the stiffness matrix of a structure is determined, the internal design forces due to any loading condition can readily be obtained.

2. STATEMENT OF THE PROBLEM AND THE SOLUTION

Present methods of structural analysis are primarily based upon the stiffness methods of analysis in which the input is a family of stiffness coefficients presented in a matrix form and the loading conditions entered in a vector form. The unknowns are displacements and subsequently internal forces. The standard mathematical representation of these three variables is:

$$\{F\} - [K]\{x\} \tag{1}$$

in which

F is an Nx1 loading vector
K is an NxN stiffness matrix
x is an Nx1 system displacement vector
N is the number of degrees of freedom

In the traditional approach to structural analysis, $\{x\}$ is the unknown, whereas in this study the unknowns are the elements of K, which in some sense represent the characteristics of the structure. A process that has been developed for other engineering disciplines, but which is being introduced in structural engineering, is generically referred to as "System Identification". It is an attractive procedure to formulate and improve mathematical models.

To illustrate the derivation of the stiffness of the structure in terms of the applied force vector and the associated and measured displacements, the following situation is used:

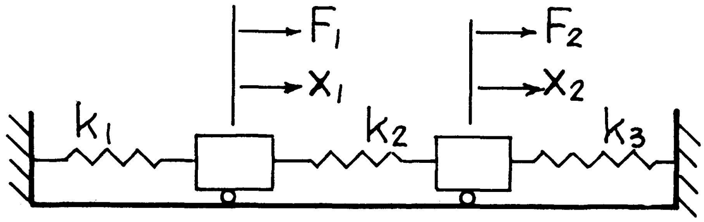

Figure 1

Figure 1 shows a two-degree of freedom system in which two lumped masses are attached to three linear springs with stiffnesses k_1, k_2, k_3 .

The force displacement relation for this situation is written in the following form:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \tag{2}$$

For an exact solution, the following statement holds true, i.e.

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} - \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

However, when this is not the case equation error vectors can be defined as:

$$\begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} - \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \tag{4}$$

To obtain an error function the right hand side of equation (4) is squared and the result is then summed over the number of degrees of freedom. For the present case, the squared error function becomes

$$E^2 = E_1^2 + E_2^2 \tag{5}$$

The problem now is reduced to that of minimizing the error function with respect to the unknown stiffnesses. This is achieved by taking the derivative of E^2 with respect to each unknown element stiffness and setting it equal to zero. This leads to a set of linear equations equal in number to the number of elements.

Taking the first derivative of equation (5) with respect to k_1, k_2 and k_3 yields the following set of equations written in matrix form

$$\begin{bmatrix} x_1 & 0 \\ x_1-x_2 & -x_1+x_2 \\ 0 & x_2 \end{bmatrix} \begin{Bmatrix} F_1-(k_1+k_2)x_1+k_2x_2 \\ F_2+k_2x_2-(k_2+k_3)x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{6}$$

which may be further reduced to

$$\begin{bmatrix} x_1 & 0 \\ x_1-x_2 & -x_1+x_2 \\ 0 & x_2 \end{bmatrix} \begin{bmatrix} x_1 & x_1-x_2 & 0 \\ 0 & -x_1+x_2 & x_2 \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} = \begin{bmatrix} x_1 & 0 \\ x_1-x_2 & -x_1+x_2 \\ 0 & x_2 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{7}$$

Defining a Jacobian matrix $[J]$ as follows

$$[J] = \begin{bmatrix} \dfrac{\partial E_1}{\partial k_1} & \dfrac{\partial E_1}{\partial k_2} & \dfrac{\partial E_1}{\partial k_3} \\[2ex] \dfrac{\partial E_2}{\partial k_1} & \dfrac{\partial E_2}{\partial k_2} & \dfrac{\partial E_2}{\partial k_3} \end{bmatrix} \tag{8}$$

It is readily noticed that equation (7) can be written in the following form

$$[J]^T[J]\{k\} = [J]^T\{F\} \tag{9}$$

From which $\{k\}$ can be solved for directly

$$\{k\} = [[J]^T[J]]^{-1}[J]^T\{F\} \tag{10}$$

The following example illustrates the solution. In this example a determinate truss configuration is chosen for simplicity in which displacements were actually computed using the standard Direct Stiffness Method. This is a numerical experiment meant to test the proposed method for the retrieval of the structure's unknown element stiffnesses. It must be mentioned, however, that for a determinate truss no such elaborate procedure is necessary because the problem in such a case is reduced to the solution of a system of linear equation.

