Mathematical analysis of structures: Usefulness and Risks

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Mathematical Analysis of Structures: Usefulness and Risks

Modèles mathématiques pour l'analyse des structures: utilité et danger

Mathematische Analyse von Tragwerken: Gebrauchstüchtigkeit und Risiko

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SUMMARY
The implication of choice of „mathematical models" of different degrees of complexity is analyzed in an effort to elucidate the risks incurred through inevitable departures from reality. This analysis is necessary in view of the ever-growing use of very complex models, more and more removed from the intuition and sensitivity of structural engineers.

RESUME
Les conséquences du choix de „modèles mathématiques" au degré croissant de complexité sont étudiées. Le but de cette analyse est de définir les risques introduits par suite de différences pratiquement inévitables entre le modèle et la réalité. Une telle analyse s'avère toujours plus nécessaire car les ingénieurs utilisent des modèles toujours plus compliqués et toujours plus éloignés de l'intuition et de la sensibilité personnelle.

ZUSAMMENFASSUNG
Es werden die Auswirkungen analysiert, die die Auswahl von mathematischen Modellen steigender Komplexität nach sich zieht. Zweck dieser Analyse ist die Untersuchung und die Bestimmung der auftretenden Risiken, bedingt durch die praktisch unvermeidbaren Unterschiede zwischen Modell und Realität. Diese Analyse wird immer nötiger, wenn man die Tatsache berücksichtigt, dass die Ingenieure zunehmend kompliziertere Modelle benützen, die sich immer mehr von der Intuition und der persönlichen Sensibilität entfernen.

FOREWORD

Electronic computers and computational codes are but the intermediaries linking mathematical models, built by us, of reality, with the numerical results on the basis of which we intend to take practical, technical decisions.
In fact, before setting up a computational numerical code we have to define, with precision, the "mathematical model", chosen as an acceptable representation (simulation) of the real situation in question. Obviously, this model will represent the fundamental variables of this situation, as well as its actions and reactions, both internal and in relation to the outside world.
Moreover, even an efficient computational code based on such a mathematical model will not by itself suffice: it will also be necessary to extract from the real world credible input data, without which the results will have no meaning or value whatever.
These concepts may be expressed in the following block diagram (fig. 1).
It follows that critical analysis of the use of computers and computational codes, however important and necessary, is not by itself sufficient to ensure correct interaction between quantitative analysis of an engineering problem and the operative technical decisions.

