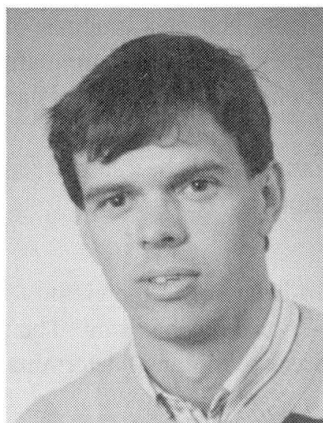# Computational Bond Models: Three Levels of Accuracy

Modèles de calcul de l'adhérance selon trois niveaux de précision

Rechenmodelle für den Verbund: drei Genauigkeitsstufen

Jan G. ROTS
Research Engineer
Delft Univ. of Technology
Delft, The Netherlands

Jan Rots obtained his civil engineering degree in 1983 and his doctor's degree in 1988, both from Delft University of Technology. He has been a member of the Civil Engineering Department at Delft University of Technology and of the Computational Mechanics Department of TNO Building and Construction Research since 1983.

SUMMARY

The paper reviews three computational bond models of decreasing degree pf precision. Firstly, a detailed resolution of bond-slip is given, which aims at supporting experimental work. Secondly, the bond-slip law is used as input to interface elements with a view to predicting crack spacing and width. Thirdly, embedded reinforcement techniques for global analysis in engineering practice are discussed. Examples demonstrate the capability of nonlinear finite element analysis to simply and correctly reveal «strut-and-tie» systems after stress redistribution.

RÉSUMÉ

Cet article passe en revue des modèles de calcul de l'adhérence, et ceci, selon 3 niveaux décroissants du degré de précision. Premièrement une résolution détaillée est donnée du problème de glissement-adhérence, qui vise à étayer un travail expérimental. Deuxièmement, la loi de glissement-adhérence est introduite dans le cas des interfaces afin de prédire la largeur et l'espacement des fissures. Troisièmement, les techniques d'enrobage des armatures de la pratique sont analysées globalement. Des exemples démontrent les capacités d'une analyse non-linéaire par éléments finis qui révèlent simplement l'efficacité des systèmes basés sur l'analogie du treillis après redistribution des effets.

ZUSAMMENFASSUNG

Der Artikel behandelt Rechenmodelle für den Verbund mit den drei folgenden Stufen abnehmender Genauigkeit. Zuerst wird eine ausführliche Lösung der Verbund-Schlupf-Beziehung gegeben, die zur Ergänzung von Versuchvorhaben dient. Zweitens wird das Verbund-Schlupf-Gesetz als Eingabe für Kontaktflächen-Elemente benutzt, mit der Zielsetzung, Rissabstände und Rissbreiten zu bestimmen. Drittens werden die Verfahren mit in Beton eingebetteten Bewehrungsstäben diskutiert, die zur baupraktischen Berechnung gesamter Systeme dienen. Einige Beispiele demonstrieren die Fähigkeiten nichtlinearer Finite Element Berechnungen, einfach und richtig Stabwerkmodelle nach Spannungsumlagerungen aufzuzeigen.

1. INTRODUCTION

Computational simulators for concrete can be applied in three different ways:

- to support experimental research,

- to develop, verify or validate design rules and hand calculation methods,

- to incidentally analyse particular structures.

The former two types of applications are not meant to be made at the desk of a practising engineer. Instead, these are made by researchers, students and post-graduate students who use the computational tools in a manner similar to experimental tools. In this way, computational models are transferred into practice in an indirect manner. The third type of application is of a direct nature.

In this paper computational bond models will be categorized in the above way. This corresponds to three levels of decreasing degree of precision:

- **Resolution of bond-slip.**
  This strategy zooms at the micro-behavior in the vicinity of the rebar, where cone-shaped secondary and longitudinal splitting cracks are crucial mechanisms. The method aims at explaining the fundamentals of traction-slip behavior and supports experimental determination of local bond-slip laws.

- **Bond-slip interface analysis.**
  This approach lumps bond-slip into an interface element, with a view to supporting the derivation of design rules on spacing and width of primary cracks in structural concrete members.

- **Embedded reinforcement with tension-stiffening.**
  For global analyses even the above approach that zooms at primary cracks becomes too delicate. Instead, the primary cracks are smeared out and techniques of automatic embedment of reinforcing elements and prestressed cables in concrete elements are necessary. In combination with tension-stiffening, these techniques can be directly used in engineering analysis of structural concrete systems.

