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Autor(en): **Tang, Wilson H. / Shah, Haresh C. / Benjamin, Jack R.**

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Statistical Evaluation of Load Factors in Structural Design

Calcul statistique des coefficients de sécurité dans la conception des ouvrages

Statistische Berechnung von Sicherheitsfaktoren im Entwurf

WILSON H. TANG
Assistant Professor
of Civil Engineering
University of Illinois

HARESH C. SHAH
Associate Professor
of Structural Engineering
Stanford University

JACK R. BENJAMIN
Professor
of Structural Engineering
Stanford University

I Introduction

The current design procedures in structural engineering involve the use of load factors to account for uncertainties that may exist in the applied loads and in the resisting strength of the structural members. The associated level of reliability depends on the probabilistic character of both loads and resistance. A procedure leading to the determination of load factors is formulated based on the following criteria: 1) The load factors should provide a desired level of structural safety with respect to all loading configuration. 2) This desired level of reliability may be determined by economic factors such as cost of construction and the cost of failure of the structural system. 3) Load factors for individual element design may be determined by the desired level of reliability of the whole system. 4) Additional information about the loading environment and the material strength characteristics can be systematically incorporated into the procedure to obtain a better set of load factors. 5) The procedure to evaluate load factors should be simple in form and easy for application. In the formulation that is presented here, dead, live, wind and earthquake loads are included. First, the load factors for a given reliability level for a structural element are calculated and second, the optimal reliability level of the structural system, its relationship with element reliability and the economic considerations in determining optimal reliability are considered. Step by step procedure is outlined and numerical example is worked out to illustrate the simplicity of the procedure.

II Determination of Load Factors for Dead, Live, Wind and Earthquake Loads

In addition to dead and live load, a structure may be subjected during its life span to loads like earthquake and wind. In previous works, Tang (7), Niyogi (3), Shah and others (4) have initiated such studies under dead and live loads. Incorporation of high wind loads and earthquake loads are especially important for structures with long service life and located around regions where occurrences of earthquakes or hurricanes or both are frequent. Given the location and the desired life span of a structure, the maximum magnitude of earthquake and wind loads that will act on the structure are random variables. In the formulation presented here, only the largest magnitudes of wind and earthquake loads are considered. The effects of repeated occurrences of minor earthquakes and hurricanes are neglected. If we exclude the possibility that earthquake and wind would occur at the same time, then the possible loading configurations are:

$$W_1 = DL + LL \; ; \quad W_2 = DL + WL \; ;$$
$$W_3 = DL + EL \; ; \quad W_4 = DL + LL + WL \; ; \qquad 1$$
$$W_5 = DL + LL + EL \; .$$

where DL, LL, EL and WL represent dead, live earthquake and wind loads respectively. If the mean values of these loads are taken as nominal loads to which load factors are multiplied, the corresponding design load values are:

$$W_1^* = \alpha_{11}\mu_1 + \alpha_{12}\mu_2 \; ; \quad W_2^* = \alpha_{21}\mu_1 + \alpha_{23}\mu_3$$
$$W_3^* = \alpha_{31}\mu_1 + \alpha_{34}\mu_4 \; ; \quad W_4^* = \alpha_{41}\mu_1 + \alpha_{42}\mu_2 + \alpha_{43}\mu_3 \qquad 2$$
$$W_5^* = \alpha_{51}\mu_1 + \alpha_{52}\mu_2 + \alpha_{54}\mu_4$$

where μ_1, μ_2, μ_3 and μ_4 are the mean dead, live, wind and earthquake loads respectively. Note that load factors

$$\alpha_{13} = \alpha_{14} = \alpha_{22} = \alpha_{24} = \alpha_{32} = \alpha_{33} = \alpha_{44} = \alpha_{53} = 0 \qquad 3$$

