Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 5 (1970)

Artikel: Time-dependent forces induced by settlement in continous prestressed concrete structures

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DOI: https://doi.org/10.5169/seals-6911

I

Time-Dependent Forces Induced by Settlement in Continuous Prestressed Concrete Structures

Forces liées au temps par suite du tassement dans les poutres continues en béton précontraint

Zeitabhängige Kräfte infolge Setzungen in durchlaufenden Spannbetonträgern

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INTRODUCTION

The reactions and internal forces induced by settlement in a prestressed concrete structure depend on creep occurring during and after the period in which the movement of support occurs. If the total settlement occurs suddenly, the forces induced will have a maximum initial value which may be calculated by an elastic analysis. Due to creep, this value will decrease continuously with time. In practice however, the settlement process takes place simultaneously with the creep over longer periods of time. Thus the forces due to settlement have initially a value of zero and later on a maximum value which is considerably smaller than the elastic. To allow accurately for differential settlement in design, it is important to take into account the time effects.

In an earlier paper[1], a step-by-step method was developed for predicting the change in reactions in continuous *reinforced concrete* beams caused by support settlement taking into account the variation with time of creep, modulus of elasticity of concrete, effective flexural rigidity and settlement. The method was verified by experiments.

Dimel[2] in a theoretical investigation analyzed the settlement effects using the "rate of creep" approach. This approach does not give accurate results because most of the creep recovery is neglected.

In the present investigation we are concerned with fully prestressed concrete continuous structures. A simplification is immediately obvious compared with the reinforced concrete beams considered earlier[1] in which development of cracks caused changes in the effective flexural rigidity. However, emphasis in the present paper is made on the difference between creep and creep recovery, or in other words, on the difference between creep due to unit stress increments and unit stress decrements. Thus, a bending moment on a prestressed concrete section produces different creep response in the fibres above and below the centroid.

THE PRINCIPLE OF SUPERPOSITION

The principle of superposition of creep of McHenry[3] leads to the following equation for the total strain at age t (instantaneous + creep) caused by a varying stress σ:

$$\varepsilon(t) = \int_{\tau_o}^{t} \frac{1}{E_\tau} \frac{d\sigma(\tau)}{d\tau} d\tau + \int_{\tau_o}^{t} f[\tau,(t-\tau)] \frac{d\sigma(\tau)}{d\tau} d\tau \qquad (1)$$

where τ_o is the age at which stressing starts and $f[\tau,(t-\tau)]$ is the creep function, that is the creep at age t due to a unit stress applied at any age τ. Eq. 1 may be put in the form:

$$\varepsilon(t) = \int_{\tau_o}^{t} \varepsilon^* \frac{d\sigma(\tau)}{dt} d\tau \qquad (2)$$

where ε^* is the total strain at age t due to unit increase in stress coming into action at age τ

$$\varepsilon^* = \frac{1}{E_\tau} + f[\tau,(t-\tau)] \qquad (3)$$

Several investigators tested the validity of the superposition principle. Ross[4] studying creep under variable stress demonstrated that the principle of superposition yielded better agreement between theory and experiment than other methods. The U.S. Bureau of Reclamation[5] in an extensive testing program also verified the general validity of this principle.

Davis[6] using mixes and stresses representative of prestressed concrete, found that creep recovery is smaller than creep. However, the superposition Eq. 1 does not differentiate between the creep caused by an increasing or decreasing stress, and therefore when there is a decrease in the stress the creep recovery will be overestimated. To correct for this, we introduce a reduction factor R equal to the ratio of specific creep recovery to specific creep and ε^* in Eq. 2 is replaced by

$$\varepsilon^*_R = \frac{1}{E_\tau} + Rf[\tau,(t-\tau)] \qquad (4)$$

in which $R = 1$ when $\frac{d\sigma(\tau)}{d\tau} > 0$ (5)

and $R = \theta(t-\tau)$ when $\frac{d\sigma(\tau)}{d\tau} <$ (6)

where $\theta(t-\tau)$ is a dimensionless function of the period $(t-\tau)$. The factor R in Eq. 4 is a function of $(t-\tau)$ only and not of both τ and $(t-\tau)$ (similar to the creep function) as may be expected. It has been found however from Davis' results as well as from tests by the present authors that the dependence of R upon τ is of minor importance.

