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Autor: Ohno, Kazuo / Obata, Mamoru

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On the Design Provisions for Thermal Stresses

A propos des réserves de configuration des tensions thermiques

Über Vorkehrungen beim Entwurf für thermische Spannungen

KAZUO OHNO Professor
MAMORU OBATA Associate Professor
Faculty of Engineering
Hokkaido University
Japan

I. General feature of the thermal stresses in rectangular rigid frames with uniform spans and heights

1) Introduction

When some temperature difference occurs between the over-ground frames and footing tie-beams, both of the beams and footing tie-beams will be subjected to certain axial forces, because they are mutually constrained by being rigidly connected to the columns. These axial forces increase with the increase of the number of spans and with the increase of relative stiffness of columns, and cause the secondary length change of beams and footing tie-beams. It is evident that the axial forces of beams and footing tie-beams in the vicinity of the central part of frames will be larger than those of the external part of the frames, and that the "slopes" and the "deflections" of members would be different at each frame joint.

Some analyses of these kinds of thermal stresses were developed by the authors under the following assumptions:

(i) the structures consist of rectangular rigid frames with uniform spans and heights, and the beams and the columns have the respectively same dimension for the same story.

(ii) the whole members of the overground frames are exposed to the same temperature and a certain temperature difference exists between the overground parts of frames and underground parts.

(iii) the length change of columns is neglected, because the axial stress of the column caused by thermal deformation is usually very small. And also the length change of members caused by shearing stress and bending moment is neglected.

2) Effect of relative stiffness ratio of beam on the thermal bending moments

The thermal bending moments of one-storied frames of 1~8 spans, the columns of which are perfectly fixed on the ground, are theoretically derived after the "slope-deflection method". The thermal bending moments can be expressed as a function of the following terms:

k_b = relative stiffness ratio of beam,
$B = 6K_0 l/Ah^2$,
$C = 6EK_0 \alpha t_e l/h$,

where

K_0 = stiffness of column (taken as the standard stiffness),
l = length of beams and footing tie-beams,
A = sectional area of beam including floor slab,
h = length of column,
E = elastic modulus of the material of frame,
α = thermal expansion coefficient of material,
t_e = effective temperature difference between the overground frame and its footing or footing tie-beam.

In the case of a frame without floor slab,
$B = (d/h)^2/2k_b$, where d = depth of beam.

Several examples of the relation between M/C and k_b are obtained by taking d/h as a parameter. When the length change of beam due to an axial force is not taken into consideration, d/h must be zero. In this case, M/C shows the largest value, if k_b is kept equal. It is also found that the value of M/C increases with the increase of the value of k_b. And M/C has a tendency to converge to a certain value for each frame, when k_b becomes comparatively large.

Fig.1 shows an example of the feature of thermal bending moments of each column of those frames analyzed here. When the other conditions are equal, the larger the number of spans, the larger the thermal bending moments of the columns. In the case of 8-spanned frames, the bending moments in columns excepting two columns at the end show approximately a linear variation. Further, the bending moment in each column tends to change by the value of d/h. The value of M/C of the end column would not always be the largest for a certain value of d/h.

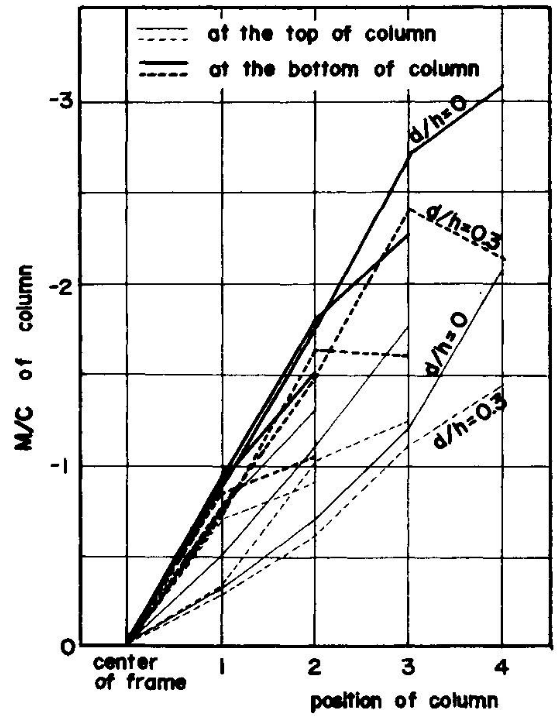


Fig.1 M/C of each column of 1-storied frames of 2 ~ 8 spans.

