# Influence of creep and shrinkage on the behaviour of reinforced concrete beams

# Influence of Creep and Shrinkage on the Behaviour of Reinforced Concrete Beams

Influence du fluage et du retrait sur le comportement de poutres en béton armé

Der Einfluss des Kriechens und Schwindens auf das Verhalten von Stahlbetonbalken

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## 1. INTRODUCTION

Recent experimental work in Australia has emphasized the importance of creep and shrinkage effects on the long-term behaviour of concrete structures. In field tests conducted in Melbourne on an experimental lightweight concrete flat plate structure, Blakey (1) has observed long-term deflections which were seven times the initial deflections. In Sydney, Taylor (2) has measured long-term deflections of from six to eight times the initial deflections in several in-service flat plate and flat slab structures. These figures are much in excess of the often quoted and often used value of 3.0 for the long-term deflection multiplying factor for concrete flexural members.

In an attempt to provide a theoretical framework for the study of creep and shrinkage effects in reinforced concrete members, a theoretical study has recently been made of the time-varying stresses and strains in a cracked beam section subjected to sustained moment (3). This analysis has also served as a basis for the theoretical investigation of long-term deflections (4).

The method of analysis of a cracked section is indicated briefly in the present paper and some results of the analysis, in particular concerning stress redistribution and increase in beam deflection with time, are summarized.

## 2. BASIC ANALYSIS

A rectangular, cracked, doubly reinforced section is considered with section width b, and the depths to the tension and compression steel areas of d and $\alpha$d, respectively. The proportion of tensile steel is p and the ratio of compression steel to tension steel is r. From equilibrium requirements at a typical time instant t, together with the usual assumptions of perfect bonding and linear distribution of stress and strain, expressions can be obtained for the extreme fibre concrete compressive stress, $\sigma_1(t)$, and for the tensile and compressive steel stresses, $\sigma_s(t)$ and $\sigma_s'(t)$, in terms of the constant applied moment M and the parameter k(t) defining the time-varying position of the neutral axis of stress,

$$\sigma_1(t) = +6\frac{M}{bd^2}\frac{1-k(t)-r(k(t)-\alpha)}{k(t)P_1} \qquad \ldots.(1)$$

$$\sigma_s(t) = -6\frac{M}{bd^2}\frac{1-k(t)}{2pP_1} \qquad \ldots.(2)$$

$$\sigma_s'(t) = +6\frac{M}{bd^2}\frac{k(t)-\alpha}{2pP_1} \qquad \ldots.(3)$$

In these equations $P_1$ is a polynomial in k(t),

$$P_1 = 3-4k(t)+k(t)^2+r\,\{3\alpha^2-4\alpha k(t)+k(t)^2\} \qquad \ldots.(4)$$

If the simplifying assumption (3) is made that the neutral axes of stress and strain coincide, the corresponding strains become,

$$\varepsilon_1(t) = +6\frac{M}{bd^2}\frac{k(t)}{2pP_1E_s} \qquad \ldots.(5)$$

$$\varepsilon_s(t) = -\frac{1-k(t)}{k(t)}\varepsilon_1(t) \qquad \ldots.(6)$$

$$\varepsilon_s'(t) = +\frac{k(t)-\alpha}{k(t)}\varepsilon_1(t) \qquad \ldots.(7)$$

Viscoelastic behaviour of the concrete is taken into account by applying a version of the Dischinger equation of state to the outermost compressive concrete fibre,

$$\dot{\varepsilon}_1(t) = \frac{\dot{\sigma}_1(t)}{E_c} + \dot{\phi}\{\frac{\sigma_1(t)}{E_c} + \frac{\varepsilon_{sn}}{\phi_n}\} \qquad \ldots.(8)$$

Expressions for $\sigma_1(t)$ and for the time derivatives $\dot{\sigma}_1(t)$ and $\dot{\varepsilon}_1(t)$ are obtained from Eqs. 1, 4 and 5 and substituted into Eq. 8 to give a first order, non-linear differential equation in k,

$$\frac{dk}{d\phi} = \frac{2npkP_1(P_2+P_1S)}{k^2(P_1-P_3)+2np\{(1+r)kP_1+P_2P_4\}} \qquad \ldots.(9)$$

In Eq.9 $k=k(t)$, $\phi = \phi(t)$ and the additional polynomials in k are,

$$P_2 = 1-k-r(k-\alpha) \qquad \ldots(10)$$

$$P_3 = 2k\{k-2+r(k-2\alpha)\} \qquad \ldots(11)$$

$$P_4 = 3-8k+3k^2+r(3\alpha^2-8\alpha k+3k^2) \qquad \ldots(12)$$

The constant term S includes both the loading effect M and the shrinkage quantity $\varepsilon_{sn}$,

$$S = \frac{1}{6}\,\frac{bd^2}{M}\,\frac{\varepsilon_{sn}E_c}{\phi_n} \qquad \ldots(13)$$

The term n is the modular ratio for instantaneous loading, $E_s/E_c$.