This paper reviews the three approaches. Most attention will be given to examples of the third category, where results support the 'strut-and-tie' philosophy for structural concrete.

2. DETAILED LEVEL: RESOLUTION OF BOND-SLIP

The first and most sophisticated computational approach to bond-slip is to simulate the behavior in the vicinity around the rebar in detail. The bond-slip, i.e. the tangential relative displacement between the deformed rebar and the concrete (measured some distance away from the rebar), is controlled by four mechanisms [12,17]: (a) elastic deformation, (b) internal conical transverse cracking behind the ribs of the rebar, (c) longitudinal cracking in response to tensile ring-stresses, (d) crushing in compressive cones radiating out from the ribs. Fig. 1 shows a result of an elastic-softening simulation which included elastic deformation, transverse cracking and longitudinal cracking. The structure is a simple reinforced tension bar (tension-pull specimen) modelled in axi-symmetry.

Initially, the external force is transferred from the steel into the concrete primarily via axial tensile stresses. On increasing load, cone-shaped transverse secondary cracks nucleate behind the ribs of a rebar. This cracking starts near the end-face of the specimen and gradually moves inwards, which parallels experimental findings [9]. After transverse secondary cracking, the direct bond action is lost and the transfer of bond forces is subsequently furnished by compressive cones that radiate out from the ribs. The radial components of the compressive cones are balanced by rings of tensile stress [17]. When the ring is stressed to rupture a longitudinal crack arises and the balance against the compressive cones is lost, which causes a further break-

down of bond. These longitudinal splitting cracks are plotted as the shaded area in Fig. 1a.

Aside from giving insight in the basic bond mechanisms, simulations provide quantitative information. Fig. 1c shows a local bond traction-slip curve extracted from the analysis. The curve shows a linear-elastic stage, a stage of decreased stiffness and a softening stage. This trilinear idea of bond curves lends support to analytical bond-models that were derived on the line of argument, and agrees with experimental findings [7]. The slip modulus of the second stage of the curves (appr. $300N/mm^3$) falls within experimental scatter [6]. For further results and details the reader is referred to [14,15], where also a numerical contribution is made with regard to the yet unresolved principal issue in bond research, namely the dependence or non-dependence of bond curves on the distance from the primary crack.