In general, for the ith combination of loading, we can write:

$$W_i^* = \sum_{j=1}^{4} \alpha_{ij}\mu_j \qquad 4$$

Stochastically, if all the load components are assumed normally distributed, the design load for the ith loading configuration is

$$W_i = \mu_{W_i} + k_i\sigma W_i = \sum_j \mu_j + k_i(\sum_j \sigma_j^2 + \sum_{\substack{k,\ell \\ k\neq\ell}} \rho_{k\ell}\sigma_k\sigma_\ell)^{1/2} \qquad 5$$

where $\rho_{k\ell}$ denotes the coefficient of correlation between the k-th and ℓ-th component. The index j depends on the ith loading configuration. Thus,

$$j = 1,2 \text{ when } i = 1 \; ; \quad j = 1,4 \text{ when } i = 3$$
$$j = 1,3 \text{ when } i = 2 \; ; \quad j = 1,2,3 \text{ when } i = 4 \qquad 6$$
$$j = 1,2,4 \text{ when } i = 5 \; .$$

In terms of the component loads, this design load may also be written as

$$W_i = \sum_j \ell_{ij} = \sum \mu_j(1 + k_iV_j) \qquad 7$$

where V_j = coefficient of variation of the jth load component. Equating 4, 5 and 7 and assuming that the resistance is Gaussian with coefficient of variation V_R and reliability coefficient k_i, (k_i measures the number of standard deviations in the standardized Gaussian distribution corresponding to the ith load), a general expression for the load factor can be obtained

$$\alpha_{ij} = \frac{1}{1-k_iV_R} \cdot \frac{\ell_{ij}}{\mu_j} = \frac{1 + \dfrac{(\sum_j \sigma_j^2 + \sum_{\substack{k,\ell \\ k\neq\ell}} \rho_{k\ell}\sigma_k\sigma_\ell)^2}{\Sigma\sigma_j} k_iV_j}{1 - k_iV_R} \qquad 8$$

If k_i^* denotes the level of the overall reliability for the ith loading combination, the relation between k_i and k_i^* can be shown to be (ref. 7)

$$k_i = \frac{\sqrt{(V_R\mu_R)^2 + \sigma_{L_i}^2}}{V_R\mu_R + \sigma_{L_i}} k_i^* \quad 9$$

where

$$\sigma_{L_i} = (\Sigma\, \sigma_j^2 + \sum_{\substack{k,\ell \\ k\neq\ell}} \rho_{k\ell}\sigma_k\sigma_\ell)^{1/2} \quad 10$$

For any desired level of overall reliability corresponding to k_i^*, k_i is determined from equation 9. Load factors are then computed by equation 8. In order to test for the sensitivity with respect to the type of distribution that is assumed, Extreme Type II (largest value) distribution has been assumed for the wind and earthquake loads, keeping the dead load and live load distributions Gaussian. It was observed that the values of design loads and load factors obtained for any desired reliability level does not change appreciably from the all-Gaussian model.

III Summary of Procedure

A step by step procedure is outlined below for the evaluation of load factors under dead, live, wind and earthquake loads. (1) Compute the magnitudes of dead, live, wind and earthquake loads acting on the structure, based on the empirical formula in the existing code. (2) Through structural analysis, obtain the design moments (stresses) due to each load component. Select the critical section for the member to be designed. (Numerical example is given in section 6.) Let μ_1, μ_2, μ_3 and μ_4 be values of such design moments (stresses) at the critical section due to dead, live, wind and earthquake load respectively. (3) Based on the available data, determine the coefficients of variation V_1, V_2, V_3 and V_4 for each load component. Compute σ_1, σ_2, σ_3 and σ_4 by using equation $\sigma_1 = \mu_i V_i$, i = 1,2,3,4. (4) From available data, determine coefficient of variation of resistance (V_R). (5) Choose the value of the overall reliability for the member to be designed, say $\bar{u}$. Determine the corresponding reliability coefficient k* from tables of normal integrals. (6) Compute the values of σ_s, μ_s, σ_L, V_L and k for each loading configuration. All ρ_{ij} except ρ_{14} may be assumed to be zero. The value of each k_i (1 = 1 to 5) is determined from the values k*, V_R and V_L, by using equation 9 or from charts similar to that in Figure 4. (7) For each loading configuration i, compute load factors α_{ij} by

