Stiffener Eccentricity in Theory of Orthotropic Plates

Excentricité des raidisseurs dans la théorie des plaques orthotropes

Steifenexzentrizität in der Theorie orthotroper Platten

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SUMMARY
The paper is concerned with the effect of stiffener eccentricity on the critical stress and on the elastic postcritical behaviour of orthotropic plates in compression. The non-linear theory takes account of initial geometrical imperfections and of large displacements. The problem has been studied on four large scale steel models. The first results have shown that effect of stiffener eccentricity is practically negligible.

RÉSUMÉ
L'article traite l'influence de l'excentricité des raidisseurs sur la contrainte critique et sur le mouvement élastique postcritique des plaques orthotropes comprimées. La théorie non-linéaire prend en considération les flèches initiales et les grandes déformations. On étudie ce problème sur quatre modèles d'acier. Les premiers résultats montrent que l'influence de l'excentricité des raidisseurs est pratiquement négligeable.

ZUSAMMENFASSUNG
Der Aufsatz beschäftigt sich mit dem Effekt der Steifenexzentrizität auf die kritische Spannung und auf das elastische überkritische Verhalten der druckbeanspruchten orthotropen Platten. Die nichtlineare Theorie berücksichtigt anfängliche Verformung und größere Durchbiegungen. Das Problem wurde an vier großen Stahlmodellen untersucht. Die ersten Ergebnisse zeigten, daß der Einfluß der Steifenexzentrizität praktisch vernachlässigbar ist.

1. INTRODUCTION

During the last seventeen years intensive research programmes, both theoretical and experimental, have been undertaken in the field of orthotropic plates in compression as a result of the four collapses of long span box girder bridges that occured between 1969 and 1971. Extensive studies have influenced the design codes, and design rules based on the postcritical behaviour of plated structures have been established.
Two most widely adopted approaches are usually considered in the study of the stiffened plate behaviour: /i/ the strut approach and /ii/ the orthotropic plate approach. The latter is analysed in the present paper.
In this approach, the stiffened plate is treated as an equivalent orthotropic plate and the elastic large deflection theory is used.

MAQUOI and MASSONNET /1971/ developed a design method that takes account of postcritical strength increases produced by membrane stresses. The main assumptions of this analysis are as follows:
- the rigidities of the stiffeners can be smeared in order to obtain a substitute plate that is then analysed by a non-linear large displacement theory;
- the postbuckling shape of the plate is represented by only its first term of the double FOURIER series expansion;
- collapse is reached when the mean longitudinal membrane stress along the unloaded edges of the orthotropic panel reaches yield stress f_y. The flexural stresses are neglected;
- allowance for plate buckling between the stiffeners is made by using an effective width approach.

Starting from the Liege method [9], an attempt [1] has been undertaken to improve the above-mentioned approach:
- it has been able to take account of several terms of a FOURIER expansion and
- to prepare design formulae which simplify considerably the mathematical procedure and present therefore the ultimate strength theory of stiffened plates in a form suitable for practical use in designing bureaux;
- simply supported orthotropic plate in compression with more complex in-plane boundary conditions has been solved in [1] and solution comprises case of isotropic plate too and/or transversely loaded plates;
- relating collapse to the maximum membrane stress - rather than the mean membrane stress along the unloaded edges - has enabled to avoid any plastic redistribution of the membrane stresses.

The theoretical and the numerical results of the thesis [1] can be found also in the chapter 7 of the book [5]. Results of a parametric study were published also in [3].

In addition, it has been analysed in the paper [2]:
- the combined effect of the membrane stresses and the flexural stresses on the limit state by using several various collapse criteria;
- the shear lag phenomenon determined according to the new edition of the code ČSN 73 6205 "Design of Steel Bridge Structures";
- the comparisons of the theoretically obtained values and the experimental data of the ultimate load tests on 34 steel girders and 34 orthotropic plates in compression.

