Towards a consistent design of reinforced concrete structures

Towards a Consistent Design of Reinforced Concrete Structures

Dimensionnement unitaire des structures en béton armé

Zum einheitlichen Bemessen von Stahlbetontragwerken

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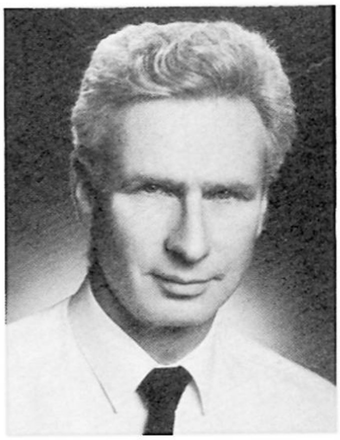

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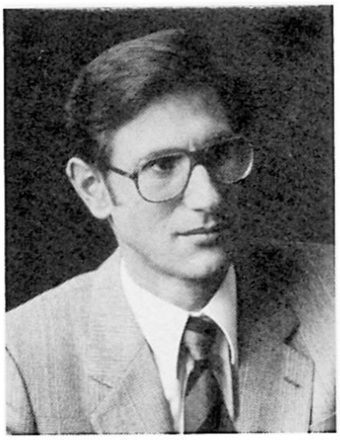

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SUMMARY
The truss model developed by Ritter and Mörsch for the design of shear reinforcement is a simple tool for explaining the internal forces in beams. It is generalized here for application to other reinforced concrete structures. Suggestions are made for the design of all members and nodes of such generalized strut models according to unified principles. Thus a consistent design concept for all kinds of reinforced concrete structures is achieved. This could be the basis for a better understanding of reinforced concrete and for simpler codes.

RESUME
Le modèle en treillis développé par Ritter et Mörsch pour le dimensionnement des armatures à l'effort tranchant dans des poutres est un moyen simple pour illustrer la distribution des forces intérieures. Il est généralisé et appliqué aux autres structures et éléments en béton armé. Si les membres et les noeuds sont dimensionnés sur la base des principes unitaires suggérés, il devrait être possible d'arriver à une conception universelle pour le dimensionnement de structures en béton armé. Cela pourrait conduire à un code plus simple et à une meilleure compréhension du béton armé.

ZUSAMMENFASSUNG
Die Fachwerkanalogie von Ritter und Mörsch liefert ein einfaches Modell zur Beschreibung des Kraftflusses in Stahlbetonbalken. Sie wird verallgemeinert und auf andere Tragglieder und Tragwerksbereiche angewendet. Wenn nun, wie vorgeschlagen, alle Stäbe und Knoten der verallgemeinerten Stabwerkmodelle nach einheitlichen Gesichtspunkten bemessen werden, kommen wir zu einem einheitlichen Bemessungskonzept für den ganzen Stahlbetonbau. Dieses könnte auch die Grundlage für ein besseres Verständnis des Stahlbetons und für einfachere Normen sein.

The ideas in M. Wicke's paper in the introductory report for this conference ("Developments in the Design of Reinforced and Prestressed Concrete Structures") are further elaborated in the following.

1. THE STRUCTURE'S B- AND D-REGIONS

Those regions of our structures, in which the Technical Bending Theory's assumption of plane strain is good enough, we nowadays design with an almost exaggerated care and "accuracy" by standard methods. We suggest to call them B-regions (B stands for beam and bending). Their internal state of stress is rapidly derived from the sectional forces and moments.

Standard methods are not applicable to all the other regions and details of a structure where the strain distribution is nonlinear e.g. near concentrated loads, corners, bends, openings, etc. (fig. 1). These we call D-regions (D for discontinuity, disturbance, detail). The D-regions are as equally important for the safety and serviceability of a structure as the B-regions, but are mostly designed and reinforced according to rules of thumb or on a purely experimental basis. This is obviously one of the sources of damage that is frequently found in D-regions.