Since the thermal stresses are deeply related with the stiffness of members as is described above, the partitions and the walls with openings which are usually treated not to share the force, must also be taken into consideration in the case of the accurate calculation.

3) The effect of the numbers of stories

Another theoretical analysis is attempted for frames with two spans and 2 ~ 4 stories, according to the "slope-deflection method". The thermal bending moments are also expressed as the functions of k_b, d/h and C, by neglecting the presence of floor slab. Some examples of the relation between M/C and k_b are shown in Fig.2. The results of analyses show that:

(i) thermal bending moments of beams and columns are the largest at the first story and decrease rapidly toward upper stories, and they can be practically neglected for the stories upper than the third floor,

(ii) in the most external column, the sign of shearing force in the second story is opposite to that in the first story,

(iii) when the number of story is larger than three and k_b is larger than about 1.0, the magnitude of thermal bending moments of beams and columns at the first and second stories do not vary so much.

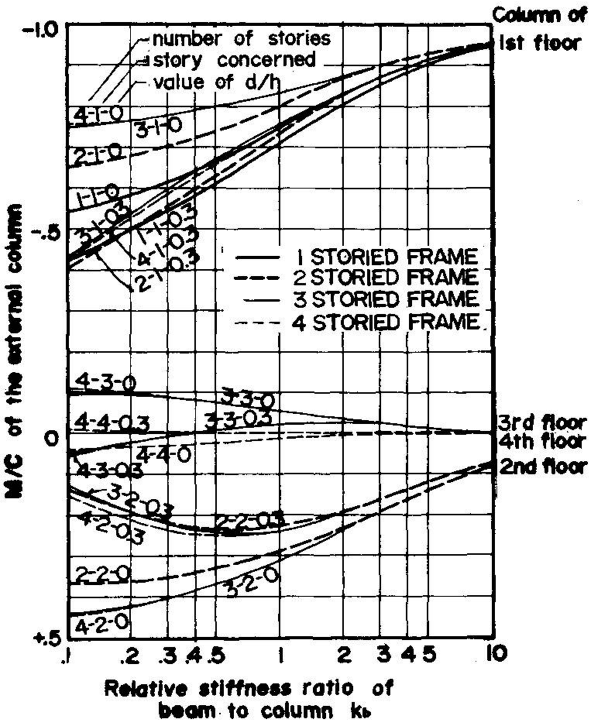


Fig.2 M/C of each column-bottom of 2 ~ 4 storied frames of 2 spans.

Consequently, it is comfirmed that the features of thermal bending moments of multi-storied frames are approximately represented by those of the two-storied frames.

TABLE 1. Shearing forces of columns of one-storied frame (ton)

condition of footing	column				
	center	No. 2	No. 3	No. 4	external
I perfectly fixed	1.16 (1.52)	2.39 (1.45)	3.75 (1.44)	5.52 (1.37)	5.45 (1.74)
II semi-fixed	1.085 (1.41)	2.228 (1.35)	3.443 (1.32)	5.513 (1.28)	4.033 (1.29)
III movable in horizontal direction	0.770 (1.00)	1.047 (1.00)	2.609 (1.00)	4.040 (1.00)	3.128 (1.00)
IV movable in any direction	0.0001 (0.0001)	0.0002 (0.0001)	0.0013 (0.0005)	0.0304 (0.0075)	0.2909 (0.093)

dimension of frame: $l = 6.5$ m $h = 3.0$ m
column 50×50 cm^2 $E = 210\ t/cm^2$
beam 30×50 cm^2 $\alpha = 12 \times 10^{-6}$
footing beam 30×70 cm^2 $t = 10$°C

The figures in the blackets show the ratios of shearing forces by taking (iii) case as a standard.