$$\alpha_{ij} = \frac{1 + (\frac{\sigma_L}{\sigma_s} kV_j)_i}{1 - k_i V_R}$$

Note that α_{13}, α_{14}, α_{22}, α_{24}, α_{32}, α_{33}, α_{44}, α_{53} are zero. (8) Compute the design load

$$W_i = \sum_{j=1}^{4} \alpha_{ij}\mu_j \quad i = 1 \text{ to } 5 .$$

The example given in Section 6 would help to illustrate the above procedures.

IV System Analysis

In general, a structure is made up of structural components called members or elements. For a floor system consisting of four T-beams, each beam is called an element. The reliability of the floor system depends on the reliability of each beam as well as the type of framing of the system. We will consider two types of systems, namely series system and parallel system.

(A) Series System - The series system is defined as one which fails if any one of its elements fails. When any element fails, that is, when its strength capacity is exceeded by the applied load, it does not continue to deform sufficiently so that its adjacent elements can take on the extra load. If we further assume for the four-beam floor system that (i) The probabilistic properties of each T-beam are identical. (ii) The event that any one beam fails is independent of that of the others. (iii) The loads are evenly distributed to each beam. Then the probability of failure of the floor system is

$$p_{FS} = 1 - (1 - p_F)^4 \approx 4p_F \quad \text{(for small } p_F\text{)} \qquad 11$$

where p_F = probability of failure of each element. This may be generalized such that for a series structural system of N elements, its probability of failure is N times that of the individual element, that is

$$p_{FS} = N \cdot p_F \qquad 12$$

(B) Parallel System - On the other hand, if all the elements are designed with sufficient ductility such that they do not lose their load capacity until all elements reach ultimate conditions, the structural system is called a parallel system. Ideally, this system will fail when all the elements fail. However in practice, even for the most ductile system, the structural system will fail when only a fraction of the total number of elements fails. In addition to the assumption listed in Case (A), if we assume for the same floor system that the system will fail if two or more beams fail, then using the concept of Bernoulli Trials,

$$p_{FS} = \binom{4}{2} p_F^2 (1 - p_F)^2 + \binom{4}{3} p_F^3 (1 - p_F) + \binom{4}{4} p_F^4 \qquad 13$$

For small value of p_F, then

$$p_{FS} = \binom{4}{2} p_F^2 = 6p_F^2 \qquad 14$$

This may be generalized for a parallel structural system with N elements, if the failure of M or more elements lead to system failure, then the probability of system failure is approximately

$$p_{FS} \approx \binom{N}{M} p_F^M \qquad 15$$

Since reliability is defined as

$$\bar{u}_S = 1 - p_{FS}, \quad \bar{u} = 1 - p_F \qquad 16$$

we can see from equations 12 and 15 that simple relations do exist between system reliability and element reliability.

V Cost Analysis

It is known that the total design load increases as the desired level of reliability for one design increases. If we assume that cost of construction is linearly proportional to this value of design load, and let c_2 be the loss when the structure fails, and assume all other costs negligible, an expected total cost function can be defined as

$$\begin{aligned} TC &= \text{cost of construction} + \text{expected cost of failure} \\ &= c_1 W^*(\bar{u}) + c_2(1 - \bar{u}_s) \qquad 17 \end{aligned}$$

where W^* = total design load which is a function of reliability; c_1 = construction cost per unit design load for the system; and $\bar{u}$ = reliability of an element.