THE CREEP FUNCTION

Using the time-creep curve recommended by the joint CEB-FIP Committee[7], the creep function is expressed as

$$f[\tau,(t-\tau)] = \frac{1}{E_\tau} \phi \zeta\rho \qquad (7)$$

where ζ and ρ are functions of τ and $(t-\tau)$ respectively and ϕ_N is the creep coefficient which is independent of time but a function of the environmental conditions, the dimensions and the composition of concrete. The committee defines ϕ_N as the ratio between the ultimate creep and the instantaneous strain which occurs in a creep test when the age at loading τ_o = 28 days. The factor ζ which accounts for age at loading is represented by a graph by the committee which may be approximated by the equation[1]:

$$\zeta = \frac{10.29}{5 + \sqrt{\tau}} \tag{8}$$

which yields the value 1.0 when τ_o = 28 days. The ratio of creep at any time after loading to the ultimate creep, if assumed to occur 2000 days after loading, can be represented in the form[1]:

$$\rho = 0.1315 \, \ell n \, [(t-\tau) + 1] \tag{9}$$

Substituting Eq. 8 and 9 into 7, we obtain the creep function

$$f[\tau,(t-\tau)] = \frac{1}{E_\tau} \phi_N \frac{1.35 \, \ell n \, (t-\tau+1)}{5 + \sqrt{\tau}} \tag{10}$$

From the test results presented later on, the recovery function R is found to be (see Fig. 3):

$$R = 0.6 + \frac{t-\tau}{40 + 3.2(t-\tau)} \quad \text{when} \quad \frac{d\sigma(\tau)}{d\tau} < 0 \tag{11}$$

Substituting Eq. 10 into 3 or 4 and the result into Eq. 2, we obtain the equation for the total strain at age t due to a stress of varying magnitude starting to act at age τ_o

$$\varepsilon(t) = \int_{\tau_o}^{t} \frac{1}{E_\tau} \left[1 + R \, \phi_N \frac{1.35 \, \ell n \, (t-\tau+1)}{5 + \sqrt{\tau}}\right] \frac{d\sigma(\tau)}{d\tau} \, d\tau \tag{12}$$

in which R = 1 for an increasing stress (Eq. 5) and is given by Eq. 11 when the stress is decreasing.

CREEP IN FLEXURE OF A PRESTRESSED MEMBER

Consider a prestressed member subjected to compressive stresses over the whole area of each cross section. Bending moment caused by settlement of a support will increase the value of the compressive stress at one face of the beam and reduce it on the other. The creep per unit change of stress will be f and Rf above and below the centroidal axis. If plane cross sections are assumed to remain plane, the stress distribution due to the moment caused by settlement will be nonlinear and a shift in the neutral axis must occur for equilibrium.

A bending moment increment ΔM produced at time τ on any section of the beam considered above will cause an instantaneous curvature

$$\Delta \left.\frac{d\alpha}{dx}\right|_\tau = \frac{\Delta M}{E_\tau I} \tag{13}$$

where α is the slope of the deflection line, x is the distance along the beam axis and I is the moment of inertia of the section. If concrete had the same response for creep and creep recovery, the curvature after a period (t−τ) would become

$$\Delta \left.\frac{d\alpha}{dx}\right|_t = \frac{\Delta M}{I} \varepsilon^* \tag{14}$$

where ε^* is given by Eq. 3. It can be proved that the difference in response in creep and creep recovery of a prestressed member subjected to an external bending moment can be accounted for with good approximation by replacing ε^* in Eq. 16 with a value

$$\varepsilon_f^* = \frac{1}{E_\tau} + \frac{(1+R)}{2} f[\tau,(t-\tau)] \qquad (15)$$

which is the average of ε^* and ε_R^* corresponding to creep and creep recovery, respectively, (Eq. 3 and 4). It thus becomes apparent that for calculation of curvature (or deflections) we may use an *effective strain* instead of the actual strain, calculated by the equation:

$$\varepsilon_e(t) = \int_{\tau_0}^{t} \frac{1}{E_\tau} [1 + \frac{(1+R)}{2} \phi_N \frac{1.35 \ \ell n \ (t-\tau+1)}{5 + \sqrt{\tau}}] \frac{d\sigma(\tau)}{d\tau} d\tau \qquad (16)$$

and this equation is applicable anywhere above or below the centroidal axis. Since a positive or negative moment will produce increase and decrease in stress at points on opposite sides of the center of gravity, the effective strain Eq. 16 can be used in both cases.