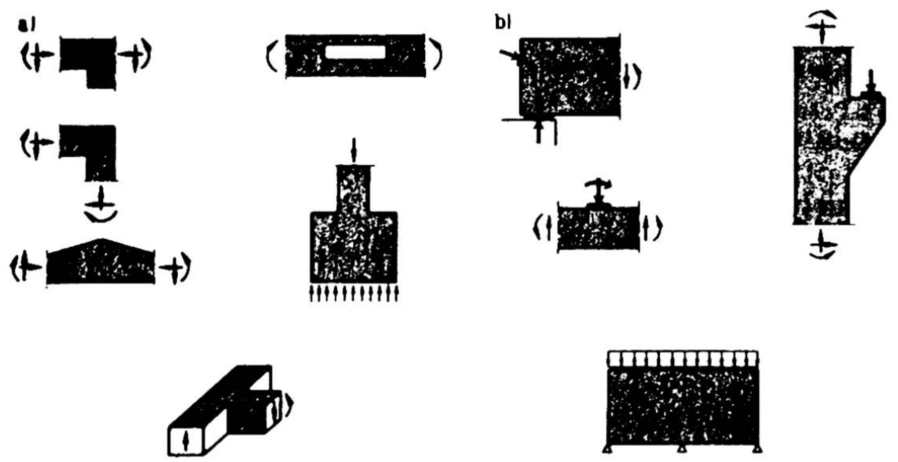

Fig. 1 (a) Geometrical discontinuities, (b) statical and/or geometrical discontinuities

A design concept should be clear and based on simple models in order to avoid rules which are not understood by the designing engineer. A consistent design concept must comprise B- and D- regions without contradictions. Considering the complexity of B-region design and the even more complex state of stress in D-regions a unified design concept will require concessions with respect to accuracy. But we think even a very simplified methodical concept will be better than todays practise of D-region design.

For this purpose it is suggested to generalize the well known truss analogy for beams (fig. 2) in order to apply it in form of generalized strut models to the more complicated regions and to the whole structure /1-3/. The strut model condenses all stresses in tension and compression members which are connected by nodes. If we design all its members and nodes with uniform criteria, we arrive at a consistent design concept which is applicable to all possible cases. On top of that it could be the basis for a better understanding of reinforced concrete and therewith for better and safer structures and simpler codes.

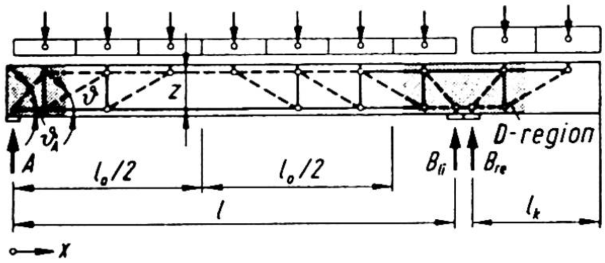

Fig. 2 Truss model of a beam with cantilever

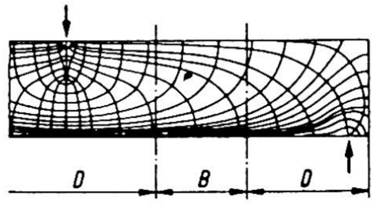

Fig. 3 Stress trajectories near discontinuities

Stresses and stress trajectories are quite smooth in B-regions compared to their turbulent pattern near discontinuities (fig.3). Stress intensities decrease rapidly with the distance from the origin of stress concentration. This behaviour allows us to classify B- and D-regions by splitting the real state of stress (fig. 4a) into two states:

The state of stress (b) complies with Bending Theory. It has to satisfy equilibrium between external loads and reactions, but may violate the actual boundary conditions.

The superimposed state of stress (c) is necessary in order to satisfy the boundary conditions. The applied group of forces is self-equilibrating. According to the principle of De St.Vénant the stresses are negligible in a distance from the equilibrating forces, which is approximately equal to the distance between the forces themselves. It defines the D-regions. However, cracked concrete members have different stiffnesses in different directions; this enlarges the D-regions in the direction parallel to the cracks considerably, which may be taken into account. The subdivision of a structure into B- and D-regions is already of considerable value for the comprehension of the internal forces in the structure. The method is applied to a frame in fig. 5.

