Seismic behaviour of guyed masts

SEISMIC BEHAVIOUR OF GUYED MASTS

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SUMMARY

The method for computation of natural vibrations of guyed masts with respect to the mass of guys is described and the possibility of solving the amplitudes excited by the motion of the ground using modal analysis is demonstrated. At last the check of stresses caused by the permanent change of the distance between shaft and guy foundation blocks is given.

RÉSUMÉ

On a développé dans cette article une méthode de la résolution des vibrations propres du pylône en prenant compte la masse reparti des haubans. On y présente aussi une méthode de la résolution des amplitudes des vibrations, qui prendent naissance dans le mouvement des appuis, par la décomposition d'après les formes propres, aussique la méthode de la résolution d'influence du changement permanent de distance parmi des foundations du pylône et des haubans.

ZUSAMMENFASSUNG

Im Artikel wird gezeigt die Berechnung von Eigenschwingungen des abgespannten Mastes mit Berücksichtigung der Masse von Pardunen, und die Lösung von seismisch erregten Schwingungen nach dem Verfahren der Zerlegung in Eigenschwingungformen. Zum Schluss wird die Schätzung von Spannungen gegeben, die durch die nachhaltige Veränderung des Abstandes zwischen Schaft und Pardunen Fundament verursacht sind.

1. INTRODUCTION

Guyed masts are structures enabling the attainment of great heights at relatively low costs and are, therefore, the tallest structures in the world in general. Their static as well as dynamic behaviour, on the other hand, is very complicated: a considerable source of difficulties in the static analysis are the nonlinearity of supports and the great deformations of the shaft. In dynamic analysis the assumption of linearity may be preserved in the majority of the cases, but great difficulties are due to a large number of natural frequencies and modes, and to their mutual proximity. Particularly if the computation considers the mass of guy cables the frequency spectrum is very dense. E.g. for a television mast with 4 guy levels and the height of 320 m altogether 14 natural frequencies were found within the limits of 0 - 1.0 Hz. This number is even higher if the mast is guyed by different cables in each level /e.g. oscillating about the equilibrium position at a certain deflection due to static wind/. A radio mast with heavy cables carrying heavy insulators will also have a major number of natural frequencies. On the other hand, in the case of a mast with light cables of man-made fibres the mass of these cables will practically play no role at all.

The seismic excitation of the mast may be due to either the motion of the shaft foundation /horizontal displacement or rotation, if the mast is clamped at its foot/, or to the horizontal movement of the cable anchorage foundation. Vertical movement is not dangerous for the shaft, it could endanger the galleries or cabins located on the mast; in the case of guy cables it would come into consideration in exceptional cases only, since it does not produce simultaneous oscillations of the shaft. The frequencies of natural seismicity /2 - 5 - 10 Hz/ are comparable with higher natural frequencies of the mast. These, however, similarly as the higher modes, cannot be estimated approximatelly; it is therefore very difficult to find some quasistatic solution. Intuitively it is possible to say that these modes, in which the higher vibrations of the cables prevail, do not manifest themselves, since the damping of the cables is in general higher.

On the other hand the shaft, usually of allwelded design or connected by means of friction bolts, has an extremely low damping, so that the modes in which the shaft vibrations prevail, can be excited even in high frequencies.

Simultaneous, but probably very little synchronous excitation of the shaft and guy-anchorage foundations should reduce the resulting excitation of the mast as a whole. Newertheless, at higher frequencies the excitation transferred by the cables can be absorbed and the vibrations of the shaft itself may ensue.

Another danger to a guyed mast during earthquake is due to the fact that a permanent change of the distance between the foundation of the shaft and the foundation of the guy cable may occur. A suitable location and foundation of the mast in uniform geological conditions should eliminate this undesirable effect; however, if it is not possible, it is necessary to foresee a certain change of these distances and estimate it in the design. The stresses due to this effect may be considered as temporary; after the earthquake the cable length may be rectified and the shaft stresses annuled.