Analytic solutions of Eq. 9 can be obtained (3). Alternatively, a simple, second order Runge-Kutta method can be used to evaluate a sequence of k values $k_o$, $k_1$, $k_2$,...$k_i$,...$k_n$, corresponding to a sequence of $\phi$ values of 0, $\Delta\phi$, $2\Delta\phi$, $i\Delta\phi$,... $\phi_n$.

The recursion equation is

$$k_{i+1} = k_i + \frac{\Delta\phi}{2}\{f(k_i,\phi_i) + f(k_i+\Delta\phi f(k_i\phi_i),\phi_i+\Delta\phi)\} \qquad \ldots(14)$$

in which $f(k,\phi)$ is the right hand side of Eq. 9. The initial value at time zero, for $\phi_o = 0$, is given by modular ratio theory as

$$k_o=\sqrt{p^2(n+(n-1)r)^2 + 2p(n+\alpha(n-1)r)} - p(n+(n-1)r) \qquad \ldots(15)$$

## 3. TYPICAL STRESS REDISTRIBUTION

It is intuitively clear that an unloading of the compressive concrete occurs with time, with a corresponding increase in the steel compressive stress. The extent of this unloading, as predicted by the above analysis, can be surprisingly large. In many not unusual circumstances, a complete unloading of the compression concrete is predicted, with a subsequent development of concrete tensile stress in the "compression" zone above the neutral axis.

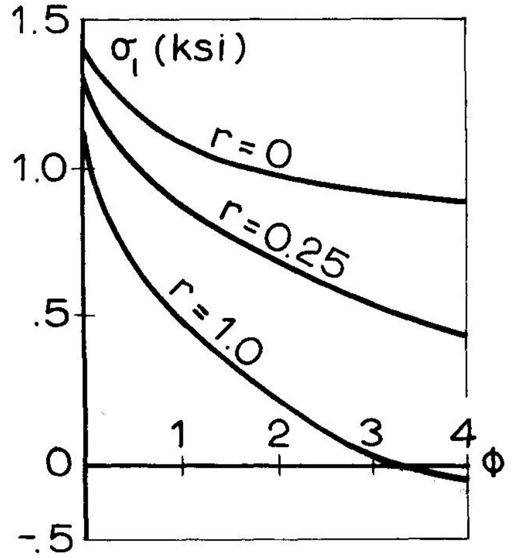


Fig. 1

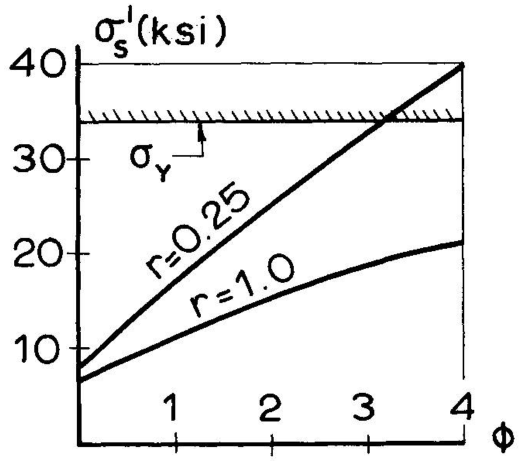


Fig. 2.

Unloading of the compressive concrete is most severe when the terms r and S are both large; that is, when the section contains large amounts of compression steel, when the concrete shrinkage strains are large, and when, at the same time, the applied moment is relatively small.

In Fig. 1 the concrete stress $\sigma_1$ is plotted against $\phi$ (which may be regarded as a transformed time scale) for three different r values of zero, 0.25, and 1.0. The curves are plotted for p = 0.015, for a moment of $M=0.25bd^2$ ksi, and for material properties of n=10, $\phi_n=4.0$ and $\varepsilon_{sn}=0.0006$. For the case r=1.0, it is seen that $\sigma_1$ reduces from an initial value of 1,100 psi compression to 96 psi tension. Some experimental confirmation of such extreme redistributions of internal stresses has been found in published test data (3).