For a series structural system, applying Equations 12 and 16, this becomes

$$TC(\bar{u}) = c_1[W^*(\bar{u}) + C(1 - \bar{u})] \tag{18}$$

where $C = \dfrac{\text{cost of failure}}{\text{cost of construction per element per unit design load}}$

In other words, the cost coefficient C is a measure of how important the consequences of structural failure are relative to the unit cost of construction. Thus, a large value of C corresponds to a case where failure involves great losses. In the numerical work which follows, the mean dead load is assumed not as a constant but to be 1 psf per 5 psf of the total design load. The expected total cost function is evaluated for the sample data used for the Gaussian model. Figure 1 shows, for a given value of C, that a distinct minimum cost does exist at a certain level of reliability. As C increases, the optimal design should have a higher level of reliability so that the expected loss due to failure is decreased. The optimal reliability level as a function of the cost coefficient for various coefficients of variation of loads and resistance is shown on Figure 2. The variation in the load does not seem to affect this optimal relation. However as the coefficient of variation of resistance increases, the optimal reliability level does decrease for any given cost coefficient C. This implies that the optimal decision, when the resistance is highly uncertain, is to cut back the cost of construction by using smaller load factors and to risk a higher probability of failure. The analysis is similar in the case of a parallel system. The total cost function for the four-beam floor system example is computed to be

$$TC(\bar{u}) = c_1[W^*(\bar{u}) + 1.5C(1 - \bar{u})^2] \tag{19}$$

The relation between the cost coefficient and the optimal level of element reliability is given in Figure 3 for various values of the coefficient of variation of the resistance. This relation is again very insensitive to the statistical variation in the applied loads. For a given value of cost coefficient C, the optimal level of element reliability is much less for the parallel system than for the series system. Therefore, for the two types of systems, namely, series and parallel, once we know the values of C and V_R, the optimal level of reliability for element design may be easily determined.

VI Numerical Example - Dead, Live, Wind and Earthquake Loads

A one-story plane frame structure is chosen to illustrate how load factors can be determined in a step by step procedure for a desired level of reliability. The frame's dimension and member stiffness are shown in Figure 5. Assume that only bending moment failure is of interest. (1) Compute the magnitudes of dead, live, earthquake and wind loads acting on the structure. Assume a tributary span of 20 feet perpendicular to the frame. The dead load is represented by a uniform load acting on the girder BC. Its magnitude is given by $w_1 = DL \times 20' = 60 \times 20 = 1.2^{K/ft}$. Similarly, the magnitude for the live load is $w_2 = LL \times 20' = 80 \times 20 = 1.6^{K/ft}$. For the earthquake load, the Seismic Engineering Association of California (SEAOC) recommends the following equation for the equivalent static lateral load. (Ref. 8)

$$Q = KC_oW_oZ_o \tag{20}$$

We may assume $Z_o = 1$ (in California), D = 1 for the type of framing and $C_o = 0.1$ for a one-story structure. Then $Q_1 = DL \times \text{floor area} \times Z_o \times K \times C_o = 60 \times 20 \times 20 \times 1 \times 1 \times 0.1 = 2.4^K$. For the wind load, an equivalent static force is given by (ref. 9)

$$F = .00256\ Cd\ AV^2 \tag{21}$$

We may assume $C_d = 1$ for flat wall, V = 80 mph for a 50-year occurrence period in San Francisco for a building height of 50 feet, and A = 10 x 20 = 200 square feet for a tributary span of 20 feet and an exposed height of 10 feet. Then $Q_2 = .00256 \times 1 \times 200 \times 80^2 = 3.3^k$. (2) Compute bending moments due to each load. The moments (in kip-ft.) at critical locations are

	DL	LL	EL	WL	DL+LL	DL+EL	DL+WL	DL+LL+EL	DL+LL+WL
A	13.4	18.0	34.3	47.1	31.4	47.7	60.5	65.7	78.5
B	-26.7	-35.7	-25.7	-35.4	-62.4	-52.4	-62.1	-88.1	-97.8 *
C	-26.7	-35.7	27.7	35.4	-62.4	- 1.0	8.7	-36.7	-27.0
D	13.4	18.0	-34.3	-47.1	31.2	-17.9	-33.7	- 3.1	-15.9
E	33.3	44.5	0	0	77.8	33.3	33.3	77.8	77.8