In conclusion, a computational resolution of bond-slip contributes to a better understanding of the basic bond mechanisms and works complementary to experimental research in this area. Simulations of this type are not meant to be made at the desk of a practising engineer. Rather, the outcome comes available to practice in an indirect way, via improved bond curves and design rules.
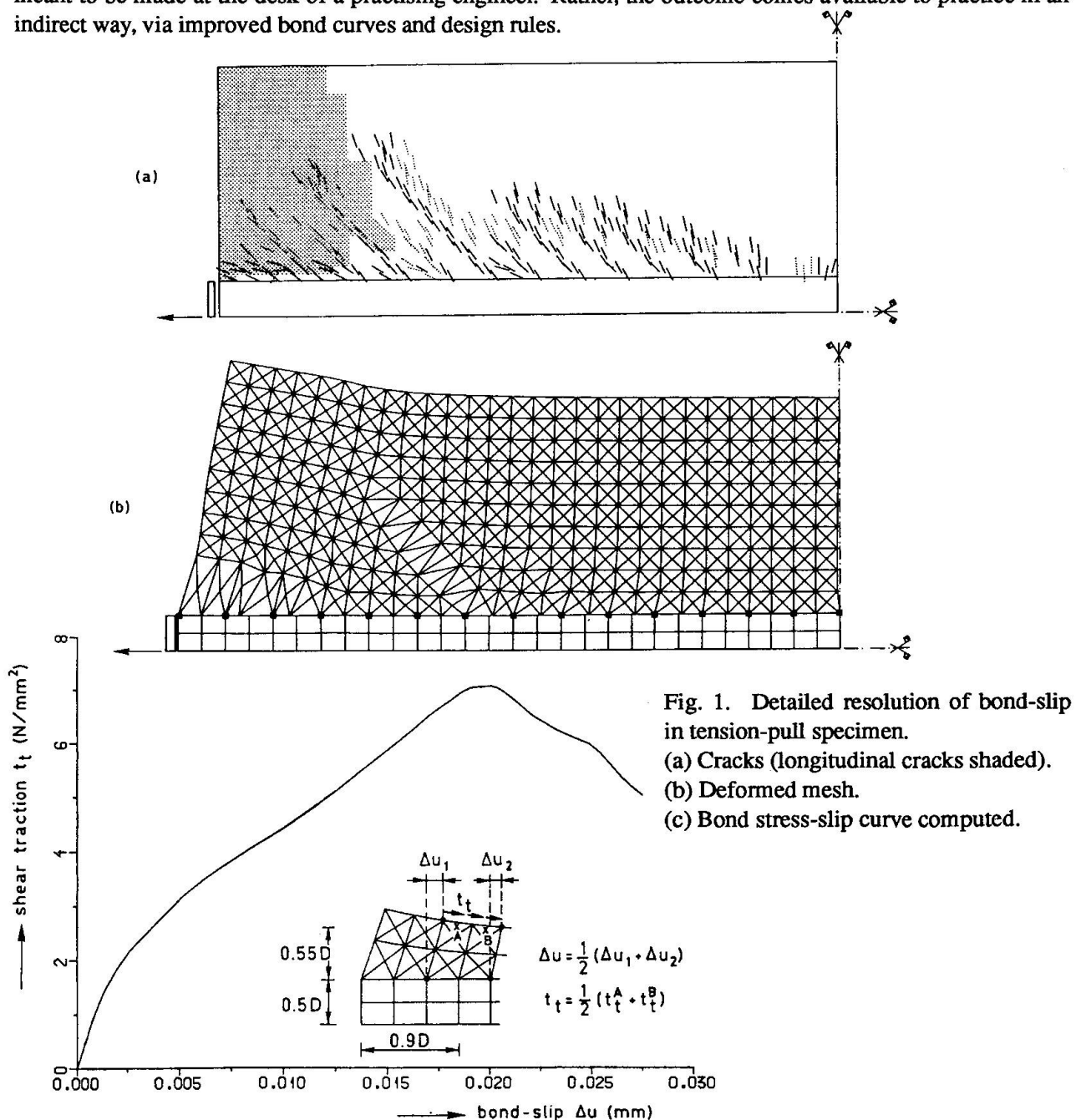

Fig. 1. Detailed resolution of bond-slip in tension-pull specimen.
(a) Cracks (longitudinal cracks shaded).
(b) Deformed mesh.
(c) Bond stress-slip curve computed.

3. INTERMEDIATE LEVEL: BOND–SLIP INTERFACE ANALYSIS

A second approach is to lump the bond-slip behavior into a fictitious interface. Consequently, the traction-slip curve, which was output from the previous analysis, is now input. This technique was introduced by Rehm [12] in order to subsequently evolve into a powerful analytical tool for predicting the spacing and width of primary cracks in structural concrete members, e.g. [11,17,3]. In analytical approaches simplifying assumptions have to be made, like the assumption of a simple and unique bond-slip curve or a sudden stress drop for the concrete after cracking. In computational studies, the procedure can be refined since more advanced bond-slip laws can be inserted for the interface elements and gradual softening curves for the cracks.