We can write an equation similar to equation 16 for the total deflection (instantaneous + creep) at age t caused by a load P of varying magnitude coming into action at age τ_0:

$$U(t) = \int_{\tau_0}^{t} u_\tau [1 + \frac{(1+R)}{2} \phi_N \frac{1.35 \ \ell n \ (t-\tau+1)}{5 + \sqrt{\tau}}] \frac{dP(\tau)}{d\tau} d\tau \qquad (17)$$

where u_τ is the instantaneous deflection due to a load P = 1 applied at age τ.

STEP-BY-STEP COMPUTATION OF REACTIONS INDUCED BY SETTLEMENT

We use a numerical procedure to solve Eq. 17 for the load P(t) representing the change in reaction at a support of a continuous prestressed structure produced by a given settlement U(t) of this support taking place between the ages τ_0 and t.

The period during which the force P is required is divided into N intervals not necessarily of equal length. The change in P caused by settlement and creep during any interval is assumed to occur in the middle of the interval. We can calculate the magnitude of $P_{i+\frac{1}{2}}$ at the end of any ith interval (age $t_{i+\frac{1}{2}}$) provided the force at the beginning of the same interval (age $t_{i-\frac{1}{2}}$) is known. The total settlement $U_{i+\frac{1}{2}}$ at the end of the ith interval is considered as the sum of increments $(\Delta\delta)_j$ in the jth interval so that:

$$U_{i+\frac{1}{2}} = \sum_{j=1}^{i} (\Delta\delta)_j \qquad (18)$$

Substituting for R from Eq. 11 into Eq. 17 and putting it in a summation form, we obtain

$$\sum_{j=1}^{i} (\Delta\delta)_j = \sum_{j=1}^{i} u_j \left\{ 1 + [0.8 + \frac{t_{i+\frac{1}{2}} - t_j}{80 + 6.4(t_{i+\frac{1}{2}} - t_j)}] \phi_N \times \frac{1.35 \ \ell n(t_{i+\frac{1}{2}} - t_j + 1)}{5 + \sqrt{\tau_j}} \right\} (\Delta P)_j \qquad (19)$$

where $(\Delta P)_j$ is the load increment in the jth interval and u_j is a flexibility coefficient, that is the instantaneous deflection which would occur at the support if the support is removed and - at its location - a unit downwards load is applied at the middle of the jth interval. The coefficient u_j can be expressed in the form $u_j = b/E_j$ where E_j is the modulus of elasticity of concrete at the middle of the jth interval and b^j is coefficient calculated by an elastic analysis. Its value depends upon the geometrical properties of the structure but not on time. Using the notation

$$a_{ij} = \phi_N \frac{1.35 \, \ell n[(t_{i+\frac{1}{2}} - t_j) + 1]}{5 + \sqrt{t_j}} \left[0.8 + \frac{t_{i+\frac{1}{2}} - t_j}{80 + 6.4(t_{i+\frac{1}{2}} - t_j)}\right] \qquad (20)$$

and putting $\Delta P_j = P_{j+\frac{1}{2}} - P_{j-\frac{1}{2}}$ and $u_j = b/E_j$ Eq. 19 becomes

$$\sum_{j=1}^{i} (\Delta\sigma)_j = b \sum_{j=1}^{i} \frac{1}{E_j} (1 + a_{ij}) (P_{j+\frac{1}{2}} - P_{j-\frac{1}{2}}) \qquad (21)$$

Rearranging, we obtain the force at the end of the ith interval

$$P_{i+\frac{1}{2}} = P_{i-\frac{1}{2}} + \frac{E_i}{1+a_{ii}} \left[\frac{1}{b} \sum_{j=1}^{i} (\Delta\delta)_j - \sum_{j=1}^{i-1} \frac{(1+a_{ij})}{E_j} (P_{j+\frac{1}{2}} - P_{j-\frac{1}{2}}) \right] \qquad (22)$$

Reasonable accuracy can be achieved by hand computation for Eq. 22 using four or five intervals. The intervals should be chosen such that they are shorter at the beginning when the rates of change of settlement, modulus of elasticity and creep are highest. For higher accuracy a larger number of intervals must be taken and use of computer becomes necessary.