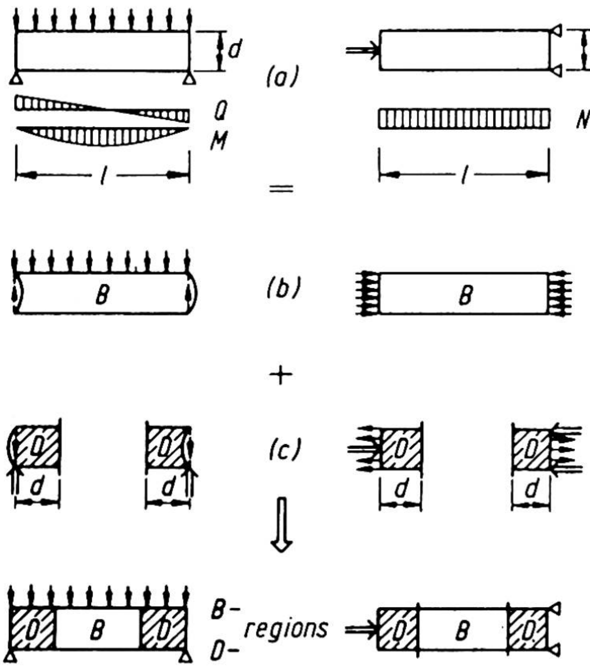

Fig. 4 Splitting of structure into B- and D-regions

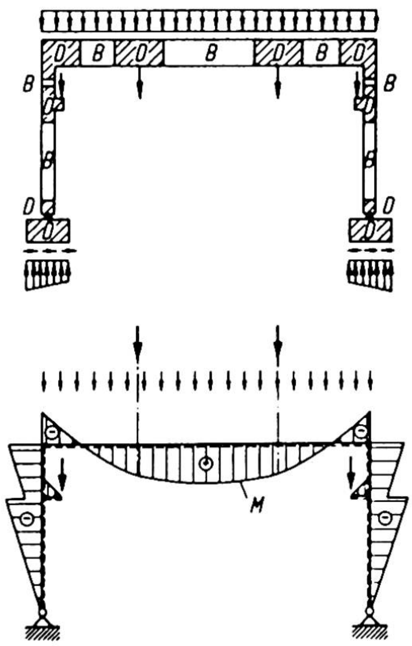

Fig. 5 Frame structure with B- and D-regions

2. MODELLING

We suggest the following basic procedure for the design with truss or strut models (fig.6):

- develop the geometry of the model by orienting the struts and ties at the stress trajectories for elastic behaviour considering the material properties, especially stiffnesses, and the detailing of reinforced concrete;
- calculate the member forces, fulfill equilibrium;
- dimension the members and nodes:
  check capacity of compression members and of possible unreinforced tension members, design reinforcement of tension members with due consideration of crack limitations.

This method implies that the structure is designed according to the lower bound theorem of plasticity. As concrete allows only limited plastic deformations the internal structural system (the strut model) has to be chosen in a way that the deformation limit (capacity of rotation) is not exceeded at any point before the assumed state of stress is reached in the rest of the structure.

This requirement is automatically fulfilled by adapting the strut model and reinforcement to the direction and size of the internal forces as they would appear from the theory of elasticity. In highly stressed D-regions we propose to proceed accordingly. In medium or minor stressed D-regions even large deviations from the ideal reinforcement pattern are no problem as the structure adapts to the assumed internal structural system. However, a minimum reinforcement which is sufficient to control crack widths has to be provided.

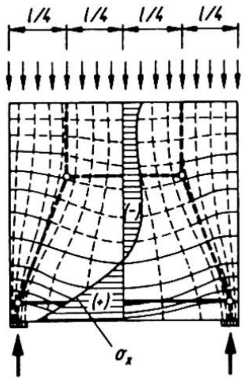

Fig. 6 The strut model approximating the stress trajectories

Whoever spends some time with developing strut models will make the useful and instructive observation that some types of D-regions appear over and over again even in the apparently most different structures. As an example fig. 7 shows D-regions in 4 different structures, altogether modelled with the same type of model (fig.6): A detail of internal forces in a beam, the distribution of cable forces in a bridge deck, a wall with big openings and a box girder with anchor loads from prestressing tendons. In all of these cases the pattern of internal forces is basically identical.