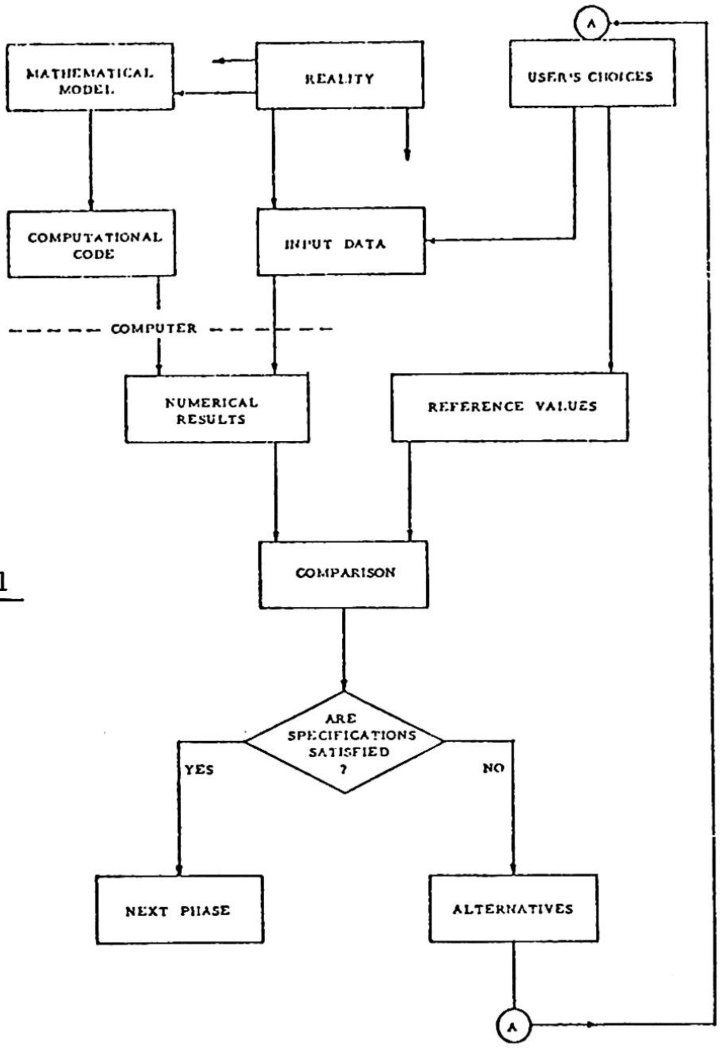

Fig. 1

It will also be necessary to examine critically the often unstated criteria underlying the choice and formulation of the basic mathematical model. The latter, it must be admitted, is not determined in isolation - at least, not for those complex problems that require the use of electronic computers; on the contrary, it is partly the result of subjective choices from among various possible alternative. And where there is a choice, there is also a probability or risk of error, or, at any rate, of a less than-optimal result.
This paper will develop some considerations that we deem relevant to this type of analysis. These preliminary thoughts obviously cannot aim to exhaust the subject, but will, it is hoped, open up a debate and stimulate awareness of often-neglected, or taken-for-granted implications that are, nonetheless, important for the consequences, whether positive or negative, stemming from them.

1. More and more often, recourse is had to so-called "mathematical forecasting models", and this includes the engineering field.
For greater clarity, we should specify what we mean by this expression. We refer to numerical simulation processes - nearly always implemented by means of computational codes on computers - where-by it is claimed that the evolution (in time and/or in space) of certain physical quantities typical of the system being examined, or checked, can be simulated.
Without excluding others, let us therefore focus our attention on two applications typical of such models:
A) - The design - or rather, the parametric optimization of the design.
B) - On-line check of an installation or process.
In case A), the designer has at his disposal certain degrees of freedom as regards the design - some dimensions, some physical constants, some characteristic times - and, by varying parametrically the corresponding model variables, he can claim to be able to investigate the effects of such variations on the behaviour of the system designed.
In this case, use of the model (by the designer) is interactive - in principle, even though, often, this is not strictly speaking so.
In case B), the model has to be implemented on a mini-or-micro-computer, connected to the physical system by adequate sensors that will feed into the model the correct information on the state of the system and on the settings given by the operators, or by the automatisms, to the control devices. A continuous comparison is then set up on some of the fundamental quantities simulated in the model and monitored in the physical system; and if the difference between forecasts and observations of the value of one or more of these variables exceed a pre-established threshold, then a warning is emitted, or else appropriate, preset action is taken.

2. Undoubtedly, the techniques referred to above have met with considerable success in application, and are showing themselves to be indispensable in the design (A) or checking (B) of complex systems, in which the conventional methods or the intuition of the engineer no longer succeed in mastering the whole situation represented by the many possible interactions. But these same successes may create a dangerous tendency to understimate - or even not to take into consideration - the ever-present uncertainties, the causes of error, and,

lastly, the risks inherent in the two types of model utilization.
That there are indeed risks is shown clarly, among other things, by a qualitative analysis such as that which follows. But such considerations are not usually brought explicitly to light, or else the various parties involved - i.e., the designer and author of the model, for example, when they are two different people - tend to have a more or less complete and inevitably different awareness and evaluation of them. From this fact there follows that the results may be badly or erroneously utilized or interpreted, and it would therefore seem worth-while attempting to clarify, as far as possible, the logical implications of the decisional processes stemming from the results of the model itself.