In sections containing smaller amounts of compression steel, the reduction in $\sigma_1$ is not so extreme; however, the compressive steel stress $\sigma_s'$ becomes larger as r decreases. Yielding under sustained service loading must be accepted as a normal occurrence when r is in the order of 0.25. This is shown in Figs. 1 and 2. In singly reinforced beams, r=0, the change in magnitude in $\sigma_1$ is less significant, as shown in Fig. 1. The time variation in the tensile steel stress $\sigma_s$ is seen in Fig. 3 to be relatively slight.

## 4. LONG-TERM DEFLECTIONS

The analysis described in Section 2 allows the radius of curvature at the cracked section,

$$\rho(t) = d/(\varepsilon_1(t)-\varepsilon_s(t)) \qquad \ldots(16)$$

to be determined at any time instant t. A useful measure of the increase in

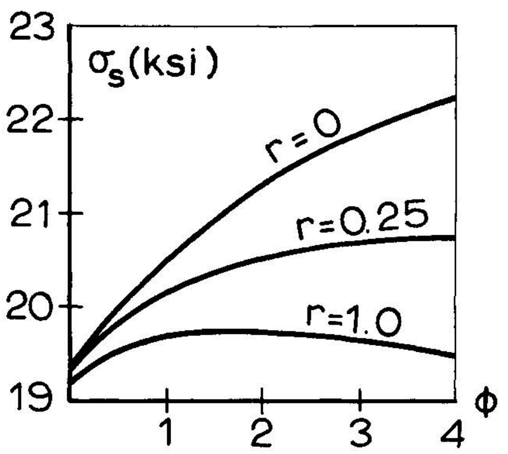


Fig. 3.

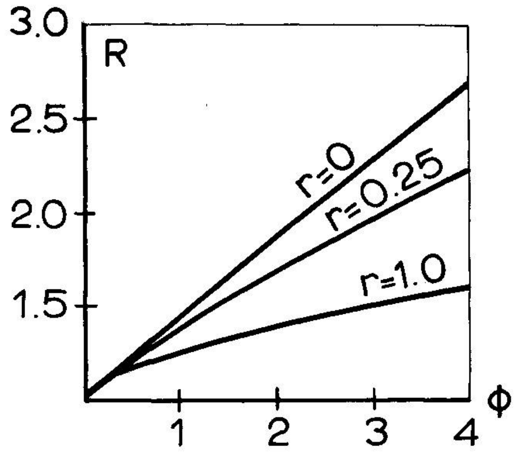


Fig. 4.

curvature in the section with time is provided by the curvature ratio,

$$R(t) = \frac{\rho_o}{\rho(t)} = \frac{\varepsilon_1(t)-\varepsilon_s(t)}{\varepsilon_1(o)-\varepsilon_s(o)} \qquad \ldots(17)$$

in which $\rho_o$ is the initial curvature at the time of load application. A more convenient expression in terms of $k_o$ and $k(t)$ is obtained from Eqs. 5, 6 and 17,

$$R(t) = \frac{3-4k_o+k_o^2 \quad +r\{3\alpha^2-4\alpha k_o+k_o^2\}}{3-4k(t)+k(t)^2 \quad +r\{3\alpha^2-4\alpha k(t)+k(t)^2\}} \qquad \ldots(18)$$

In Fig. 4 the variation in R with increasing $\phi$ is shown for the sections considered previously in Figs. 1, 2 and 3. Although the presence of compression steel causes unloading of the compression concrete, it is seen also to be very effective in controlling the increase in curvature and hence in reducing long-term deflections.

In a reinforced concrete beam under sustained loading the radius of curvature $\rho$ is a function of both time t and distance x along the span. It is therefore necessary to write $\rho(x,t)$ and $R(x,t)$. The deflection curve at time t is

$$y(x,t) = \int\int \frac{1}{\rho(x,t)}\,dx\,dx = \int\int R(x,t)\,\frac{1}{\rho_o(x)}\,dx\,dx \qquad \ldots(19)$$

If the simplifying assumption can be made that the curvature ratio is independent of moment, and hence of x, Eq. 18 reduces to the very convenient form,

$$y(x,t) = R(t)y_o(x) \qquad \ldots(20)$$

in which $y_o(x)$ is the initial deflection curve at time of loading and R(t) is, in effect, a time-increasing deflection multiplying factor. (It is tacitly assumed that the bending moment diagram does not change; i.e., the loading remains constant and there is no significant moment redistribution in time).

The above derivation of Eq. 20 draws attention to an important assumption underlying the use of deflection multiplying factors, namely that the curvature ratio R at any section is independent of, or at least insensitive to, the applied moment M. Inspection of Eqs. 9 and 18 shows that R can be independent of M only when the term S, defined by Eq. 13, is zero; non-zero values of S in Eq. 9 imply that k(t) is a function of M for all $t > 0$.