EXPERIMENTAL INVESTIGATION

The experimental investigation consisted of 2 parts: (a) Creep and creep recovery tests on 3" x 9¼" cylinders to modify the principle of superposition and (b) Continuous prestressed concrete beams subjected to various rates of settlement to observe the development of the reactive forces.

All the tests were carried out in the laboratory environment where the temperature varied between 68° - 70°F and the relative humidity between 50 and 60 per cent.

The same concrete mix was used throughout, having proportions by weight of water:cement:aggregates = 1:1.85:8.5 and a maximum aggregate size 3/4 in. The weight of the fine aggregates was 36 per cent of the total aggregate. High early strength cement was used.

Both creep specimens and beams were demolded one day after casting, cured for 2 days in a steam room and then transferred to the testing room .

The compressive strength was measured on standard cylinders. The average strength at age 28 days was $f'_{c28} = 6360$ psi and the variation of the strength with time could be approximated by the function[8]

$$f'_{c\tau} = f'_{c\,28}/(0.875 + 3.5/\tau) \qquad (23)$$

Creep specimens of Group I were loaded with a constant stress of 1000 psi at ages 7, 14, 53 and 93 days and the strains were recorded to the age of 300 days (see solid lines in the graph of Fig. 1). The creep curves in this figure are in good agreement with Eq. 10 with ϕ_N between 3.3 and 3.7.

Other specimens (Group 2) were loaded with a stress of 1000 psi at 7 days and unloaded or subjected to a stress of 2000 psi at ages 14, 53 and 93 days. The curve A in Fig. 2 is based on the creep readings of all the specimens in Group 1 and 2 loaded at 7 days. The solid lines below and above curve A are the recovery and the 2000 psi creep curves. These curves were translated vertically so they branch out from curve A. This was necessary in order to be able to check the principle of superposition.

In both Fig. 1 and 2, the instantaneous strains at loading or unloading and the shrinkage were deducted from all readings before they were plotted.

The difference between curve A and the recovery curves in Fig. 2 is plotted in Fig. 1 (dotted curves) in order to compare creep with creep recovery. The average of the ratio R of the creep recovery to creep at the same $(t-\tau)$ value was calculated from the three pairs of curves in Fig. 2 and plotted in Fig. 3. The ratio R was also deduced from Davis'[6] results and plotted in the same graph. The dotted curve in this Fig. 3 follows the selected equation for R (Eq. 11).

All beams had rectangular cross section 4" x 8", length 2ℓ = 6 ft. and were post-tensioned at 7 days by one unbonded concentric 3/4" bar with an initial force of 32.0 kip. Concentric prestressing was chosen for all beams because the forces induced by settlement of supports are not affected by the magnitude and eccentricity of the prestressing force.

In each experiment a pair of identical beams were placed one above the other (Fig. 4) separated at the ends by rollers. The two beams were identical in all respects and thus no relative displacement occurred between them due to differential shrinkage within each beam or due to self weight. By means of threaded bars, the beams of each pair were forced to approach each other at the centre an arbitrarily chosen distance. The relative deflections were controlled by dial gauges and the force causing the deflection was measured by electrical strain gauges*

The first pair of beams, Test 1, was subjected to a total relative deflection of 0.30 in. and this deflection was maintained throughout the test. In tests 2, 3 and 4, the deflection was introduced in 7 equal intervals each of 0.05 in. The age of beams at the time of application of the deflection is given in Table 1. The force holding the beams in the deflected position was recorded over 200 days in all tests.