3. It is obvious, in the first place, that any mathematical model is, of its very nature, imperfect. The operation involved in representing a physical reality, which is always extremely complex and open to every kind of interaction, with a finite series of numerical variables and mathematical relationships between the latter, already involves a considerable degree of abstraction, and therefore inevitable departures from reality.
At best, we shall find that the model is, if we look at it clearly incomplete, because it will ignore variables or "secondary" phenomena in relation to the quantities on which the user wishes to concentrate his attention, but that also interact to some extent with the phenomenon modelled.
Moreover, very often the laws and relationships applied as a basis for the model will only be valid as an initial approximation. To this must be added the fact that the inputs of the model themselves (initial conditions and/or boundary conditions of the system) will be incomplete and affected by inevitable errors.
There may also be errors in implementing the model (programming of the computational codes), or in its use.
The consequence will obviously be that the outputs of the model will also be affected by errors and approximations.
The forecasts of the model, on which the user's decisions are based, may therefore sometimes lead to mistaken, or at any rate less-than-optimal choices.
Reducing the matter to its simplest terms, we may say that very often, both in case A) and in case B), what the user wants is to be able to recognize, based on the responses of the model, whether the situation considered on each separate occasion is safe (S) or unsafe (U). In case A), each indication U will be followed by a different choice of free parameters - within the framwork bounded by the limitations of the problem - until an S verdict is obtained.
In case B), each U verdict will have to be followed by appropriate operative decisions calculated to restore the system to an S situation.
However, with regard to what we have said above, the verdict issued on the basis of the model's responses will not always correspond to the real situation.
In particular, a real situation U may sometimes correspond to an S verdict, while, on other occasions, the opposite will occur.
If we assume that the model is used repeatedly in homogenous circumstances, in which the inputs are made to vary in all possible ways, these mistaken verdicts will be issued with a certain frequency (or probability), which we may consider to be represented in the followin possible "table of decisione" by the user:

real situation / verdict issued on basis of model	S	U		
S	$(1-\eta)(1-\beta)$	$\alpha\eta$	$(1-\eta)(1-\beta) + \alpha\eta$	(erroneous decisions in the thick frames)
U	$(1-\eta)\beta$	$(1-\alpha)\eta$	$(1-\eta)\beta + (1-\alpha)\eta$	$\alpha \ll 1$ risk of error S-U
	$1-\eta$	η	1	$\beta \ll 1$ risk of error U-S

N. B. In the case of the design, it should be assumed that $0 < \eta = 1-\varepsilon$ with $\varepsilon \ll 1$.
In the case of on-line checking, usually $0 < \eta \ll 1$.

4. At this point, it is clear that using a mathematical model involves two distinct types of risk:

1 - Type a. risk (probability α): the user considers U to be a S situation (and therefore neglects to take corrective decisions); correspondingly, he will have to bear the costs of any consequences of not having recognized the lack of safety.

2 - Type b. risk (probability β): the user considers S to be U situation, and therefore takes corrective action that is not necessary; in this case, he will have to bear the additional cost of the corrective action, and the cost of any harmful consequences of such action, which might, in an extreme case, bring the system, which was initially in S conditions, towards a U condition.

While the cost connected with the correct decisions (SS and UU) can be evaluated and introduced into financial optimization processes, the costs associated with incorrect decisions (SU or US) are not usually explicitly recognized, still less evaluated. The only evaluation that the user makes is generally an intuitive evaluation of the "degree of trust" to be placed in the model used, and therefore of the more or less wide "safety margin" that will have to be taken in relation to the indications given by the model.

In his turn, the author of the model instinctively tries to protect himself by adopting "conservative assumptions" which may lead him to accentuate the probability of a U-S error in order to diminish the probability of an S-U error. But this choice of pessimistic assumptions is made on a qualitative- intuitive basis, and in the case of complex systems may not lead to the desired results, as we shall be pointing out later on.

It is fairly obvious, and is definitely borne out by the results, that this way of proceeding is often, in practice, acceptable; nor, incidentally, would it be easy to minimize the overall risk run by the user, given the fact that the frequencies (or probabilities) listed in the "table of decision" are not known in advance. But the mere putting of the question in logical, rational terms helps those concerned to sensitize the problem and to evaluate it in as homogenous a way as

possible. Moreover, both the attitude of anyone who, whether consciously or unconsciously, places blind faith in the model, and the opposite attitude, are fairly widespread but both clearly mistaken on the basis of what ha so far been acknowledged.
It is therefore worth-while trying to analyze to the full these interactions between the model, reality, and the user, for this may certainly lead to models being used with more awareness and sense of responsability than hitherto.