The term S provides a measure of the relative effect of shrinkage (load independent) deformations and creep and elastic (load dependent) deformations on long-term deflections. Since S is zero only when $\varepsilon_{sn}$ is zero, it can be concluded that <u>the use of deflection multiplying factors can be strictly valid only when shrinkage strains are zero</u>. Nevertheless, it also follows that multiplying factors should give reasonably accurate results provided the long-term increase in deflection is caused primarily by creep effects rather than by shrinkage effects, i.e. provided S remains small.

The simplified form of Eq. 20 allows the deflection multiplying factor for a beam to be calculated by carrying out an analysis of time-varying behaviour, as described in Section 2 of this paper, on a typical section (for example, the section of maximum moment). Numerous computer calculations have been made for a variety of sections and material properties (4). The results substantiate the above conclusion that R is insensitive to M provided S remains small. The results further show that R becomes particularly insensitive to M as the amount of compression reinforcement in the section increases. Average values of $R_n$, the long-term deflection multiplying factor (or, more correctly, the long-term curvature ratio), have been taken from these calculations and are presented in Table 1.

One final note of warning is warranted. In many practical situations shrinkage can have a decisive effect on long-term deflections, thus invalidating the use of multiplying factors. Often, for example, the sustained loading on a beam is a small proportion of the full service load. Not infrequently (particularly in Australia) concretes with comparatively high shrinkage characteristics are used in construction. In such instances, calculations based on long-term deflection multiplying factors can seriously underestimate long-term deflections. The experimental work of Blakey (1) and Taylor (2) provides adequate proof of this.

Table 1: LONG-TERM DEFLECTION MULTIPLYING FACTORS

| $\phi_n$ | Singly Reinforced Beams r = 0 | | | Doubly Reinforced Beams | | | |
|---|---|---|---|---|---|---|---|
| | Values of p: | | | Values of r: | | | |
| | .01 | .02 | .03 | .25 | .50 | .75 | 1.0 |
| 2 | 2.0 | 2.4 | 2.8 | 2.1 | 1.9 | 1.7 | 1.6 |
| 4 | 2.4 | 3.1 | 3.6 | 2.4 | 2.0 | 1.8 | 1.6 |
| 6 | 2.9 | 3.7 | 4.4 | 2.7 | 2.1 | 1.8 | 1.6 |

## 5. ACKNOWLEDGMENT

The work described in this paper forms part of a general study of creep and shrinkage effects in concrete structures. The initial phase of the investigation has been carried out in the School of Civil Engineering, University of New South Wales, under the sponsorship of the Building Research Division of the CSIRO. The help and assistance of Mr. T.W. Leong, Research Assistant, is gratefully acknowledged.

The author also wishes to acknowledge with thanks a research stipendium provided by the Alexander von Humboldt Foundation, which has enabled work on the investigation to be continued during 1970 at the Technological University, Braunschweig.

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SUMMARY

A theoretical analysis of the time-varying stresses and deformations in a cracked, reinforced concrete beam section subjected to sustained moment is briefly described. Numerical calculations indicate that the stress redistribution which occurs with time in doubly reinforced sections can be very considerable. In not unusual circumstances the initial compressive stresses in the concrete above the neutral axis can reduce to zero and even become tensile. The theoretical analysis also provides a means of evaluating long-term deflections for reinforced concrete beams under sustained loading.

RESUME

L'auteur décrit brièvement une analyse théorique de la variation en fonction du temps des tensions et des déformations dans une poutre en béton armé fissurée et soumise à un moment constant. Des calculs numériques montrent que la redistribution des tensions qui apparaît avec le temps dans des sections doublement renforcées peut être considérable. Dans des circonstances normales, les tensions de compression au-dessus de l'axe neutre peuvent être réduites jusqu'à zéro et même devenir négatives (traction). L'analyse théorique permet aussi l'évaluation à long terme de la flexion pour des poutres en béton armé sous charge permanente.

ZUSAMMENFASSUNG

Eine theoretische Untersuchung der zeitabhängigen Spannungen und Deformationen in einem gerissenen Stahlbetonquerschnitt unter konstantem Moment wird kurz beschrieben. Numerische Beispiele zeigen beträchtlich grosse Spannungsumlagerungen mit der Zeit, besonders in Querschnitten, die Druckarmierung erhalten. In nicht ungewöhnlichen Fällen können die Druckspannungen im Beton über der Nulllinie vollkommen verschwinden und sogar negativ (Zug) werden. Die zeitabhängigen Durchbiegungen in Stahlbetonträgern unter Dauerlast werden auch kurz behandelt.