Fig.2 can be also applied for the top story, the roof floor of which is subjected to a different temperature.

4) The effect of the fixing conditions of foundation

The authors' observation on the movement of an actual one-storied building showed a considerable amount of displacement of foundation. A theoretical analysis is attempted on four kinds of fixing conditions of foundation:
(i) footing is perfectly fixed,
(ii) footing cannot move, but can rotate corresponding to the stiffness of footing tie-beam (semi-fixed condition),
(iii) footing can move in the horizontal direction and can rotate in the same way as (ii),
(iv) footing can move in any direction and can rotate in the same way as (ii).

Table 1 shows the results of numerical computations with the theoretical formulae under these four conditions. It can be seen that the thermal bending moments would remarkably decrease with the decrease of restraint of the footing. The semi-fixed condition is recommended for the practical estimation of the thermal bending moments.

II. The charts for estimating thermal bending moments

The thermal bending moments of 240 rectangular frames of two-storied buildings were numerically computed for various parameters: total number of spans, span length, story height, k_b, k_f (relative stiffness ratio of footing tie-beam), B and M/C. The charts for practical estimation of thermal bending moments are prepared in Fig.3 based on these results. In the charts, bending moments at the third floor are neglected, since they are very small.

For the application of the charts, determine the parameters k_b, k_f, k_f/k_b, B and C at first. Second, determine the coefficients η_1, η_2, from the charts.

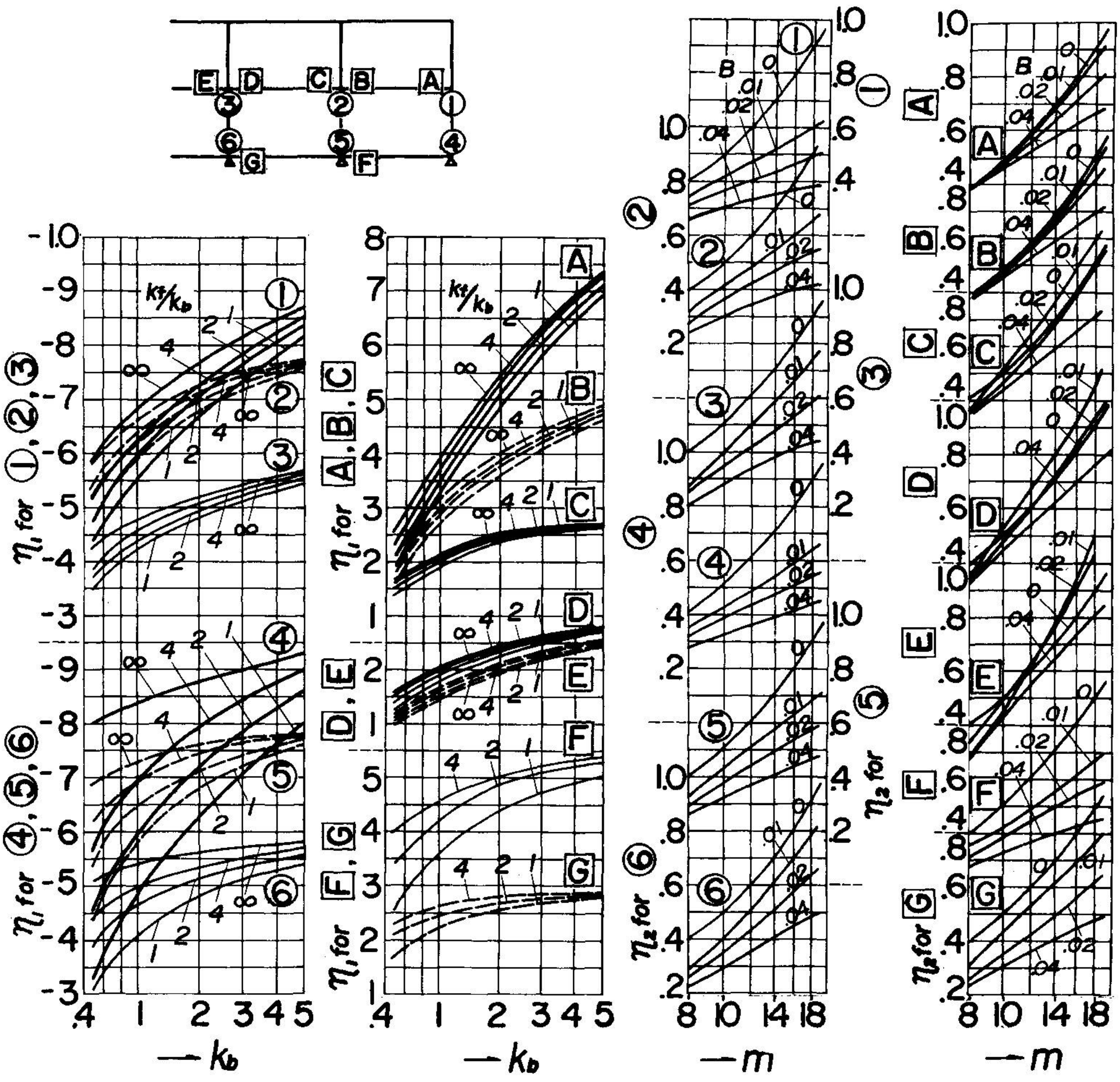
Then, the bending moment will be given by