During stochastic excitation of the limited white noise type the response of the mast consists of the vibrations in natural modes, whose frequencies are very near one to the other. In this way beats occur and the case may arise in which the top of the mast is suddenly displaced by a considerable amount in a certain moment. As a rule this effect will not be dangerous for the mast; however, if it has been provided at the top with a pendulum vibration absorber of a considerably lower frequency than the frequency of the afore mentioned lurch, the absorber may break away. In such a case the absorber should be therefore provided with buffers limiting its displacements.

2. DETERMINATION OF NATURAL VIBRATIONS OF A GUYED MAST

For the solution of natural vibrations of a guyed mast a computer programme has been in use in our institute for a number of years, modelling the mast as a continuous beam on elastic supports with constant normal force and bending stiffness in every span, and solving it by the slope-deflection method [1]. /This programme was written by J.Náprstek/. Recently it was completed by the introduction of the mass of guy cables, using Koloušek's solution [2], further elaborated and experimentally verified by Davenport [3].

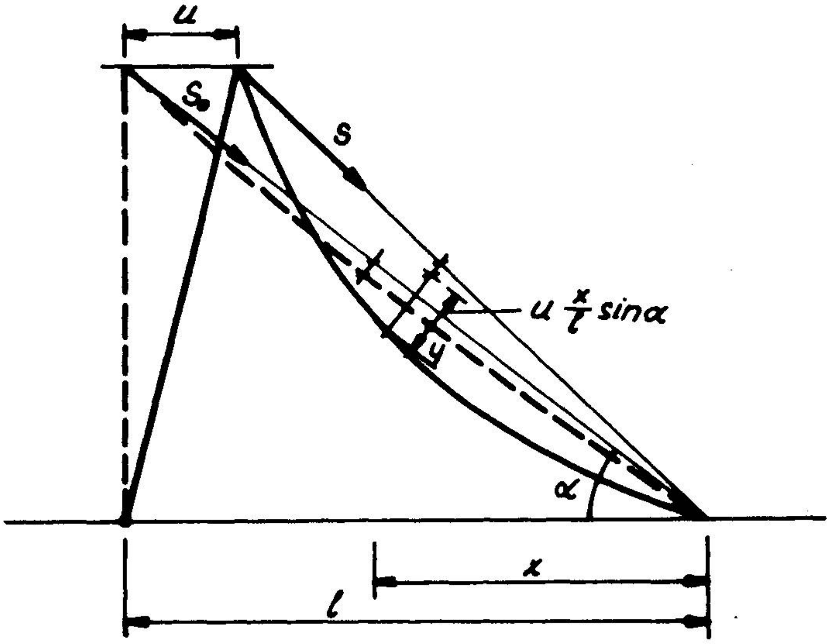

Fig. 1 Displacement of one guy of the support

According to these for the cable amplitude can be written /Fig.1/

$$y(x) = u\frac{x}{l}\sin\alpha + uL\left[\frac{1}{1-\Omega^2}\sin\frac{\pi x}{l} + \frac{1}{3(9-\Omega^2)}\sin\frac{3\pi x}{l} + \frac{1}{5(25-\Omega^2)}\sin\frac{5\pi x}{l} + \cdots\right]$$

$$L = -\frac{4}{\pi}\cos\alpha\,\frac{1-\dfrac{\pi^2 S_0^2\,\mathrm{tg}\,\alpha}{2EA\mu g l}\,\Omega^2}{\dfrac{\pi^2 S_0^2}{EA\mu g l} - \dfrac{\mu g l}{S_0}\,\eta(\Omega)} \qquad /1/$$

$$\Omega = \frac{l}{\pi\cos\alpha}\sqrt{\frac{\mu}{S_0}}\,\omega \quad ; \quad \eta(\Omega) = \left[1 - \frac{2}{\pi\Omega}\,\mathrm{tg}\left(\frac{\pi\Omega}{2}\right)\right] : \Omega^2$$

and for the horizontal force in the upper end of the cable

$$R = -u\,\frac{S_o \cos^3\alpha}{l}\;\frac{1-\mu q l\,\Omega^2 \operatorname{tg}\alpha\;\eta(\Omega) : 2S_o}{\frac{S_o}{EA}-\left(\frac{\mu q l}{\pi S_o}\right)^2 \eta(\Omega)} \qquad /2/$$