Take location B for our further analysis. (3) Compute mean and standard deviation for each component.

$$V_3^2 \approx V_{z_o}^2 + V_k^2 + V_{c_o}^2 + V_{w_o}^2$$

$$= (0.1)^2 + (0.08)^2 + (0.2)^2 + (0.08)^2 = 0.0628 \text{ (say)}$$

$$\mu_1 = 26.7 \quad V_1 = 0.083 \quad \sigma_1 = 2.22$$
$$\mu_2 = 35.7 \quad V_2 = 0.25 \quad \sigma_2 = 8.93$$
$$\mu_3 = 35.4 \quad V_3 = 0.2 \quad \sigma_3 = 7.08$$
$$\mu_4 = 25.7 \quad V_4 = 0.25 \quad \sigma_4 = 6.43$$

The index 1, 2, 3, 4 correspond to dead, live, wind and earthquake load contributions respectively. The values of the coefficient of variation V_1, V_2, V_3, V_4 are assumed for numerical illustration. The correlation coefficients ρ_{ij} are assumed zero, except for $\rho_{14} = 0.5$. (4) Assume coefficient of variation of resistance, say, 0.1 in this case. (5) Choose the desired overall reliability u = 0.9999. (6) Compute the values of σ_s, μ_s, σ_L, V_L, k*, k.

Case	Loading Combination	σ_s	μ_s	σ_L	V_L	k*	k
1	DL + LL	11.15	62.4	9 2	0.147	3.72	2.68
2	DL + WL	9.3	62.1	7.41	0.12	3.72	2.7
3	DL + EL	8.65	52.4	7.8	0.15	3.72	2.68
4	DL + LL + WL	18.23	97.8	11.6	0.12	3.72	2.7
5	DL + LL + EL	17.58	88.1	11.85	0.134	3.72	2.7

(7) Compute load factors α_{ij} and design load w_i.

Case	Loading Combination	α_{i1}	α_{i2}	α_{i3}	α_{i4}	W_i(k-ft)
1	DL + LL	1.61	2.13	0	0	119
2	DL + WL	1.61	0	1.96	0	113
3	DL + EL	1.61	0	0	2.08	96.8
4	DL + LL + WL	1.57	1.97	1.85	0	177
5	DL + LL + EL	1.58	1.97	0	1.97	163

VII Conclusion

An approach is presented in this paper to formulate a procedure for quantitative evaluation of load factors for dead, live, wind and earthquake loads. Various models which describe the statistical characteristics of the loads and resistance are studied under the formulation. Load factors obtained in each case for any desired level of reliability do not differ appreciably. The Gaussian model for both the loads and resistance appear to be the most convenient one to work with. Step-by-step procedure are outlined to illustrate the simplicity in the evaluation of load factors. From system and cost analyses, the optimal level of reliability for the structural member design are mainly determined by two parameters. They are the cost coefficient C which represents the ratio of cost of failure and the cost of construction, and the coefficient of variation of resistance V_R. Once this desired reliability level is given, together with the coefficients of variation of each load component, the corresponding load factors may be computed by the procedures formulated.

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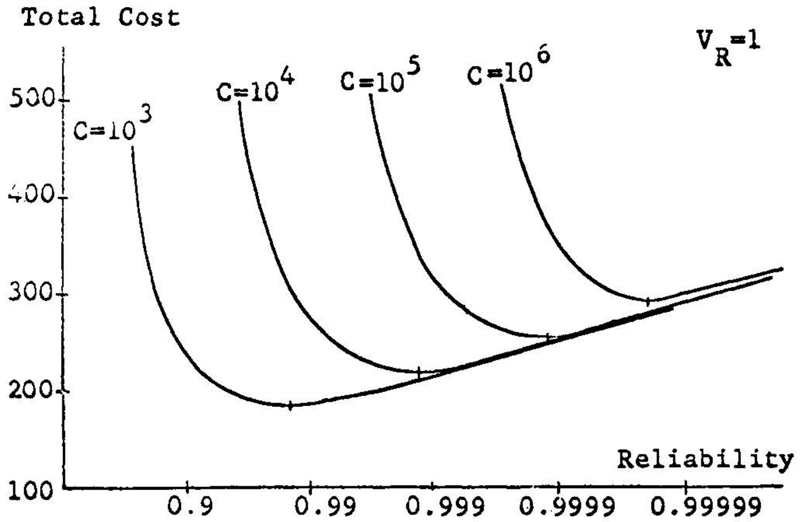