$$M = \eta_1 \times \eta_2 \times C$$

The bending moments at the outside end of footing tie-beams and those at the column bottoms in the second story can be determined according to the equilibrium of moment at each joint.

In the case of the frames with an even number of spans, the moment of center columns must be zero. In the case of the frames with an odd number of spans, the bending moments of members excepting the two columns at the end span can be determined under the assumption that their magnitudes are in proportion to the distance from the imaginary center column which will be conveniently assumed to exist at the midpoint of the frame.

When one intends to estimate the bending moments of one-storied frames, the coefficients η_1 and η_2 for two-storied frames with the same number of spans in the charts must be applied. Calculate all of the bending moments. Then distribute again the obtained bending moment at the column bottom in the second story to the beams connected at the joint proportionally to the respective stiffness ratio.



The charts for η_1 The charts for η_2

Fig.3 Charts for estimating the thermal bending moments

$M = \eta_1 \times \eta_2 \times C$: thermal bending moment
$B = 6K_0 l/Ah^2$
$C = 6EK_0 \alpha t_e l/h$
k_b = relative stiffness ratio of beam
k_f = relative stiffness ratio of footing tie-beam
m = number of spans

Note: For the columns of the 2nd story,
(i) bottom moment must be determined by the equilibrium at each joint,
(ii) top moment can be neglected.

III. The temperature difference to be employed for design

It is already known by theoretical analysis on structures of perfect elasto-plastic material that a structure does not collapse due to a monotonous increase or decrease of temperature so far as the temperature difference is not large. On the other hand, it has also been reported that a repetition of temperature changes may sometimes cause a cyclic collapse or incremental collapse of structures. Therefore, the value of amplitudes of repeating temperature changes are most important for elasto-plastic structures.

In the case of reinforced concrete structures, it has been proved experimentally that the elasto-plastic theory can be applied under some limited conditions. However, the theory can not be applied when a local failure of concrete structure would be caused by the shear failure. In such a case, the absolute value of temperature change, which gives the maximum stresses in the frame, becomes important.

In respect to the temperature difference for practical design, the concept of effective temperature, which is derived by the results of observations on the actual structures, is recommended. Probably in summer or in winter the mean temperature of structural members of the building which has not air conditioning in rooms would almost be equal to the mean atmospheric temperature. On the other hand, the result of the observation shows that the maximum magnitude of daily movement of the reinforced concrete building corresponds to the value estimated for about two-thirds of the daily difference of atmospheric temperature. Therefore, the "effective temperature" in annual range may be expressed by the following form (Fig.4):