5. Let us now look at the advantages and disadvantages that may be the result of a growing degree of sophistication of the model, from the simplest to the most complex, which, notionally, in a sort of "passage to the limit", we might think of as being "as near as one likes" to an unattainable "perfect model" that accords completely with reality.
In particular, we are interested in making a critical appraisal of whether or not it is always justified to rely on models of an intermediate degree of sophistication, as we may consider many of the models at present in existence or being developed, with greater trust than on more schematic models (see fig. 2 on pag. 7).
Generally speaking, the procedure whereby we pass by degrees from the simplest to the most sophisticated model can be presented in simple terms as above. We begin with a very schematic formulation, in which only the fundamental relationships between the more important quantities are taken into consideration; and, among any different alternative formulations there may be, the choice falls, provided it is the same in other respects, either on the simplest from the formal and mathematical point of view, or on the most "conservative" in relation to the desired "safety margin" of the responses supplied in relation to reality: that is, the formulation that is presumed to make $\beta > \alpha$ (see figure and degree of sophistication 0).
Then, if we wish to improve the model - usually because we instinctively judge to be excessive the risk of β introduced by a simple model (and the costs incurred, therefore, likewise to be excessive), either because of too conservative a design or of the introduction of superfluous gadgets and safety devices - we gradually add to the basic mathematica formulation other variables and other "secondary" relationships: that is, relating to effects that are known to be present in the system, but that are regarded as bearers of consequences "of the second order" for the behaviour of the "main" variables. This verdict, too, and the decision as to where we should stop in considering subsequent secondary effects - since it will usually be impossible to consider them all - are taken on a qualitative and intuitive basis, at best with the help of experience of analogous cases.
In this way it is possible to commit involuntary errors of strategy, since, while it may be justifiable to ignore all secondary effects when they introduce consequences that, quantitatively, are not too unbalanced and whose sign is such that they mostly cancl each other out - in this case, to introduce just one, or only a few of the secondary effects may produce a model that is further from reality, in its responses, even than the simplest model.
This is shown in simplified form in the figure;

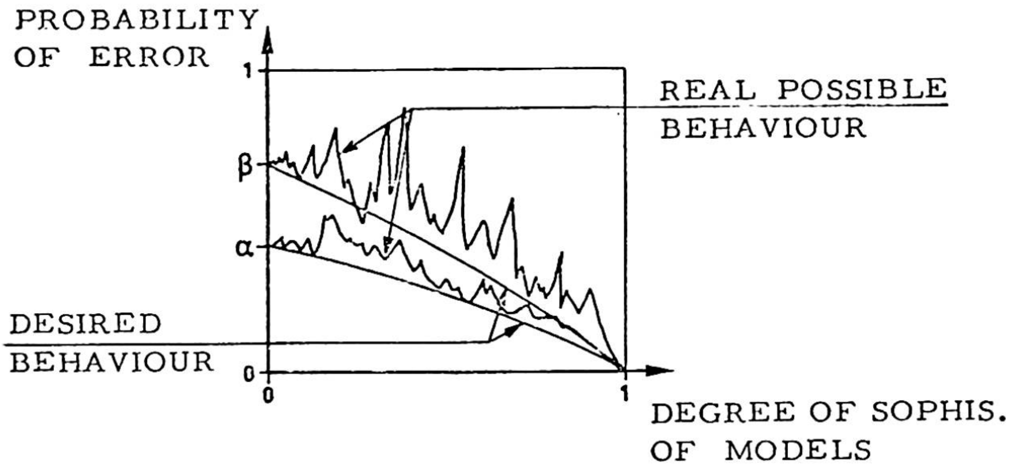

Fig. 2

while it would be desirable for both the probabilities of error α and β to decrease monotonically down to zero, with the indefinite increase in the degree of sophistication of the model, it may happen that one or the other, or both these probabilities of error show a rising (or alternately rising and falling) trend, up or down to intermediate degrees of sophistication, to tend towards zero only with the use of very complete models.