Fig. 2 shows an example of a long-embedment tension-pull specimen. The steel is modeled by truss elements, the bond-slip layer by interface elements and the surrounded concrete by continuum elements. The bond traction-slip curve for the interface elements was taken according to [6,10]. The tensile strength for the concrete was assigned a Gaussian distribution, which is consistent with the physical process in heterogeneous materials and essential to prevent a wide band of elements under homogeneous stress from cracking simultaneously. The load-elongation response shows four local maxima corresponding to the successive development of four primary cracks. The serrated type of curve is in agreement with experimental results [8] and justifies engineering models [3]. Beyond formation of the fourth primary crack, the crack pattern was fully developed (stable crack spacing) and the solution could be continued up to yielding of the reinforcement without further physical changes. During that stage only the width of the primary cracks increased.
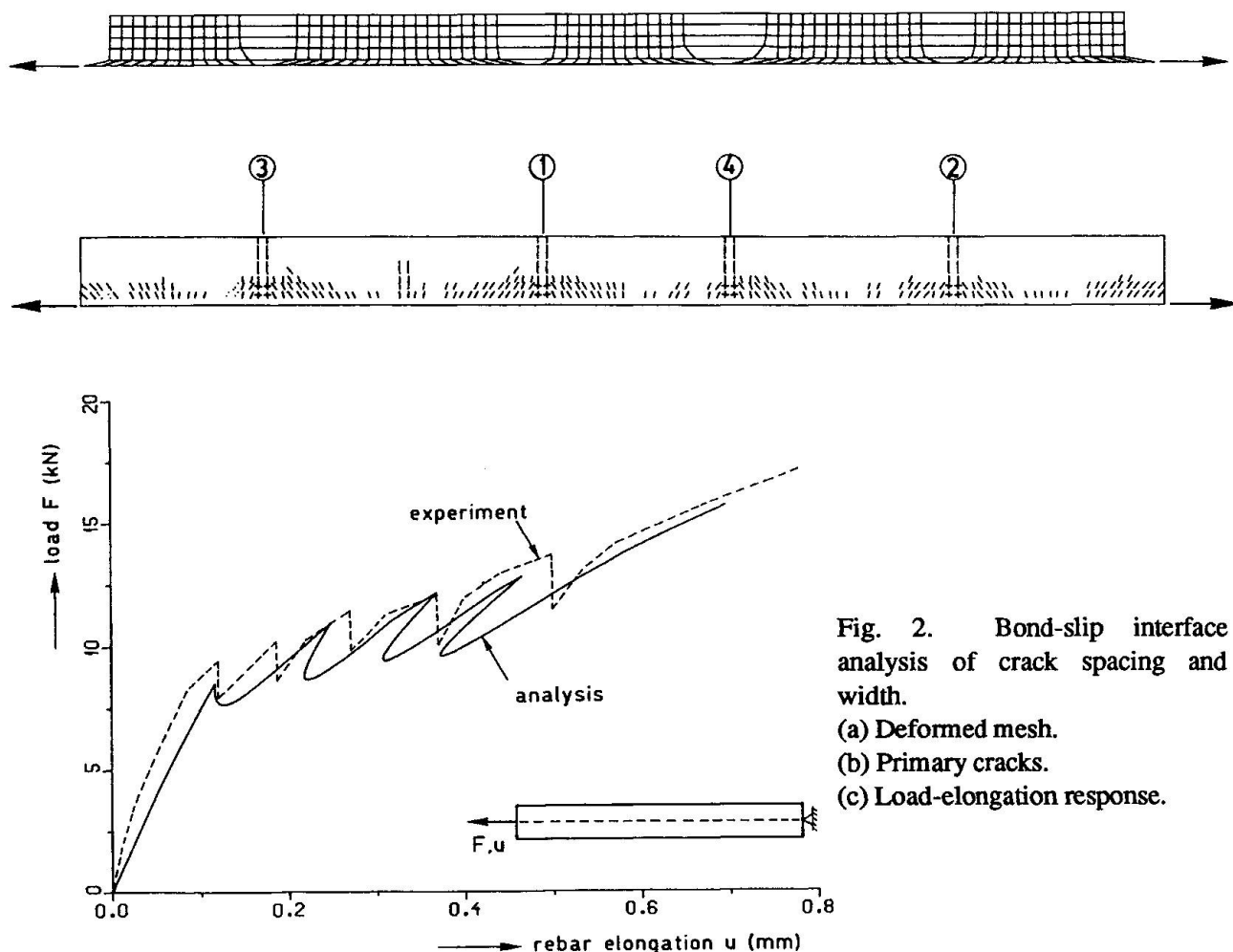

Fig. 2.   Bond-slip interface analysis of crack spacing and width.
(a) Deformed mesh.
(b) Primary cracks.
(c) Load-elongation response.

Analyses of this kind apply to cases where information is required regarding crack spacing and/or crack width. This can be the case either in research studies, where adequate design formula for spacing and width are to be developed, or directly in practical analysis of structural details like anchorages, where bond plays a paramount role. An illustrative example of the first category is the combined experimental, computational and analytical study on the control of crack width in deep reinforced concrete beams [2]. Examples of the second category have been given in e.g. [18].

4. GLOBAL LEVEL: EMBEDDED REINFORCEMENT

The third type of approach is full embedment of reinforcements in concrete elements (Fig. 3). With this procedure, the reinforcing elements do not have separate nodes and displacement degrees of freedom, but the strain in the reinforcement is calculated from the nodal displacements of the mother element in which it is embedded. The method generally implies overall perfect bond, i.e. the average strain in the cracked concrete equals the average strain in the reinforcement. For this reason, primary cracks are not treated individually, as though under a magnifying glass, but are smeared out with a view to global analysis of structural concrete systems. The joint action of cracked concrete and reinforcement must then be accounted for via a tension-stiffening model.