Table 1 - Age of Beams at Application of Deflection

Test No	Age in days at which increment applied, for deflection increment No:						
	1	2	3	4	5	6	7
1	11						
2	11	11 1/3	11 3/4	12½	13¼	15	18
3	10	12 ½	16	20	25	33	47
4	13	18¼	25	35	49	64	88

The time-force curves are shown in Fig. 5. The greatest force recorded was P = 2880 in Test 1, immediately after introducing the total deflection. If the deflection in this test had been the same as the total deflection in the other tests the force P would have been P = 2880 x 0.35/0.30 = 3360 lb. The

* Similar test arrangements is explained in more detail in Ref. 1.

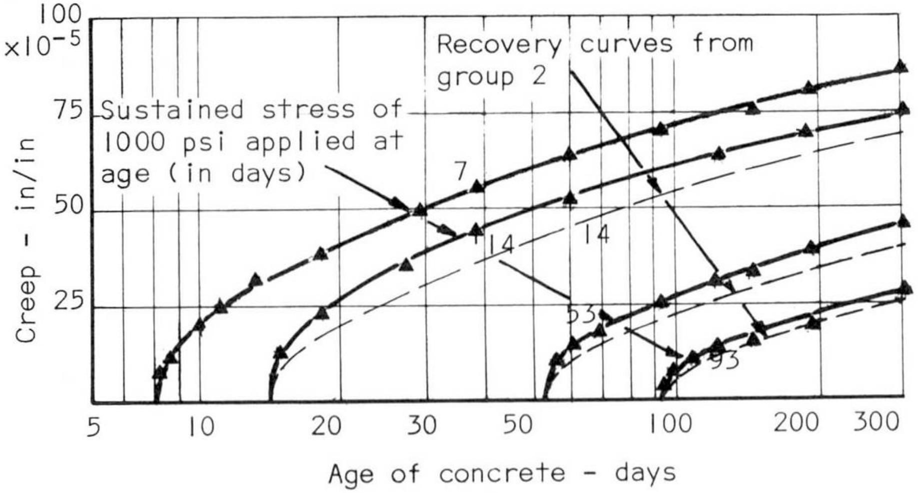

Fig. 1 Creep curves for concrete loaded at various ages and comparison with creep recovery

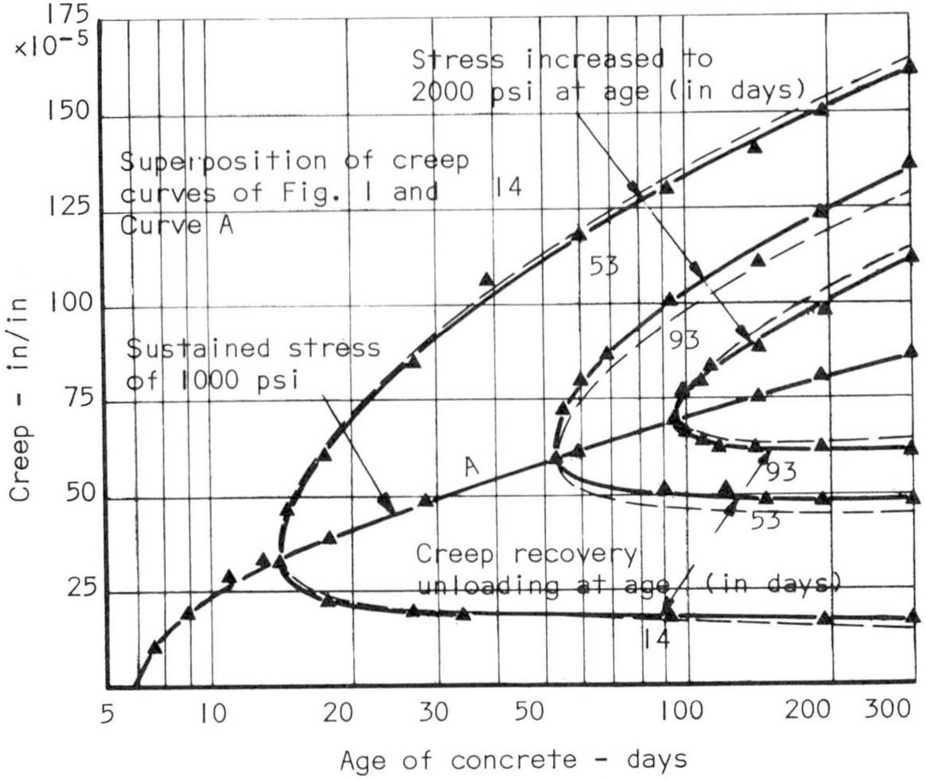

Fig. 2 Verification of the principle of superposition (specimens of group 2)