The aim of the present paper is to demonstrate a solution of the following system of two simultaneous differential equations describing of slender orthotropic plates with unsymmetrically arranged stiffeners and with an initial deflection

$$\bar{D}_x \frac{\partial^4 w}{\partial x^4} + 2\bar{H}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \bar{D}_y \frac{\partial^4 w}{\partial y^4} + \frac{\nu}{1-\bar{\nu}^2}\left(e_x \frac{S}{S_y}\frac{\partial^4 \Phi}{\partial x^4} + e_y \frac{S}{S_x}\frac{\partial^4 \Phi}{\partial y^4}\right) - \frac{e_x + e_y}{1-\bar{\nu}^2}\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} - \frac{\partial^2 \Phi}{\partial x^2}\frac{\partial^2 (w_0 + w)}{\partial y^2} - \quad (X = 0)$$

$$- \frac{\partial^2 \Phi}{\partial y^2}\frac{\partial^2 (w_0 + w)}{\partial x^2} + 2\frac{\partial^2 \Phi}{\partial x \partial y}\frac{\partial^2 (w_0 + w)}{\partial x \partial y} - q = 0 \tag{1a}$$

$$\frac{1}{S_y}\frac{\partial^4 \Phi}{\partial x^4} + \frac{2}{\bar{S}}\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{1}{S_x}\frac{\partial^4 \Phi}{\partial y^4} = (1-\bar{\nu}^2)\left\{\left|\frac{\partial^2 (w_0 + w)}{\partial x \partial y}\right|^2 - \frac{\partial^2 (w_0 + w)}{\partial x^2}\frac{\partial^2 (w_0 + w)}{\partial y^2} - \right.$$

$$\left. - \left[\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2 - \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}\right]\right\} + \nu S\left(\frac{e_x}{S_y}\frac{\partial^4 w}{\partial x^4} + \frac{e_y}{S_x}\frac{\partial^4 w}{\partial y^4}\right) - (e_x + e_y)\frac{\partial^4 w}{\partial x^2 \partial y^2} \tag{1b}$$

Equations (1) generalize the well-known equations of FÖPPL-von KÁRMAN-MARGUERRE and HUBER´s equation. Earlier solutions [1,2,3,5,9] neglected the terms of the equations (1) that contain excentrities e_x, e_y.

2. SOLUTION TO THE SYSTEM OF EQUATIONS

2.1 General

The system (1) is going to be solved by P.F. Papkovich´s method, which means that the compatibility equation (1b) will be solved exactly while the approximative method of B.G. GALERKIN will be employed in the solution to the equilibrium equation (1a).

2.2 Solution to the Compatibility Equation

The functions of the initial and additional deflections are supposed to have the form of a series in which all terms fulfil boundary conditions

$$\bar{w}_0 = \sum_m \sum_n \bar{w}_{0mn} \sin m\pi\xi \sin n\pi\eta , \qquad \xi = \frac{x}{a}, \quad \eta = \frac{y}{b}, \quad \bar{w} = \frac{w}{t}, \quad \bar{w}_0 = \frac{w_0}{t} \tag{2a}$$

$$\bar{w} = \sum_m \sum_n \bar{w}_{mn} \sin m\pi\xi \sin n\pi\eta \tag{2b}$$

The stress function is written as follows

$$\Phi^* = \Phi^*_0 + \Phi^*_p , \qquad \Phi^* = \frac{\Phi}{S_y(1-\bar{\nu}^2)t^2} \tag{3}$$

$\emptyset^*_0$ being the general solution and $\emptyset^*_p$ a particular integral of the compatibility equation.

After solution of the homogeneous compatibility equation we obtain

$$\Phi^*_0 = \Phi^*_{00} + \Phi^*_{0i} + \Phi^*_{0j} = -\frac{\lambda^2}{2}(\bar{N}^*_x \eta^2 + \bar{N}^*_y \xi^2 \alpha^2) + \sum_i R^*_i(\eta)\cos i\pi\xi + \sum_j S^*_j(\xi)\cos j\pi\eta \tag{4}$$

where $R^*_i(\eta) = A^*_i \cosh i\beta\eta + B^*_i \sinh i\beta\eta + C^*_i \cosh i\delta\eta + D^*_i \sinh i\delta\eta$

$$S^*_j(\xi) = E^*_j \cosh j\gamma\xi + F^*_j \sinh j\gamma\xi + G^*_j \cosh j\varepsilon\xi + H^*_j \sinh j\varepsilon\xi \tag{5}$$

The constants A^*_i, B^*_i, C^*_i, D^*_i, E^*_j, F^*_j, G^*_j, H^*_j are determined from the in-plane boundary conditions. In the case of boundaries being regarded as inflexible in the plate plane $A^*_i = B^*_i = C^*_i = D^*_i = 0$ (boundaries parallel to axis X) and/or $E^*_j = F^*_j = G^*_j = H^*_j = 0$ (boundaries parallel to axis Y).

The notation, which is not defined in the present paper, can be found in [1,2,5].