The clear advantage is that the lines of the finite element mesh do not need to coincide with the lines of the reinforcement, which is a must when real practical cases with a diffuse set of reinforcing bars, grids and prestressed tendons are analysed.

For practical use of embedded reinforcement techniques in nonlinear finite element analysis, three aspects are essential [4]. Firstly, various embedment combinations must be made available to model the variety of structures and geometries in practice:

- bars in Euler-Bernouilli beam elements
  e.g. 2D and 3D frames

- bars in Mindlin beam elements
  e.g. edge-beams along plates and shells

- bars and grids in plane stress elements
  e.g. panels, deep beams

- bars and grids in axi-symmetric elements
  e.g. storage vessels with radial, tangential and longitudinal reinforcement

- grids in plane-strain elements
  e.g. tunnels

- bars and grids in plate and shell elements
  e.g. slabs, shell roofs, cooling towers

- bars and grids in solid elements
  e.g. massive structures, complicated details

Secondly, options of geometrical preprocessing must be available. It is beyond realism, to let the analyst specify all intersections between reinforcement and element boundaries. Rather, the analyst would like to specify the end-points of the reinforcement, the shape and, depending on the type of shape (straight, parabole, circle etc.) additional information like e.g. a midpoint, see Fig. 3. The preprocessor should then automatically generate all intersections with the lines in the element patch. Once these intersections are known, the reinforcement portions in the elements can be evaluated and the linkage of reinforcement strains to element nodal displacements can be made.
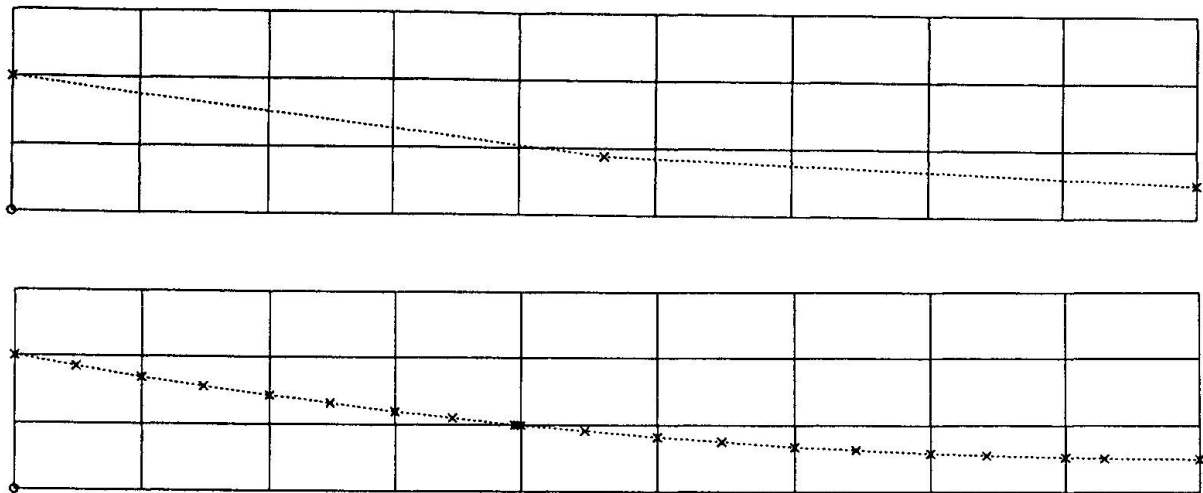
Fig. 3. Embedded reinforcement in finite element analysis.

    (a) Simple input of global tendon location, e.g. via three points.

    (b) Automatic positioning of reinforcement portions in elements, obtained via geometrical preprocessing routines.

Thirdly, options must be available for prestress, both pre-tensioned and post-tensioned. With pre-tensioning, the reinforcement is assigned an initial stress and the perfect bond assumption is used. With post-tensioning, the reinforcement is given an initial stress distribution (which should guarantee the design value of the jacking force and, simultaneously, account for the losses due to friction, elastic shortening and anchorage slip) and the analysis starts with a no-bond assumption. At this stage, the stiffness of the reinforcement is not included, but only the equivalent nodal element loads due the prestress are placed on the system. After grouting, the no-bond assumption is replaced by perfect bond and the stiffness of the reinforcement is included.