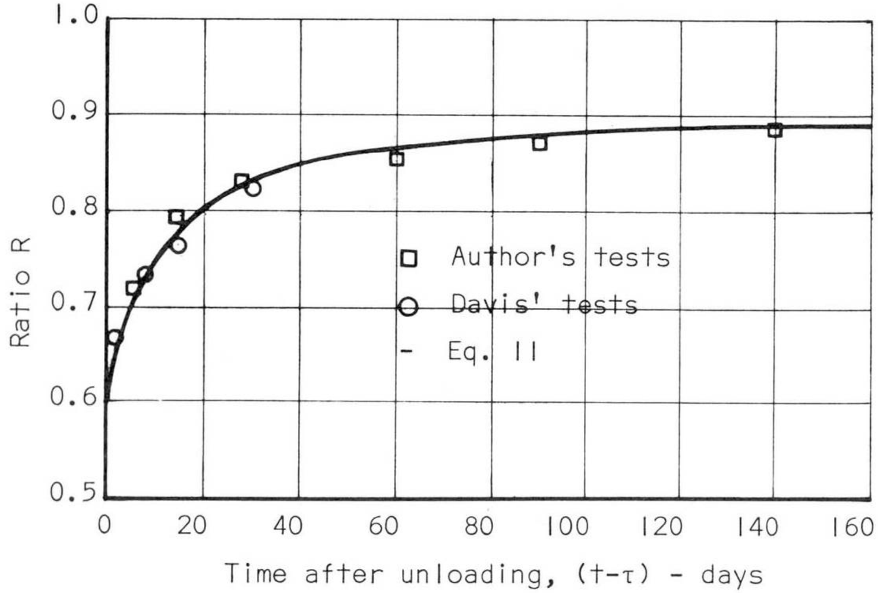

Fig. 3 The factor R relating creep recovery to creep

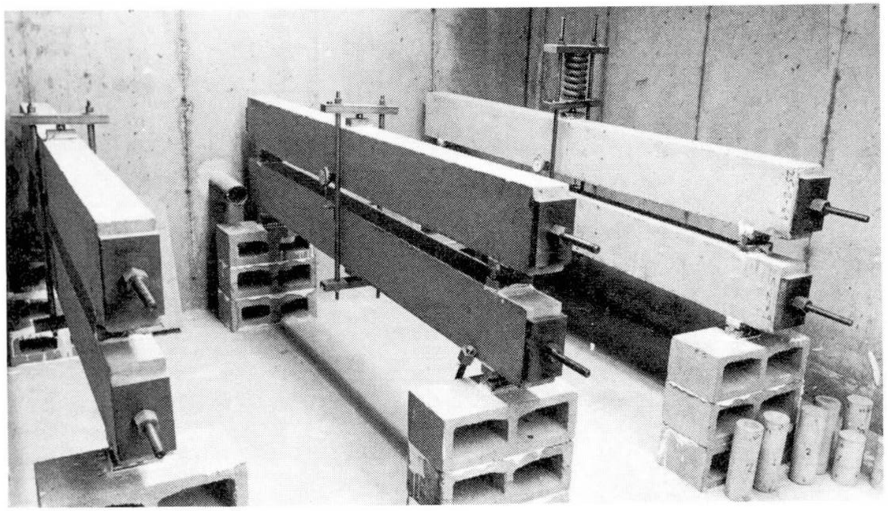

Fig. 4 Test arrangement

smallest maximum value of P(=1500 lb) was recorded in test 4 in which the deflection was introduced over the longest period.

In addition to the controlled-deflection tests, the deflection due to a sustained load of 2560 lb was recorded between τ_0=14 days and t=314 days. The creep deflection curve was found to follow closely Eq. 17 with ϕ_N = 3.6.