FIG. 1 TOTAL COST VS. ELEMENT RELIABILITY--SERIES SYSTEM

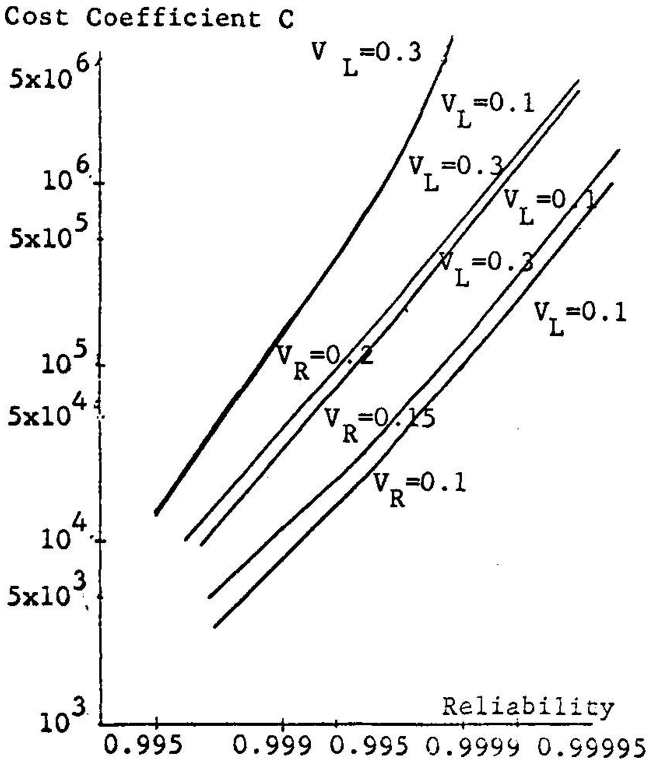

FIG. 2 COST COEFFICIENT VS. OPTIMAL ELEMENT RELIABILITY--SERIES SYSTEM

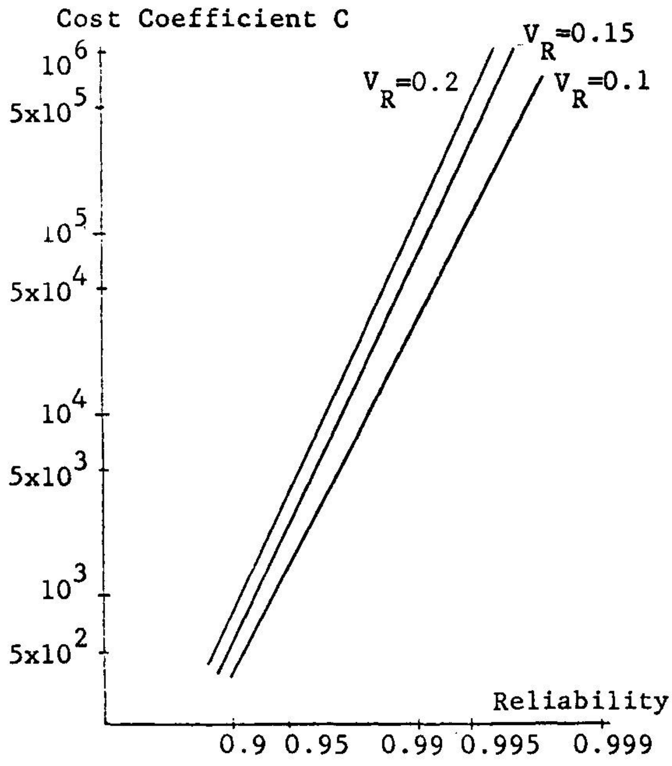

FIG. 3 COST COEFFICIENT VS. OPTIMAL ELEMENT RELIABILITY--PARALLEL SYSTEM