In the latest release of the DIANA finite element program [5] all above listed embedment combinations are available. Furthermore, geometrical preprocessing routines have been included for 2D, $2\frac{1}{2}$D (in-plane and out-of-plane location of curved tendons) and also a limited set for 3D. In the program, the two prestress options are available, whereby it is noted that automatic determination of the initial stress distribution for post-tensioning is currently under development.

5. EXAMPLE 1: CONCRETE WALL IN FLAT

A second example concerns a concrete wall in a twelve-storey appartment building which was already under construction up till the seventh floor when the surveyor noticed that the reinforcement in the 'tie' at the first floor was underdimensioned. To decide on the way of repair, the engineering firm in question called for a review analysis at TNO. Fig. 4 shows some key-results thereof.

The ground level was open and two colums of different dimensions supported the wall. In the mesh, only five storeys were taken into account, while the remaining storeys were represented by a load on top. The figure compares the principal stresses in the wall in the linear-elastic stage and in the final stage at load factor 1.7. It clearly reveals a 'strut-and-tie' system whereby in the linear-elastic stage the tie is formed by the concrete and in the final stage by the reinforcement since the concrete has cracked. Note the change in width and inclination of the strut with increasing load. These are definitely factors that should be accounted for in strut-and-tie design models.
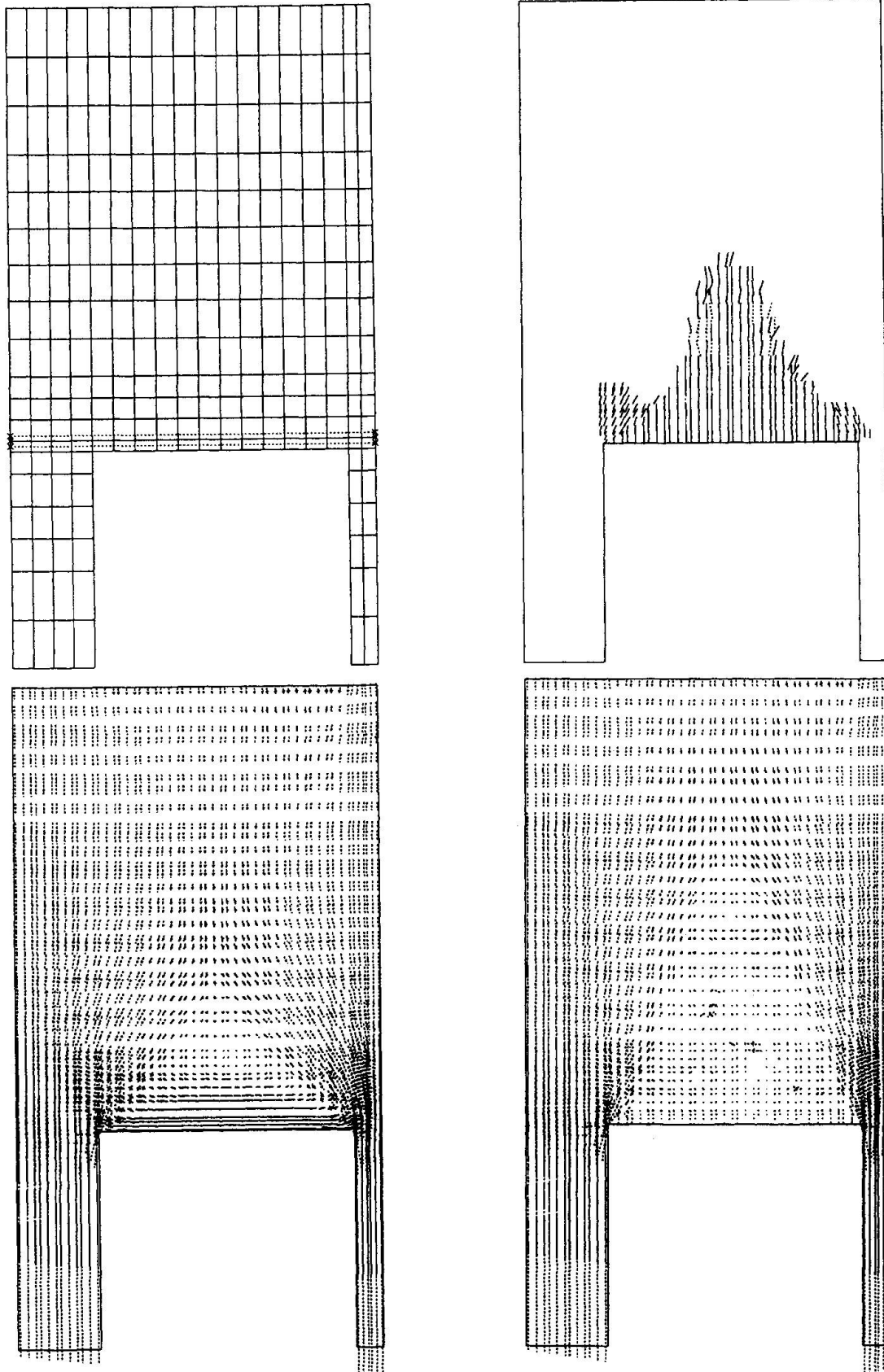
Fig. 4. Review-analysis of concrete wall in appartment building.
(a) Mesh with embedded reinforcement.
(b) Crack pattern in final stage.
(c) Principal stresses in linear stage.
(d) Principal stresses in final stage.