The reaction of a cable support depends on the frequency of motion. For 3 cables in one level it equals 1.5 times the force per one cable /2/, for 4 cables double the force. If the cables in one support are different, their forces must be composed generally in the resultant reaction. An example of the "spring constant" $D = R : u$ of such a reaction is shown in Fig.2. With the excep-

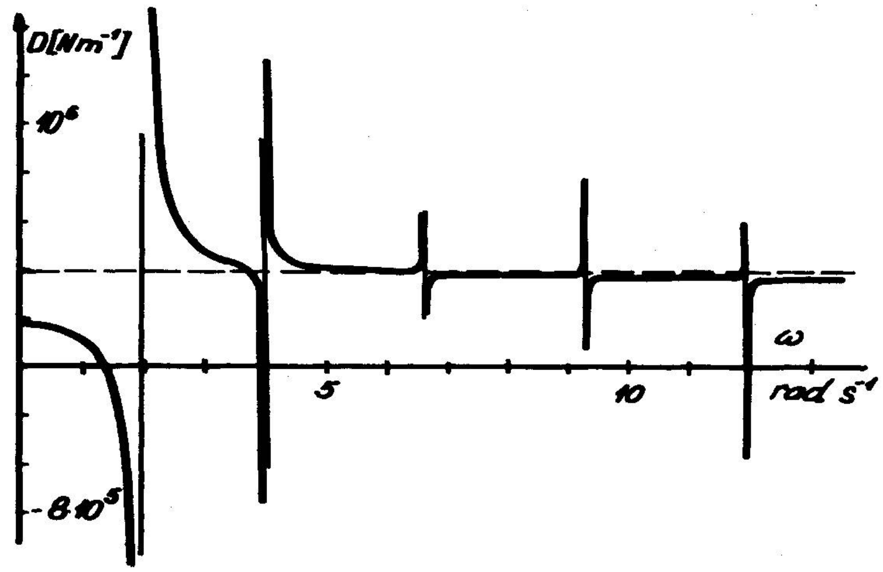

Fig. 2 Dynamic stiffness of a guy support

tion of the first or the first two the discontinuity points are determined by the frequency parameters $\Omega = 3, 5, 7, \ldots$ For higher frequencies the cable stiffness is determined mostly by its axial deformation

$$R = -u\,E\,A\,\cos^3\alpha \;:\; l \qquad /3/$$

The solution of natural frequencies is carried out as follows: for a chosen value of circular frequency first the support stiffnesses are determined, then the terms of the determinant of the system of equations of motion and the value of this determinant /its degree is twice the number of spans of the mast/. By the repetition of this procedure the frequency is found for which the determinant equals zero. The number of discontinuity points of the determinant is increased by the discontinuities of each cable. From Fig.2 can be seen, that these affect a very narrow band and in computation can be lost even with a very small step /$\Delta\omega = 0.01$ rad.s^{-1}/. In such a case the zero point approaches the asymptote, i.e. the natural frequency of the mast is very near to the natural frequency of the cable; it is therefore of some importance only for this one cable and with reference to the mast as a whole they play no practical role. In most cases it will be sufficient to consider the stiffness of supports /2/ only until the 2nd natural frequency of the support and further on the constant stiffness /3/ can be used. To the natural frequencies determined in this way the natural frequencies of the isolated cables should be added assuming that in these vibrations the mast remains at rest. After the determination of natural frequencies also the ratio of node displacements and rotations is obtained, from which the natural mode of the shaft and that of the cables /1/ is determined, and the generalised mass

$$m_{(j)} = \int \mu \, v^2_{(j)}(x)\,dx \qquad /4/$$

where the integral includes the whole mast and all guys.

3. SOLUTION OF RESPONSE OF A GUYED MAST TO SEISMIC EXCITATION

For the solution of the response of the structure to random as well as deterministic stationary excitation the modal analysis is most suitable, as a rule. Particularly in the case of little-damped structures it is always possible to consider one natural mode only whose frequency is near to the excitation frequency. By a suitable procedure it is possible to get over the fact that the amplitude of a structure with a moving support cannot be expressed by means of the base system of natural modes.