With an unlimited number of examples it could be shown, that tracking down of the internal forces by truss models results in safe structures and quite often provides simple solutions for problems which seem to be rather complicated or reveal weak points which were not obvious. Let us investigate just two examples:

It is well known, that the support reaction A of the beam in fig. 8 must be suspended beside the recess. But the complete strut model reveals clearly, that this is not sufficient: The stirrups have to carry approximately 1,5 A, for the geometrical configuration given. And stirrup stresses may increase furthermore with an additional horizontal reaction at the support.

The girder in fig. 9 obviously produces a vertical tension force T at the bend of the compression cord. But where does it go? The straight horizontal tension cord cannot equilibrate it. The model a) shows, that stirrups in the web are necessary throughout this web with zero shear force. Looking at fig. 9b we realize, that the compression cord is narrowed by the stirrups, resulting in a concentration of compression stresses over the web. Furthermore, tensile stresses in the transverse direction of the flange appear.

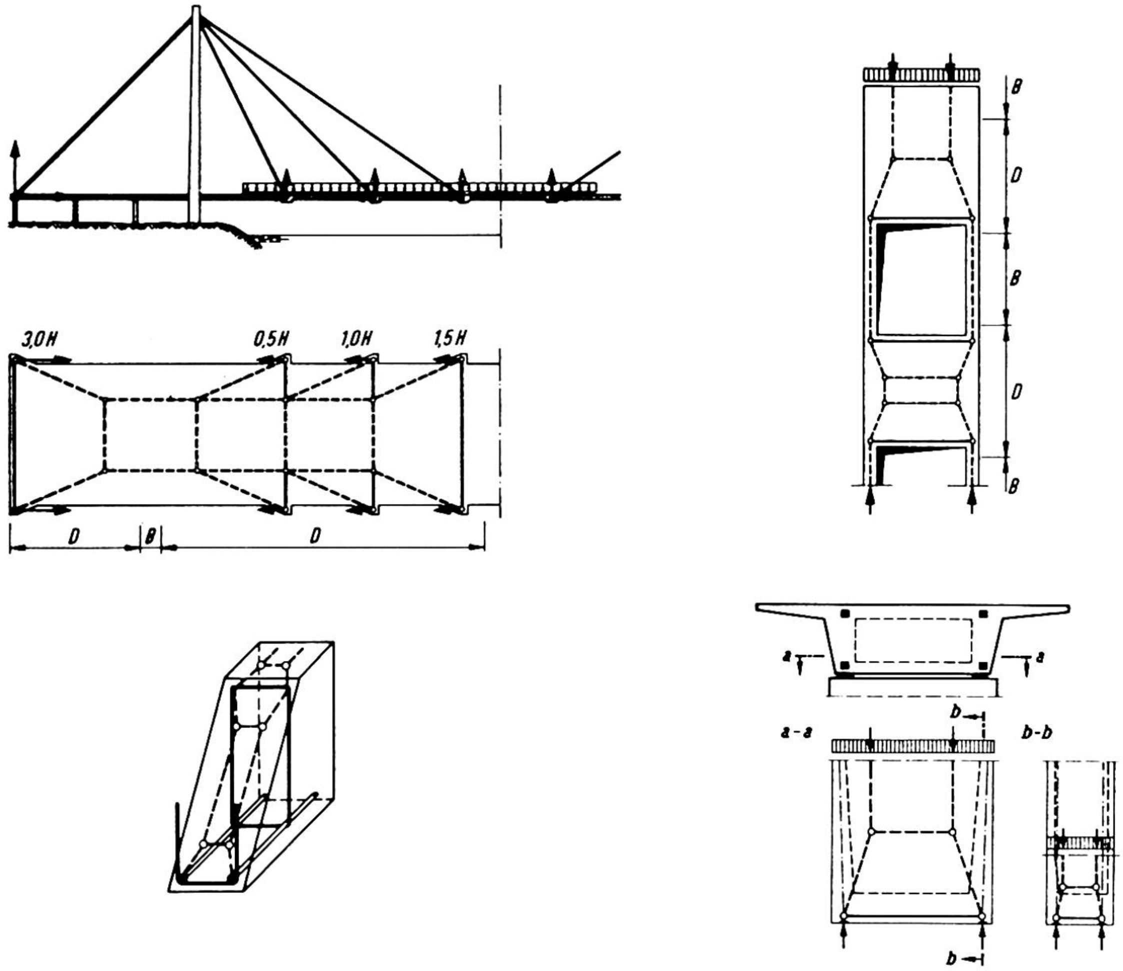

Fig. 7 One single type of a D-model appears in 4 different structures

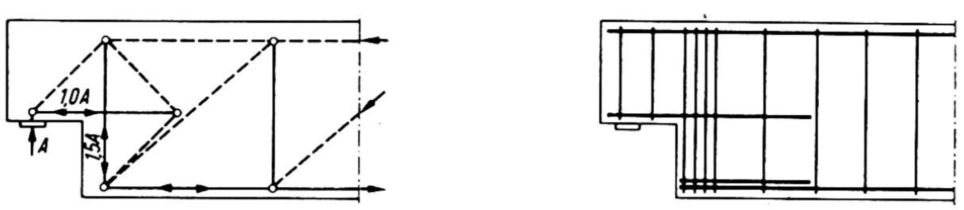

Fig 8 Beam supported in a recess

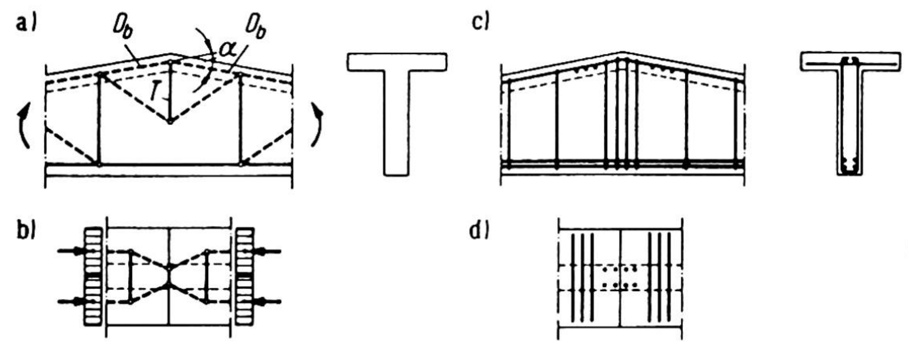

Fig. 9 Girder with bent top flange

3. DIMENSIONING

After a suitable strut model is found, its individual members and nodes have to be designed for the forces they carry. Tension members in the model will pose no problems, if we provide reinforcement or prestressing for the tension force.