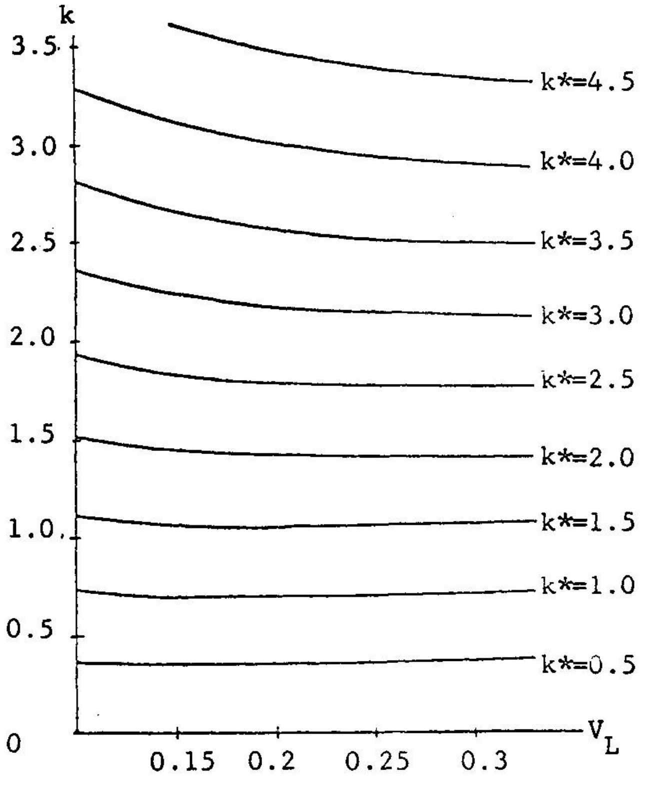

FIG. 4 CHART FOR DETERMINING RELIABILITY COEFFICIENT k--V_R=0.15

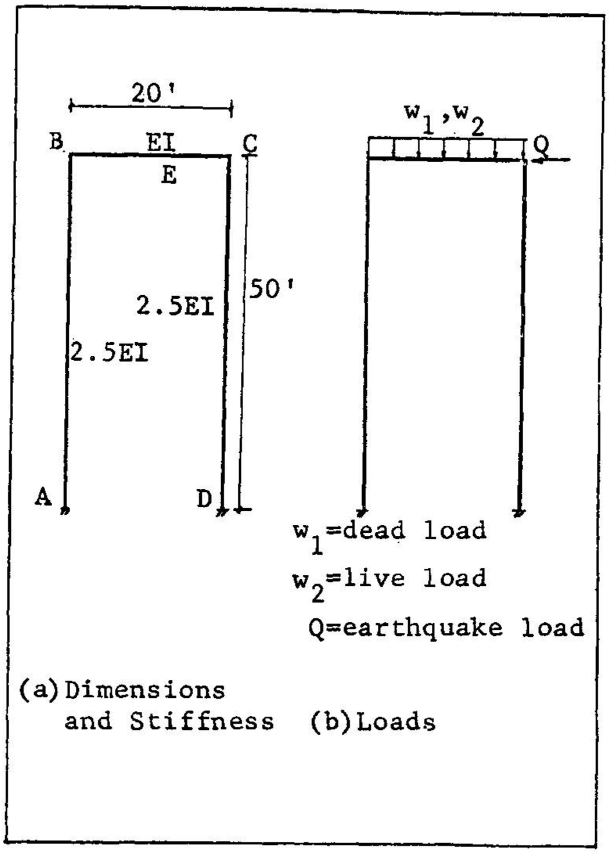

FIG. 5 PLANE FRAME EXAMPLE

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