Starting from the general equation of motion of a prismatic beam

$$\mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu\,\omega_b \frac{\partial v(x,t)}{\partial t} + EJ \frac{\partial^4 v(x,t)}{\partial x^4} = 0 \qquad /5/$$

let us expand the solution according to the modes of natural vibrations and supplement it with zero term corresponding with the given excitation of zero frequency and unit displacement, thus having the meaning of the static deflection of the structure caused by unit displacement of the given support. Then it is

$$v(x,t) = v_o(x)\, q_o(t) + \sum_j q_{(j)}(t)\, v_{(j)}(x) \qquad /6/$$

Substituting the solution /6/ into /7/, multiplying the whole equation by $v_{(k)}(x)$ and integrating over the whole member we obtain

$$\ddot{q}_{(k)}(t) + 2\beta\omega_{(k)} q_{(k)}(t) + \omega_{(k)}^2 q_{(k)}(t) = -[\ddot{q}_o(t) + 2\beta\omega_{(k)}\dot{q}_o(t)] \qquad /7/$$

where it has been introduced

$$v_{(k)}^{IV}(x) = \frac{\mu\,\omega_{(k)}^2}{EJ} v_{(k)}(x) \;;\quad v_o^{IV}(x) = 0 \;;\quad \omega_b = \beta\,\omega_{(k)}$$

$$p_{(k)} = \frac{\int v_o(x)\, v_{(k)}(x)\, dx}{\int v_{(k)}^2(x)\, dx}$$

In /7/ $q_o(t)$ - time function of the support movement

β - relative damping

$v_o(x)$ - deflection line caused by unit displacement of the support .

Supposing harmonic excitation $q_o(t) = q_o \exp(i\omega t)$, /7/ will change into

$$\ddot{q}_{(k)}(t) + 2\beta\omega_{(k)}\dot{q}_{(k)}(t) + \omega_{(k)}^2 q_{(k)}(t) = q_o\omega^2 p_{(k)} - 2 q_o \beta\,\omega_{(k)}\omega\, p_{(k)}\, i \qquad /8/$$

The second term on the right hand side can be neglected for little damped structures. From /8/ the generalised coordinates are

$$q_{(k)} = \frac{q_o\omega^2 p_{(k)}}{\sqrt{(\omega_{(k)}^2 - \omega^2)^2 + 4\beta^2\omega_{(k)}^2\omega^2}} \doteq \frac{q_o p_{(k)}}{\left(\frac{\omega_{(k)}}{\omega}\right)^2 - 1} \qquad /9/$$

in the near vicinity of resonance

$$q_{(k)} = \frac{q_o\, p_{(k)}}{2\beta} \qquad /10/$$

This result means that the effect of the movement of the support on the structure can be replaced by the effect of loading, proportionate with the static defledtion curve due to the given displacement. The solution itself can be effected using modal analysis, and the resulting amplitude is superimposed to the original static deflection line. For little damped structures in /6/ one term prevails and for the resulting amplitude can be written

$$v(x) = v_o(x) + q_o\, p_{(j)} : 2\beta \qquad /11/$$

Numerical example

Let us determine the amplitude of vibrations of a beam clamped on both ends, whose lower end moves harmonically $v_a(t) = 1 \sin \omega t$. Natural frequencies of a clamped beam are

$$\omega_{(j)} = \lambda^2_{(j)} \sqrt{EJ : \mu l^2}$$

where $\lambda_{(j)} = 4.730$, 7.853 , 10.996 , 14.137 , ...

The corresponding natural modes are

$v_{(j)}(x) = (\mathrm{sh}\lambda - \sin\lambda)(\mathrm{ch}\lambda x{:}l - \cos\lambda x{:}l) - (\mathrm{ch}\lambda - \cos\lambda)(\mathrm{sh}\lambda x{:}l - \sin\lambda x{:}l)$

The static bending line for $v_a = 1$ is

$v_o(x) = 2\, x^3{:}l^3 - 3\, x^2{:}l^2 + 1$, $\quad q_o = 1$

The amplitude of the beam at the frequency $\omega = 4.50^2 \sqrt{EJ : \mu l^2}$ is described by generalised coordinates determined according to /9/, in this particular case neglecting the damping, viz.

$q_{(j)} = 2.989$, 0.0464 , 0.00791 , 0.00222 , $0{,}000803$, ...