Main conclusions from the analysis were:

- The amount of additional reinforcement in the tie, which was accounted for in the analysis, proved to be sufficient.

- The average crack strain at service load 1.0, divided by an assumed crack spacing, led to crack widths that were well within code limits.

- The maximum compressive stresses in the narrow strut remained below the prescribed strength value in the code.

Based on this review analysis, a low-cost repair could be undertaken, consisting of a limited amount of additional horizontal reinforcement in the top layer at the floor close to the wall. Inclusion of additional vertical reinforcement or local thickening of the wall, options that were originally hinted at by the surveying committee, could be circumvented. Even more important was the fact that due to the quick solution of the problem, construction of the building could be continued without interruption, which saved significant costs.

6. EXAMPLE 2: CROSS SECTION OF TUNNEL STRUCTURE

The second example relates to a damaged part of an existing tunnel structure, which was submitted to consultancy in the Netherlands. This led to a review analysis of the shear capacity and of the bond, the results of which are described in detail in [13]. Herein, only a brief extract of the results is shown, namely the plots of the principal stress trajectories in the linear-elastic stage and the ultimate-load stage (Fig. 5). In the linear-elastic stage the concrete tensile stresses are found to make a substantial contribution to the transfer of the shear force. At ultimate load these tensile stresses have entirely disappeared because of cracking, and a pronounced thrust arch can be observed which is tied by the the midspan tensile reinforcement. The example strengthens the conclusion in the preceding section as to the capability of nonlinear finite element analysis of predicting 'strut-and-tie' systems in structural concrete after significant stress redistribution.
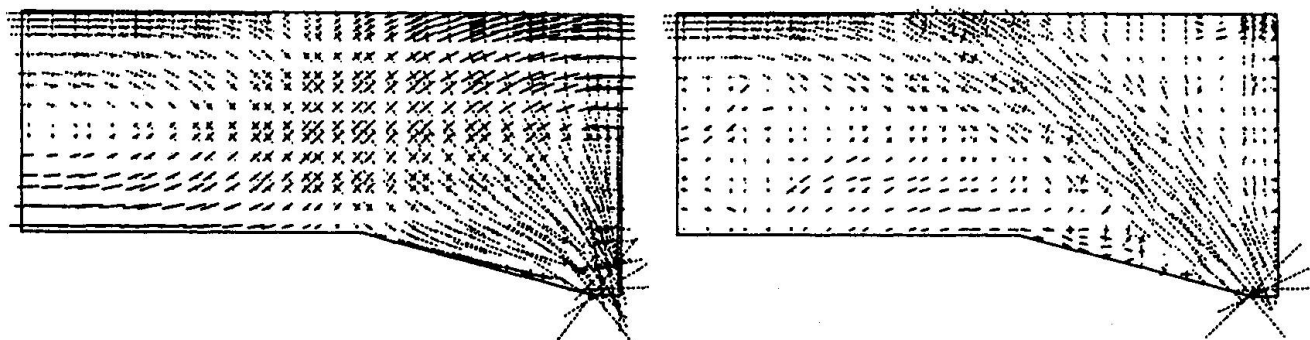

Fig. 5. Transition from linear-elastic behavior to 'strut-and-tie' system in tunnel roof.
    (a) Principal stresses in linear-elastic stage.
    (b) Principal stresses at ultimate-load stage.