However, in many cases we have to introduce tension members into our models in such places where normally no reinforcement is positioned or cannot be inserted for practical reasons. It is one of the advantages of the proposed procedure to reveal in which places the equilibrium of a structure relies on the tensile strength of the concrete. No lap splice, no bond anchor, no frame corner, no plate without shear reinforcement, not even a compression bar could work without the tensile strength of the plain concrete. If we are striving for a unified design concept and want to simplify our codes, we should not deny the concrete tensile strength and refer to it obliquely by terms such as bond or shear. Instead we must introduce and specify the tensile strength as a design quantity.

We propose to use the RVE method (representative volume element) as a practical means of smoothening out stress peaks. Thereby all kinds of tensile stress distributions can be judged by comparison with one single material property, the tensile strength of axially loaded prisms. However, the tensile strength of the concrete may be utilized for maintaining equilibrium only in such cases, where local cracking does not cause progressive failure of the whole tension zone. Existing microcracks from secondary stresses and local faults from concreting could be taken into account by assuming a representative fault area of e.g. $A = 4\ d_K^2$ = (d_K = maximum aggregate size).

The compression bars of our model are in reality 2 or 3-dimensional stress fields. The compression stresses are constricted near the nodes and spread out in between, thereby creating transverse compression and tension stresses. If we characterize the geometry of a plane bar by an effective width b_{ef} and node dimensions a_1, a_2 (fig. 10) a conservative value of the bar's critical load krit F can be taken from fig. 11. The continous lines give the average σ_o and p_a at failure of the compression bar due to transverse tension. The broken lines are postulated upper bounds for the stresses p_a in plane and 3-dimensional stress fields, respectively.