The particular integral, with regard (2), can be written in the following way:

$$\Phi_p^* = \Phi_{pw}^* + \Phi_{pe}^* = \tag{6}$$

$$= \frac{\alpha^2}{4} \sum_m \sum_n \sum_r \sum_s (\bar{w}_{mn}\bar{w}_{rs} + \bar{w}_{mn}\bar{w}_{0rs} + \bar{w}_{rs}\bar{w}_{0mn}) \times$$

$$\times \sum_{i=1}^{2} \sum_{j=1}^{2} (-1)^{i+j} g[(-1)^i r, (-1)^j s] \cos[m + (-1)^i r]\pi\xi \times \cos[n + (-1)^j s]\pi\eta + \tag{6a}$$

$$+ (1-\bar{\nu}^2)\alpha^2 \sum_m \sum_n \bar{w}_{mn}\, \bar{g}_e(m,n) \sin m\pi\xi \sin n\pi\eta \tag{6b}$$

where the functions $g(t_1,t_2)$, $g_e(m,n)$ are defined as follows:

$$g(t_1,t_2) = \frac{mt_2(nt_1 - mt_2)}{(m+t_1)^4 + 2\varkappa\omega^2\alpha^2(m+t_1)^2(n+t_2)^2 + \omega^4\alpha^4(n+t_2)^4} \tag{7a}$$

$$\bar{g}_e(m,n) = \frac{\frac{\nu\bar{e}_x}{\delta_y\alpha^2} m^4 - (\bar{e}_x + \bar{e}_y) m^2 n^2 + \frac{\nu\bar{e}_y\alpha^2}{\delta_x} n^4}{m^4 + 2\varkappa\omega^2\alpha^2 m^2 n^2 + \omega^4\alpha^4 n^4}, \qquad \bar{e}_x = \frac{e_x}{t} \tag{7b}$$

2.3 Solution to the Equilibrium Equation

Let us solve the equation (1a) by GALERKIN´s method. The system of GALERKIN´s equations is then

$$\int_0^1 \int_0^1 X \sin p\pi\xi \sin q\pi\eta \, d\xi \, d\eta = 0, \quad (p = 1,2,3,\dots; q = 1,2,3,\dots) \tag{8}$$

After integration (8) is transformed in a system of algebraic cubic equations, which is solved by NEWTON-RAPHSON´s method. Details will be published in a journal.

3. NUMERICAL RESULTS

3.1 Critical Stress

The critical stress of an orthotropic plate is given by

$$\sigma_{cr,e} = \underbrace{\frac{\pi^2 E\psi}{12(\delta_x - \nu^2)\lambda^2\alpha^2}}_{\sigma_{cr}\,[1,9]} \cdot \left[1 + \underbrace{\frac{12\,\delta_y\alpha^2(1 + 2\varkappa\omega^2\alpha^2 + \omega^4\alpha^4)}{\psi(1-\bar{\nu}^2)} \cdot \bar{g}_e^2(m=1,\ n=1)}_{\text{influence of eccentricities } e_x,\, e_y} \right] \tag{9}$$

The influence of the stiffener eccentricity e_x ($e_y = 0$) on the critical stress is studied in the Tab. 1 on the four models.

Table 1		N1	N2	N4	N3
relative rigidity	$(\psi - \bar{\psi})/\bar{\psi}$	5.214	0.533	0.058	-0.254
relative eccentricity	e_x/t	5.600	2.560	1.524	0.990
ratio of stresses	$\sigma_{cr,e}/\sigma_{cr}$	1.035	1.031	1.020	1.013

Details of the four large scale steel models N1, N2, N3, N4 (span 8.5m, depth 0.5125m) and of the test setup will be published in [4]. The full description of the tests can be found in [8].