7. EXAMPLE 3: DEEP BEAM

The third example concerns the reinforced deep beam which is also discussed in the key-note paper by Schlaich [16]. In that paper it was proposed to orientate the geometry of a 'strut-and-tie' model at the elastic stress fields, while this orientation can be adjusted upon approaching failure. It is interesting that a somewhat similar procedure was followed in [14] for nonlinear finite element analysis. The beam was first analysed in a global sense using smeared cracks. Subsequently, based on the crack pattern obtained, a simpler model was

made with a single predefined discrete crack, incorporated via interface elements. All nonlinearity was lumped into this discrete crack, surrounded by elastic elements for the strut and embedded rebar elements for the tie. Fig. 6 shows results of this 'predictor-corrector' approach, which turned out to work also for shear-critical problems [1]. Assuming sets of predefined discrete cracks in essence comes close to the yield line theory in plasticity where one imagines mechanisms of yield lines. This approach possibly has potential not only for review analysis, but also for dimensioning of so-called D-regions, where D stands for discontinuity, disturbance or detail.
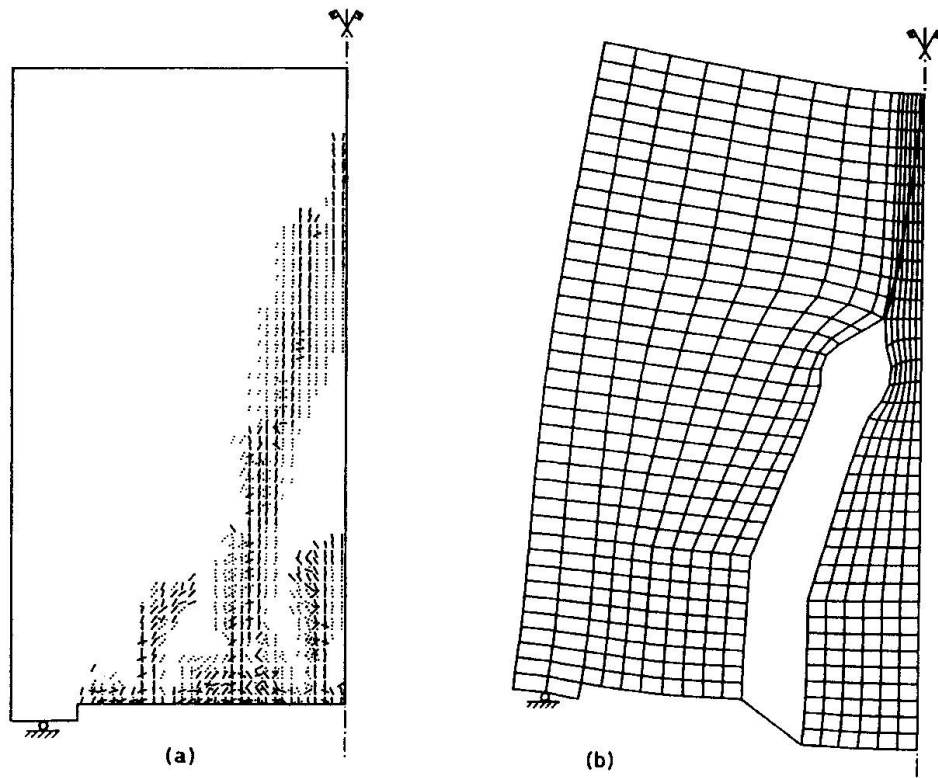

Fig. 6. Deep beam. Two possible strategies in nonlinear finite element analysis.
(a) Smeared crack analysis for prediction of crack path.
(b) Discrete crack analysis with predefined mechanism.

CONCLUDING REMARKS

It is concluded that the strength of very sophisticated bond models lies more in research than in practice. The utility of such models is of an indirect nature. These models for instance lead to a better understanding of basic bond mechanisms and better design rules for crack spacing and width. Global models with embedded reinforcement techniques are directly applicable in engineering practice. Examples have been presented that simply and correctly reveal 'strut-and-tie' systems after stress redistribution.

ACKNOWLEDGEMENTS

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