The excitation frequency $7.60^2 \sqrt{EJ : \mu l^2}$ gives the coordinates

$q_{(j)} = -0.7762$, 2.739 , 0.0811 , 0.0195 , ...

An exact solution of the same example, obtained from a general equation

$v(x) = C_1 \,\mathrm{ch}\,\lambda x{:}l + C_2 \,\mathrm{sh}\,\lambda x{:}l + C_3 \cos \lambda x{:}l + C_4 \sin \lambda x{:}l$

with boundary conditions

$x = 0 :\ v = 1 ,\ v' = 0 \qquad x = l :\ v = 0 ,\ v' = 0$

yields the results whose graphic representation does not differ.

4. EFFECT OF THE CHANGE OF SUPPORT DISTANCE

A change of the distance between the foundation of the shaft and that of the cable can occur either by gradual displacing of the equilibrium position during seismic vibrations, or suddenly with a geological disturbance between both fundations. Further on the time-history of this displacement is not considered. It is only

assumed that this change is not so sudden as to produce impact phenomena in the cable. The method of solution used is analogous with that used in [4].
Let us consider one support of a mast, consisting of a windward and leeward cables /see scheme on Fig.3, notation according to Fig.1/. The foundation of the leeward guy has been moved from the shaft by $\Delta\ell$: the leeward guy is stretched, its sag is reduced and the support will displace in the same direction. This displacement reduces again the force in the leeward guy and increases the force in the windward one. If the mast is statically determined, this displacement will continue untill both forces are equal which will occur obviusly with the displacement $u = \Delta\ell : 2$.
If the mast is statically indeterminate, the displacement of the support causes the deformation of the shaft and consequently the plastic or elastoplastic reaction depending on the magnitude of the displacement. The new equilibrium position is determinated by the condition that the force in the windward guy and the reaction of the deformed shaft may equalize the force in leeward cable. If the displacement of the guy foundation procedes in a non-negligible velocity, the displacement of the support will continue by inertia beyond the equilibrium position so far, until the kinetic energy in this equilibrium position is annuled by a further deformation of guys ahd shaft. The process is then repeated giving rise to vibrations, which will be obviously soon damped.
The mast with 3 guys in one level can be solved similarly: instead of 2 windward guys one identical guy can be considered, its end displacement is, however, only 1/2 of the displacement of the support. The shortening of the distance ℓ will manifest itself by the reduction of the force in leeward guy and the support will be drawn towards the windward guy. This effect will probably be less dangerous, the solution would be similar.

The displacement of the support u brings about in the windward and leeward cables the forces related as follows

$$u = \frac{\ell}{\cos^2\alpha}\left[\frac{(\mu g \ell)^2}{24}\left(\frac{1}{S_{ow}^2} - \frac{1}{S_w^2}\right) + \frac{S_w - S_{ow}}{EA}\right] \qquad /12/$$

$$u = \frac{\ell}{\cos^2\alpha}\left[\frac{(\mu g \ell)^2}{24}\left(\frac{1}{S_\ell^2} - \frac{1}{S_{o\ell}^2}\right) + \frac{S_{o\ell} - S_\ell}{EA}\right] \qquad /13/$$

where μg - weight of unit length of the guy

EA - tensile stiffness of the guy

S_{ow} , S_{ol} - initial forces in windward, leeward guys

S_w , S_l - forces in the guys at the displacement u

In original equilibrium position $S_{ow} = S_{ol} = S_o$.

Owing to the displacement of the foundation of the leeward guy in leeward guy S_o changes into S_{ol} , for which holds /13/with

$u = -\Delta l$, $S_{ol} = S_o$, $S_l = S_{ol}$

Using dimensionless expression

$$\psi = \frac{\mu g l}{S_o} \quad ; \quad \varphi = \frac{EA}{S_o} \quad ; \quad \varkappa = \frac{S_{ol}}{S_o} \quad ; \quad \lambda = \frac{\Delta l}{l}\cos^2\alpha$$