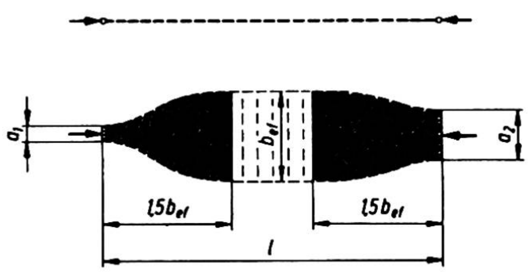

Fig. 10 The compression bar

This concept of dealing with compression in concrete obviously covers standard cases of design, e.g. compression cords of beams and locally loaded concrete as well as the many cases where no generally agreed upon method for the design of compression zones exists, as for corbels, frame corners, deep beams etc. The concept is also applicable to the design of compression struts in beams which today implicitly is taken care of by allowable shear stresses.

The nodes are indispensable parts of the structure. The strut forces resp. stresses are applied at the periphery of the nodes. Although there exist many different types of nodes the balance of the forces takes place in most cases in a concrete node region stressed by compression only (fig.12). The design of these nodes becomes very simple, if a hydrostatical state of stress is assumed in the node region, which means equal stresses in all directions of the struts. The hydrostatical state of stress can be plane or three-dimensional according to the type of the node. The width of the struts at the node are then directly proportional to their forces. From this it becomes clear that there is a close relation between the structural detail, i.e. the design of the nodes and the dimensions resp. capacities of the compression bars. Fig. 13 shows a corbel as an example which can be designed completely with this method.

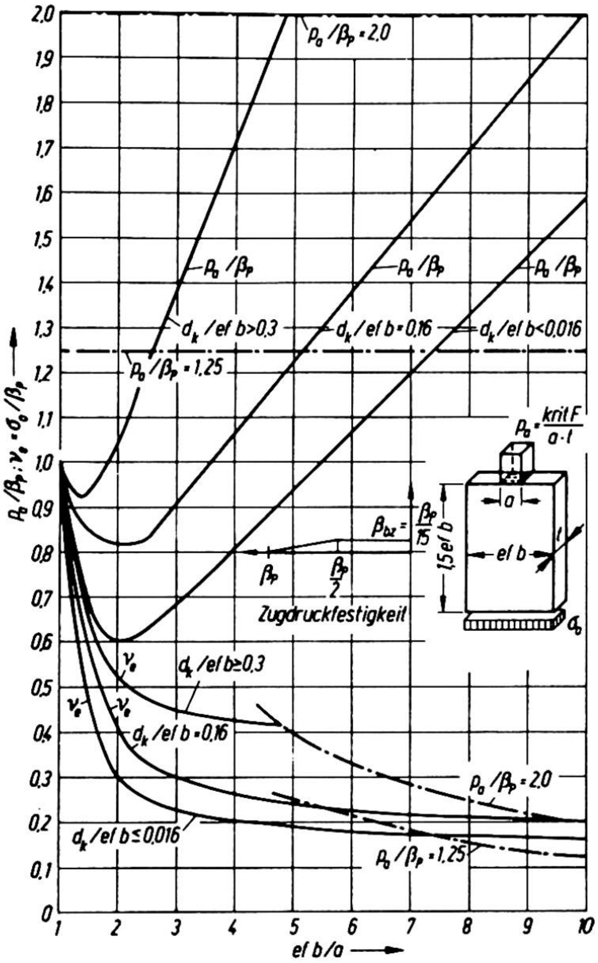

Fig 11 Diagram for evaluating the critical load of plane compression bars (β_p = prism strength)