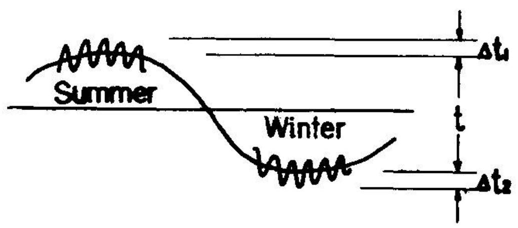


Fig.4

$$t_e = t + 2(\Delta t_1 + \Delta t_2)/3$$

where, t : the largest annual amplitude of daily mean temperature,

Δt_1 : one half of the daily amplitude at the annual maximum point of daily mean temperature,

Δt_2 : one half of the daily amplitude at the annual minimum point of daily mean temperature.

From summer to winter, the structures tend to contract following the fall of temperature, and the axial stresses of beams also shift to tension. In the case of reinforced concrete, tensile cracks will occur in the beams and floors in this process and the deformation of the structure will be somewhat relaxed. Dry shrinkage of concrete would further this tendency of cracking.

On the other hand, the stresses due to a temperature change in a long term or the stresses caused by dry shrinkage should be affected by the creep properties of concrete. When the effects of creep of concrete is taken into account, the stress analysis for

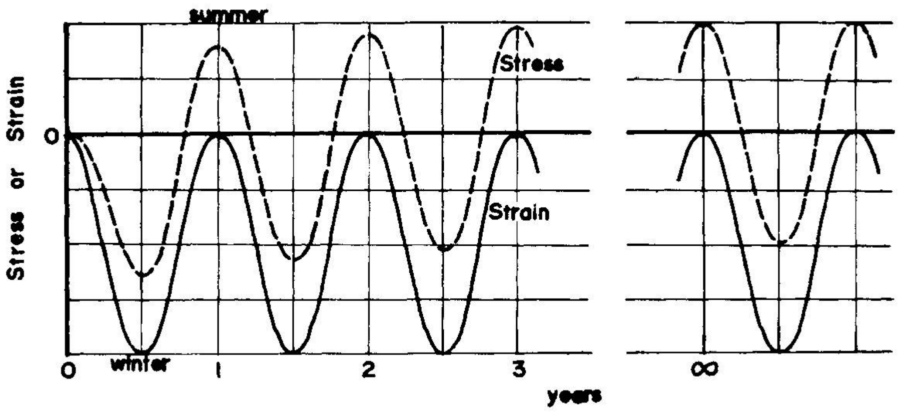


Fig.5

repeating temperature changes lead to the conclusion that the thermal stresses of buildings will approach to the values which can be obtained from the elastic analysis for one half of the effective temperature as is shown in Fig.5. A large part of this process is attained in the first year after construction.

When both of the stresses due to dry shrinkage and the temperature difference are to be taken into consideration, the following two kinds of stresses must be superposed:

(i) the stresses due to dry shrinkage including the effects of creep,

(ii) the thermal stresses for one half of the effective temperature.

Summary

General features of thermal stresses of the rectanglar rigid frames to be caused by a temperature difference between the overground part and the foundation is described. Some charts for estimation of thermal bending moments were proposed. Also, the temperature which should be applied for the computation of thermal stresses is discussed based on the measurements of thermal length change of the actual buildings.

Résumé

Nous avons décrit les caractéristiques générales des forces thermiques agissant dans un cadre rigide rectangulaire et causées par une différence de température entre la partie supérieure et la fondation. Quelques diagrammes se rapportant à l'estimation des moments de flexion dus à la température sont présentés. Aussi la température qui devrait être appliquée pour le calcul des forces thermiques est soumise à une analyse basée sur les mesures des variations thermiques de longueur dans les constructions actuelles.

Zusammenfassung

Es werden die Hauptmerkmale thermischer Spannungen rechteckiger, steifer Rahmen beschrieben, welche aus Temperaturdifferenz zwischen Ueber- und Unterbau entstehen. Für die Schätzung der thermischen Biegemomente werden einige Schaubilder vorgeschlagen. Ebenso wird die für die Rechnung der thermischen Spannungen anzunehmende Temperatur anhand von Messungen an bestehenden Brücken diskutiert.