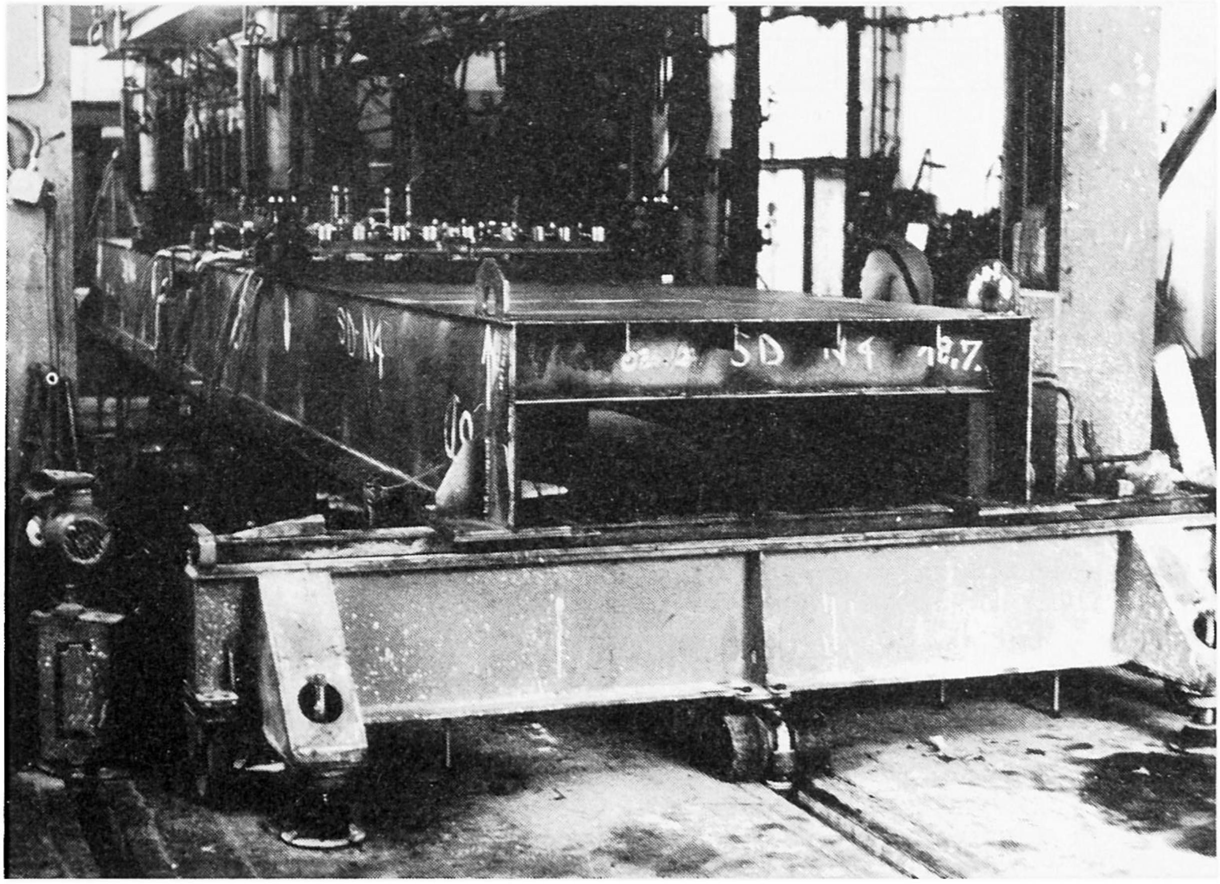

Fig. 1 KVOČÁK's test girder N4 [4,8]

3.2 Postcritical behaviour

The approach proposed in this paper enables to distinguish between two directions of orthotropic plate buckling: /i/ in the direction of the flange plate ($e_x < 0$) and /ii/ in the direction of the longitudinal stiffeners ($e_x > 0$). The former case is more danger (Tab. 2), but both cases differ only slightly from the case with $e_x = 0$. The results presented in the Tab.2 were calculated for one term of series (2), viz. w_{11}, w_{011} and they practically don't differ from those calculated for five terms of series (2), viz. w_{11}, w_{13}, w_{31}, w_{33}, w_{15} and w_{011}. $\bar{\sigma}_{x,e}$ ($e_x \neq 0$) and $\bar{\sigma}_x$ ($e_x = 0$) are the average stresses in the orthotropic plate at the moment when the maximum membrane stress attains the yield stress f_y. On the question of the in-plane boundary conditions of the orthotropic plate it had been assumed that transverse edges are "constrained", i.e. that they can freely move but remain straight. The longitudinal edges can deflect in the plane of the plate.

Table 2		N1	N2	N4	N3
$\sigma_{cr,e}/f_y$		6.429	1.581	1.079	0.756
w_0/t		0.500	0.600	0.600	0.730
$\frac{\bar{\sigma}_{x,e}}{\bar{\sigma}_x}$	$e_x < 0$	0.990	0.986	0.902	0.856
	$e_x = 0$	1.	1.	1.	1.
	$e_x > 0$	1.008	1.012	1.057	1.134

4. CONCLUSIONS

The study have been started, which enables to evaluate the influence of the stiffener eccentricities e_x, e_y on the critical stress (Tab. 1) and/or on the limit load of the orthotropic plate in compression (Tab. 2). The first results of the study have shown that the basic system of non-linear equations (1) can be simplified by neglecting the terms that contain eccentricities e_x, e_y.

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