For the verification of the method of analysis, the force P was calculated by Eq. 22 and plotted in Fig. 5(dotted lines). The flexibility coefficient $u_j = (2\ell)^3/48\ E_j I$, thus $b = \ell^3/6I$ = 547 ft^{-1}. The modulus of elasticity E was determined from elasticity tests on standard cylinders used in creep tests and by relating the force applied to the beams and their initial deflection. These results where found to be in good agreement with the ACI-equation[9] $E_\tau = 58{,}000\sqrt{f'_{c,\tau}}$ (psi) where $f'_{c,t}$ is given by Eq. 23 as a function of τ.

Calculated and measured forces in Fig. 5 show good agreement. Since the creep and elastic properties were taken the same for the analysis of all tests, although they were not identical for all specimens, small differences must also be expected for the force-time curves of Fig. 5.

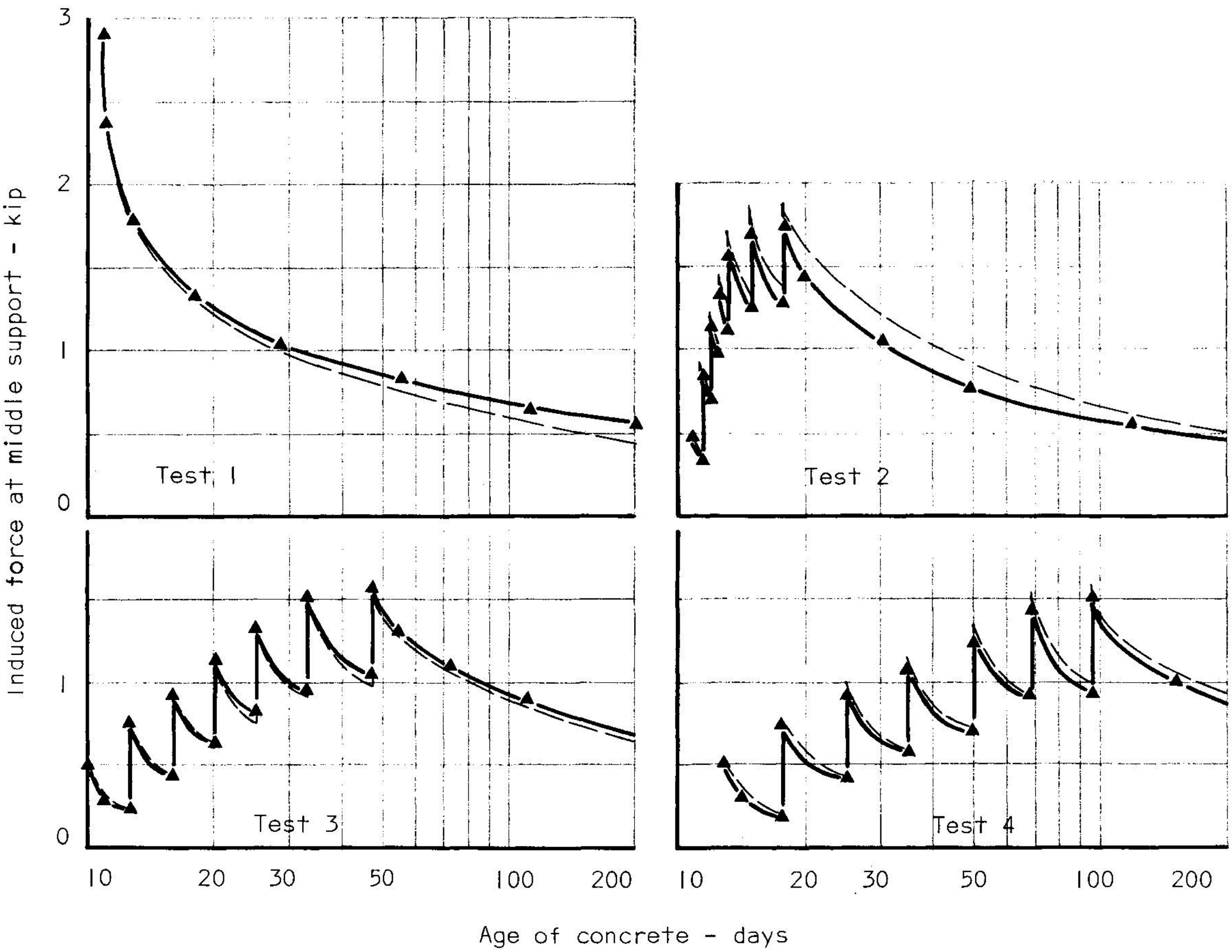

Fig. 5 Time-force relation for test beams

NOTATION

a_{ij} = dimensionless coefficient defined in Eq. 20

b = coefficient of units length^{-1} to be determined by an elastic analysis equals uE