/13/ can be rewritten

$$\varkappa^3 + \varkappa^2\left(\frac{\psi^2\varphi}{24} - 1 - \varphi\lambda\right) - \frac{\psi^2\varphi}{24} = 0 \qquad /14/$$

and consequently

$$\gamma_l^{\,3} + \gamma_l^{\,2}\left(\varphi\varepsilon + \frac{\psi^2\varphi}{24\varkappa^2} - \varkappa\right) - \frac{\psi^2\varphi}{24} = 0 \qquad /15/$$

Similarly for the force in windward cable from /12/ holds

$$\gamma_w^{\,3} + \gamma_w^{\,2}\left(\frac{\psi^2\varphi}{24} - \varphi\varepsilon - 1\right) - \frac{\psi^2\varphi}{24} = 0 \qquad /16/$$

In these expressions has been further introduced

$$\gamma_l = \frac{S_l}{S_{ol}} = \frac{S_l}{\varkappa S_o} \quad ; \quad \gamma_w = \frac{S_w}{S_{ow}} = \frac{S_w}{S_o} \quad ; \quad \varepsilon = \frac{u\cos^2\alpha}{l}$$

Solving /15/ and /16/ the reaction of the support can be obtained

$$R = (S_l - S_w)\cos\alpha$$

or

$$\varrho = \gamma_l - \gamma_w \quad ; \quad \varrho = \frac{R}{S_o\cos\alpha} \qquad /17/$$

An example of the calculated relation between the reaction R and displacement u of the support for various initial foundation shift Δl is shown in Fig.3.

For the analysis of stresses in the mast it is necessary to determine, apart from the force of the support R(u), also the reaction of the shaft to the displacement of the support under consideration P(u). It means to solve the mast without this support, loaded in this place by a horizontal force of various magnitudes. Obviously, this relation will have to be solved even beyond the yield limit considering the formation of plastic hin-