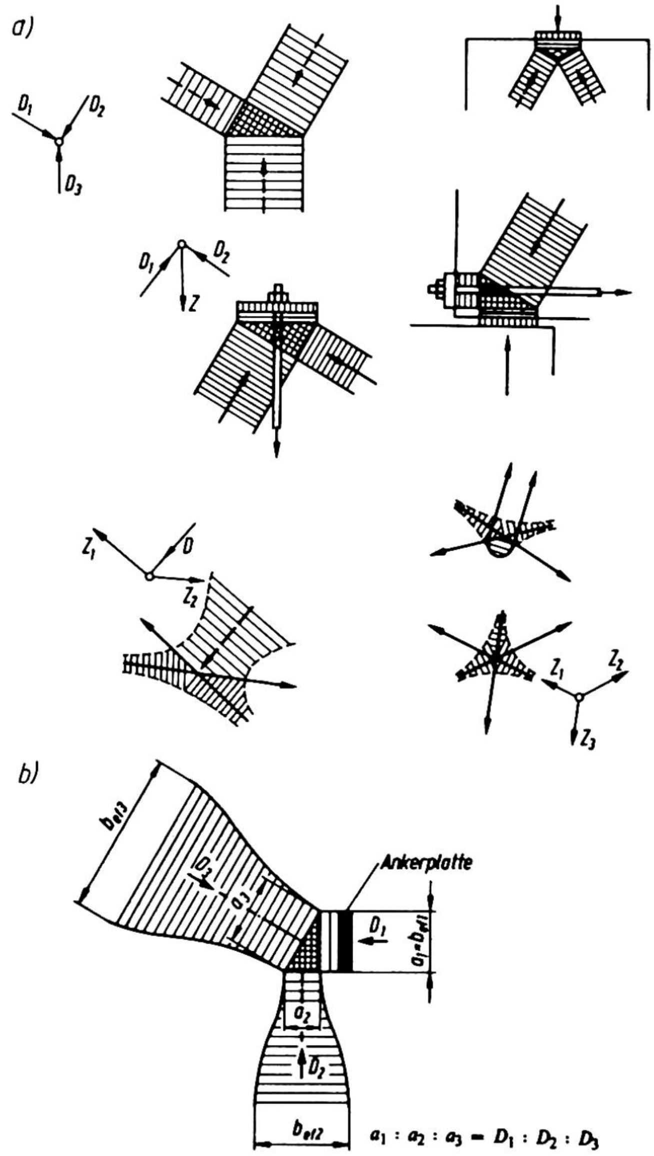

Fig. 12 (a) Different types of nodes
(b) Node with hydrostatic stress

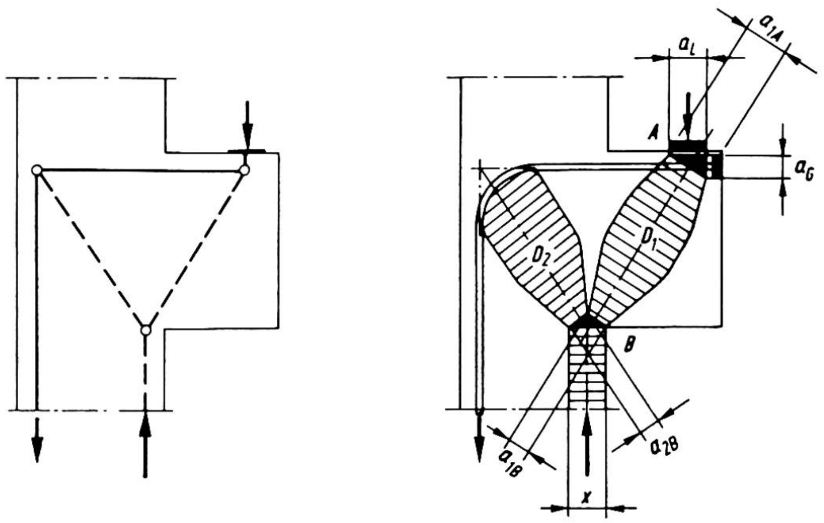

Fig. 13 Compression bars (fields) of a corbel

4. CONCLUSIONS

Calculations with the described strut models and comparison with experimental results show that this method is not only simple and clear but also leads to sufficiently accurate results.
If the design concept developed for the D-regions is applied to the B-regions it is able to explain also their well known behaviour. For example it can be shown that the low stirrup stresses in the low shear range are a consequence of the tensile strength of the concrete. Furthermore a close look at B-regions (e.g. beam section in fig. 7) discloses micro D-regions within the B-regions, where the method again is applicable and reveals for example the influence of detailing and spacing of stirrups on the load carrying capacity of B-regions. Many of the empirically derived code rules could be abandoned or improved by consistent application of the described method, but this is beyond the scope of this paper and was discussed elsewhere /1-3/.

Therefore it is felt that a consistent design concept for all kinds of r.c. structures and details can be developed. This could become the basis for a better understanding of reinforced concrete and for simpler codes.

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