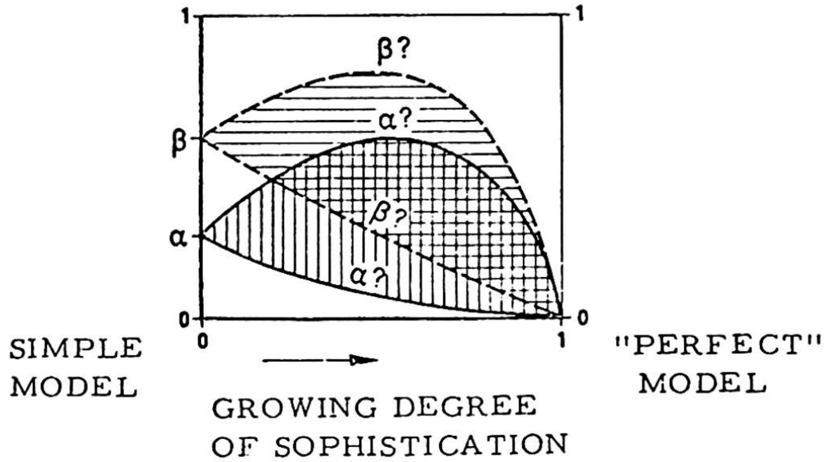

Fig. 3

Now, an increase in probability β of a type b error (U-S) usually leads to additional costs, but only rarely to an actual decrease in safety. Vice-versa, an increase in the probability α of a type a error (S-U) leads directly to a decrease in safety, and therefore to a risk of accidents, possibly serious ones, whose additional costs are usually orders of magnitudes higher than those involved in evaluations that are too cautious.

What has been said is certainly not intended as an a priori criticism of the more sophisticated models, which it is often impossible to do without; it is only desired to show that it is necessary - and, in a sense, urgently so - to find the means to evaluate whether the complications gradually introduced in a model actually fulfil the desired purpose (which, in the terms introduced above, may be summed up as the wish to diminish the probabilities β and, especially, α) or otherwise, and to decide at what point of sophistication it is advisable to stop.

6. Some devices that can be adopted to test the reliability of the model are suggested by mere common sense. For example, it is important to check that the model is stable, in the sense of making sure that, if we feed into it one, or rather, if successively, several secondary effects that were previously ignored, the responses given will not show variations such as to change the operative decisions based on them. Obviously, an unstable model in the sense defined here is not reliable; but unfortunately it is not certain that a model whose "stability" has been ascertained within a limited range, will be reliable throughout the range - and therefore, ultimately, also for the maximum degree of sophistication, in practice unattainable except as an extrapolation, that accurately coincides with reality.
The same stability requirements must obviously apply with regard to possible variations in the input parameters, which are to be expected in view of the uncertainties and degree of indetermination of our knowledge of the real conditions reflected by the value of these parameters.
At the same time, however, the model must be sufficiently sensitive - that is, it must show sufficient variation of the outputs - and especially of the safety indices - in relation to those variations in input or model structure that are known for certain to lead to significant variations in the degree of safetu of the real result. (°)
This implies, among other things, the need not to be content with a single application of the model, which may meet only immediate requirements, but to explore, with the model, several situations close to the one of interest; also, whenever at all possible, it implies the need to vary the structure itself of the model by repeating the application each time with the same input data.
These variations, both in the data and in the structure of the model may usefully be made by a "straddling" approach: by attempting, that is, to see to it that the "real" situation, as far as we may know it or guess it, is sure to be within the range of variations investigated.
Another step dictated by common sense is that of "validating" the model, as far as possible, by applying it to actual situations that have already been produced and documented.
Clearly, this operation involves both the suitability of the mathematical model and the correctness of the computational numerical code. In principle, therefore, the possibility cannot be entirely ruled out that a conceptual error in setting up the model and a programming error will, quite by change, cancel each other out. This eventuality, however, which is in itself improbable, may be virtually ruled out, if the aforementioned "validation" gives positive results, not just for a single real situation, but for a series of different ones - and, if possible, in

(°) It is superfluous to add that the sensitivity must have the correct sign: that is, when unfavourable variations of the model or the inputs occur, it must show a variation in the safety indices that, in addition to being of significant extent, disclose an increase in the degree of risk, and not vice versa.