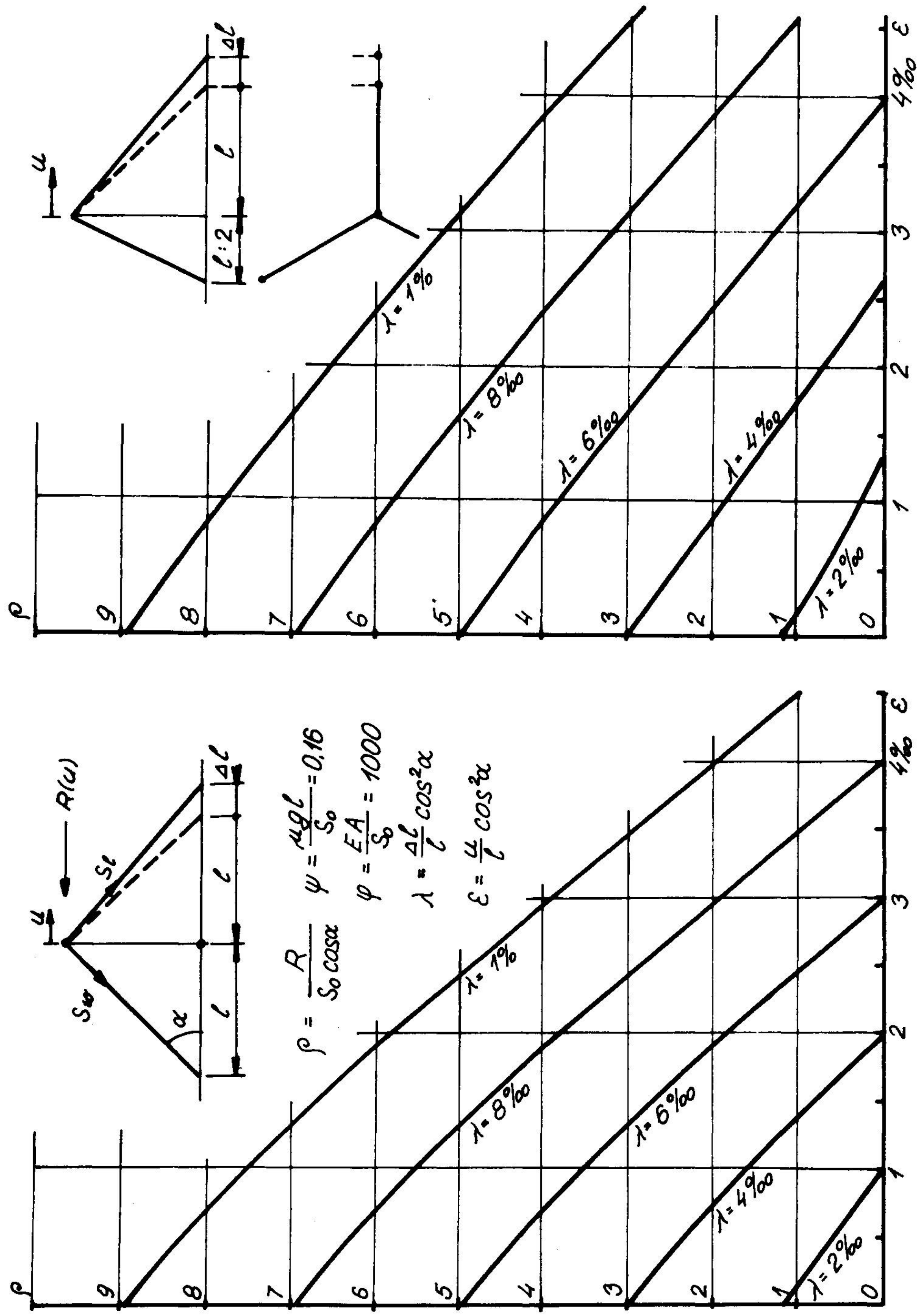

Fig. 3 Support reaction and displacement for different shifts of the guy foundation block

ges and the possibilities of mast failure. Both these relations will be plotted in one diagram /Fig.4/. At the point of intersection of both lines there lies the new equilibrium position of the support. The extreme displacement of the support is given by the condition of equal aereas between both lines before and beyond the equilibrium position. From the position of these points on the curve P(u) the danger of failure or the extent of damage can be judged.

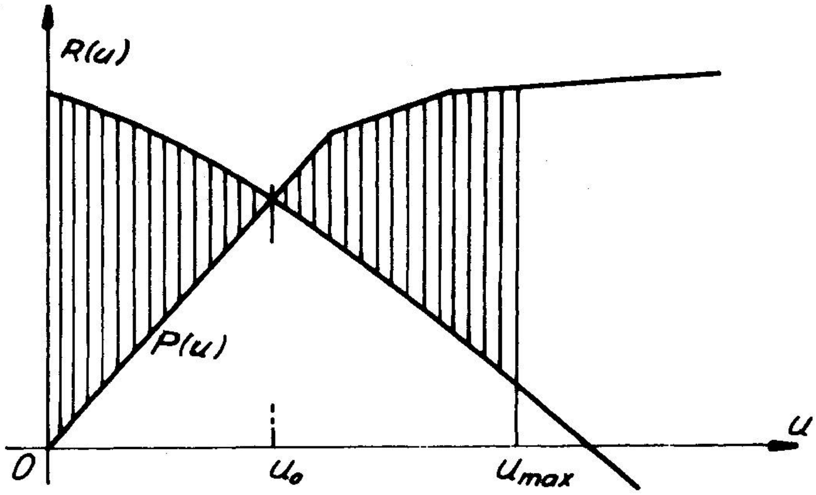

Fig. 4 Guy support force and restoring force of the mast

5. CONCLUSION

Generally small seismic effect can be expected from guyed masts, newertheless vibrations of the shaft in some higher mode can occur. The solution of such a case must be carried out exactly, since a simplified approach can hardly find higher modes.
In bad geological conditions the effect of the change of mutual position of foundations should be assessed.

REFERENCES

1. Koloušek, V.: Dynamics in Engineering Structures. Butterwords London, Academia Praha, 1973
2. Koloušek, V.: Solution statique et dynamique de pylones d'antenne. Mémoires AIPC V, Zurich 1947
3. Davenport, A.G, Steels,G.N.: Dynamic behaviour of massive guy cables. Proc.ASCE, ST 2, April 1965, p. 44 - 70.
4. Fischer, O.: Effect of rope failure on a guyed mast. Proc. conf. IASS-Cable structures, Bratislava 1975, p.282 -288.