δ = settlement

E = modulus of elasticity of concrete

i,j = number of intervals referred to in the step-by-step calculation, when used as subscript refers to the age at the middle of the interval

$i-\frac{1}{2}$ and $i+\frac{1}{2}$ = subscripts referring to the beginning and end of the ith interval respectively

ℓ = span of beam

P = force

R = ratio of specific creep recovery to specific creep

σ = stress

t = age of concrete, days

τ_o = age of concrete at beginning of stressing or of differential settlement

τ = age at any time between τ_o and t

U = deflection or settlement

u = flexibility coefficient,that is deflection per unit force calculated by elastic analysis

ϕ_N, ζ, ρ = coefficients (see Eq. 7)

ACKNOWLEDGEMENT

This project was supported by a grant from the National Research Council of Canada, which is gratefully acknowledged.

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SUMMARY

The variation of reactions in continuous prestressed concrete structures due to differential settlement of the supports is affected by creep, change in modulus of elasticity of concrete and the shape of the time-settlement curve. Experimental and predicted time-reaction curves of five pairs of continuous prestressed concrete beams subjected to various settlement rates are presented together with data on creep and creep recovery of compression specimens loaded and unloaded at various ages. An accurate prediction of the time-dependent change of reactions induced by settlement, requires consideration of all the time-dependent effects along with a modification of the generally accepted principle of superposition for creep to account for the fact that creep recovery is smaller than creep. A modified superposition equation is derived which leads to excellent agreement between calculated and measured changes of reaction. The analysis involves step-by-step calculation.

RESUME

La variation des réactions d'appui dans les poutres continues en béton précontraint par suite des tassements differentiels des appuis est produite par le fluage, les variations du module d'élasticité du béton et la courbe du tassement en fonction du temps. Cinq diagrammes expérimentaux et prévisibles ont été tracé, se rapportant à cinq poutres continues en béton précontraint qui étaient soumises à des intensités de tassement différentes. Les caractéristiques du fluage ainsi que la réduction du fluage pour les supports chargés et déchargés ont aussi été décrites à des moments différents. Une prévision exacte des changements des réactions d'appui en fonction du temps, par suite du tassement, exige que l'on tienne compte de tous les éléments existants avec une modification du principe généralement admis de la superposition lors du fluage pour expliquer que la diminution de fluage est plus petite que le fluage même. Une équation modifiée sur la superposition est formulée conduisant à une parfaite concordance entre les variations mesurées et calculées des réactions. Les calculs exigent un processus méthodique.

ZUSAMMENFASSUNG

Die Schwankung der Auflagerkräfte in durchlaufenden Spannbetonbalken infolge differentieller Setzung der Auflager wird durch Kriechen, Aenderung im Elastizitätsmodul des Betons und Zeit-Setzungs-Kurve verursacht. Beschrieben werden zusammen mit experimentellen und vorausgesagten Zeit-Reaktions-Kurven fünf verschiedener durchlaufender Spannbetonbalken, die unterschiedlichen Setzungsraten unterworfen wurden, Kriechdaten sowie Kriechminderung belasteter und unbelasteter Stützen zu verschiedenen Zeitpunkten. Eine genaue Voraussage der zeitabhängigen Aenderungen der Auflagerreaktionen infolge Setzung erfordert die Einbeziehung aller zeitabhängiger Wirkungen nebst der Modifikation des allgemein anerkannten Prinzips der Ueberlagerung für Kriechen, um der Tatsache Rechnung zu tragen, dass Kriechminderung kleiner ist denn Kriechen selbst. Eine abgeänderte Ueberlagerungsgleichung ist hergeleitet worden, die zu ausgezeichneter Uebereinstimmung von gemessenen und berechneten Aenderungen der Reaktionen führt. Die Berechnung erfordert schrittweises Vorgehen.