For an indeterminate truss the inverse of $[J]^T[J]$ upon which the solution hinges is not guaranteed. To circumvent such a situation and to assure the existence of a solution two or more loading cases must be used and the squared error function given in equation (5) can be formally written as

$$E^2 = \sum_{n=1}^{NLC} \sum_{i=1}^{N} \left[F_i^n - \sum_{\ell=1}^{N} K_k \chi_\ell^n \right]^2 \tag{11}$$

in which N is the number of degrees of freedom and NLC is the number of loading conditions.

From which the solution for the element stiffness may be written as

$$\{k\} = \sum_{n=1}^{NLC} \left[[J]_n^T [J]_n \right]^{-1} \sum_{n=1}^{NLC} [J]_n^T \{F\}_n \tag{12}$$

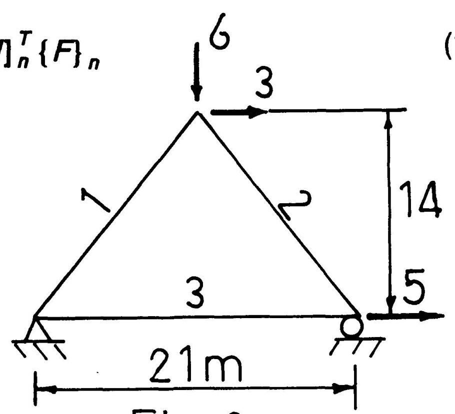

Fig 2

3. EXAMPLE

The determinate truss shown in figure 2 is used to test the procedure. The truss is composed of 3 elements of cross sectional area equal to 25 cm^2. The modulus of elasticity is $200 \times 10^6 \dfrac{kN}{m^2}$. The truss has 3 unrestrained degrees of freedom with the following reduced stiffness matrix derived with the standard Direct Stiffness Method for pin jointed trusses and written in terms of the unknown element stiffnesses.

$$[K] = \begin{bmatrix} 0.36k_1 + 0.36k_2 & 0.48k_1 - 0.48k_2 & -0.36k_1 \\ 0.48k_1 - 0.48k_2 & 0.64k_1 + 0.64k_2 & 0.48k_2 \\ -0.36k_2 & 0.48k_2 & k_3 + 0.36k_2 \end{bmatrix}$$

The applied loads are written in the following standard load vector

$$F = \begin{Bmatrix} 3 \\ -6 \\ 5 \end{Bmatrix} kN$$

Therefore the error vector of equation can be written as

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} - \begin{bmatrix} 0.36k_1 + 0.36k_2 & 0.48k_1 - 0.48k_2 & -0.36k_1 \\ 0.48k_1 - 0.48k_2 & 0.64k_1 + 0.64k_2 & 0.48k_2 \\ -0.36k_2 & 0.48k_2 & k_3 + 0.36k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

From which the following Jacobian matrix can readily be deduced

$$[J] = \begin{bmatrix} 0.36x_1 + 0.48x_2 & 0.36x_1 - 0.48x_2 - 0.36x_3 & 0 \\ 0.48x_1 + 0.64x_2 & -0.48x_1 + 0.64x_2 + 0.48x_3 & 0 \\ 0 & -0.36x_1 + 0.48x_2 + 0.36x_3 & x_3 \end{bmatrix}$$

Upon performing the operation as defined by the derived formula (10) the element stiffness are retrieved i.e. $k_1 = k_2 = 28571.4 \frac{kN}{m}$ and $k_3 = 23809.5 \frac{kN}{m}$ which are exactly the same as can be computed using EA/L . It must be reiterated that the displacements x_1, x_2 and x_3 supposed to measured, were in this numerical experiment computed using standard computer programs.

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