relation to different jobs or installations.
In the absence of sufficiently well-documented actual situations, the validation may be thought through other comparisons. For example, use may be made of the results of tests on small-scale physical models, to which the mathematical model in question may reasonably be applied; or even, for want of better, of the results of numerical structure analyses, conducted with completely different methods from the method proposed and, if possible, already independently validated.
Other fairly obvious qualitative criteria can be stated.
For example, it is highly advisable, in the process of subsequent enrichment and progressive sophistication of the mathematical model, to be able to establish and follow a sequence, in descending order of importance, of the various factors successively introduced into the simulation. Indeed, there will obviously be no point in introducing the weaker intercations until the stronger one have been satisfactorily represented. But it may very often be difficult to judge in advance which of the two influences leads to the more significant effects. In such a case, the only course remaining will be to implement both on the model, in any order, and to ascertain the extent of the effects afterwards.
It is likewise evident that, in constructing the model, it will not be enough to trust in intuition or experience in order to decide which factors to introduce into the model at each phase of development; this approach will have to give way to a detailed and exhaustive analysis of the possibilities of interaction, both internal and external, of the system. These possibilities will have to be catalogued, and if possible represented in their interrelationships and their order of importance in a kind of genealogical-tree-like or block structure (see the "fault tree" for analysis of plant reliability).

7. From what has so far been said, it may be concluded that the construction of a reliable mathematical model (in relation to the purpose for which it is required) is an "art" to a great extent associated, in the present state of affairs, with the experience and with the intuition of the individual.
It has also been shown that, for every model that can be conceived, risks of error will be incurred between which it would be necessary to strike an optimal balance that it would be difficult to state.
It should not be ruled out that scientific support for better, more rational model construction might come from specialized studies on operating strategies base on incomplete or insufficiently known data (e.g., the theory of "fuzzy sets and systems" etc.).
Until these studies reach the stage of being more utilizable and become more widely known among technicians, we can advance a few criteria that are certainly useful, but not decisive for checking, on each separate occasion, whether the model it is intended to use is adequate or not.

7.1 For each mathematical model implemented in a computational code, there should be available an eshaustive list of basic assumptions for the model, and an analytical examination should be made of the suitability or otherwise of each assumption for the case in question. Should such suitability be lacking, even only in the case of one assumption, then the applicability of the model is impaired.

7.2 A list should be drawn up of the possible causes of departure in the behaviour of the job or installation under examination, or of its environment, from the aforementioned assumptions, and at least a qualitative degree of probability should be assigned to such departures, as well as the degree of gravity of the consequences. This will be a help in seeing whether one can stop there - that is, whether the work or installation can be examined with the model chosen, assuming, obviously, that the condition stated in 7.1 above has been checked - or whether it is advisable to give consideration to other models, that will include the effects ignored by the first model.
By way of an example, for a structure situated in a non-seismic area, it is physically possible that, at a future date, that structure may be subjected to the effects of an earthquake. If the structure is particularly vital, it will be advisable to make a seismic analysis, despite the low probability of such an occurence in the area (see structures of nuclear plants).

7.3 For each input variable, it will not be sufficient to assign a "design" or "check" value, but will be necessary to define a variation interval that can reasonably be assumed, and the domain defined by the ranges of variation of the different quantities will have to be appropriately investigated with repeated applications of the model.

7.4 For each model, it will be necessary to document a case-test that is sufficiently significant and will show the realibility of the results, by way of comparison with the real behaviour of installations already constructed, of small-scale models, or at least of theoretical situations sufficiently similar to the case in question, for which the "exact solution" would be known.

7.5 For each important installation, it will be necessary to set up a file documenting the successive phases of the study on that installation, starting with the design and including both the observations made in service (and their interpretation with adequate models) and any subsequent checks made with models other than the one designed.

Appropriate internationational organizations (including IABSE) might usefully make themselves responsible for standardizing - as far as possible - criteria for drawing up such files and keeping them up to date, as well as promoting the build-up of case history records, access to which will be extremely valuable to the progress of the profession and for the work of model rationalization that is the aim of this memorandum.