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Analysis of Interconnected Space Frames

Calcul des systèmes hyperstatiques tridimensionnels

Berechnung von räumlichen Netzwerken

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Introduction

A method of analysis is presented which is applicable to any system of interconnected frames regardless of their configuration or orientation in space. Loads may be applied in any direction in space, and the effect of temperature change or movement of supports can be readily included. Each common joint of the interconnected frames can be free to move in any direction, and the individual frames can consist of straight or curved components. A numerical example is included to illustrate the method.

The procedure makes possible the partitioning of a structure in such a way that the extent of the computations can be controlled. For complex structures for which the number of simultaneous equations to be solved could exceed the capacity of the digital computer available, partitioning reduces the equations to sets which can be readily handled. The coefficients of the sets of equations are elements of stiffness matrices that can be conveniently determined by means of a numerical procedure [1, 2].

General Procedure

In this section the procedure of analysis is outlined for a three-dimensional, interconnected framework consisting of components, straight or curved, of arbitrary configuration and orientation. All moments, forces and stiffnesses are referenced to an orthogonal coordinate system. The structure is considered to be composed of individual branches or circuits, the ends of which are either at supports or at the junction of other branches, and we proceed as follows:

1. Assume a solution which satisfies geometry.
2. Determine the resulting errors in statics.
3. Impose unit rotations and displacements at each joint in each reference direction, successively, with all other joints restrained, and determine the resulting moments and forces at every joint.

4. Solve for the joint rotations and displacements necessary to adjust the errors in statics.
5. Determine the moments and forces corresponding to the rotations and displacements of step 4.
6. Combine the moments and forces of the assumed solution with those of step 5 to obtain the final moments and forces.

In general, the solution assumed in step 1 is that the joints common to individual branches are restrained against rotation and translation. For example, Fig. 1 shows a generalized loaded structure made up of three-dimensional frames which are rigidly connected at the common joints, A, B, and C. It is assumed that these joints are completely restrained, and the corresponding fixed-end moments, m_0, and fixed-end forces, v_0, are determined as explained in the next section. In the case of support movement or volume changes, however, it would be necessary to assume a solution having joint translations consistent with the imposed movements.

At each common joint fixed-end moments and forces on the ends of all branches at the joint are added algebraically, the positive direction of the coordinate system indicating the sense of positive forces and positive moment vectors (right-hand rule). The resulting moments, $\sum m_0$, and the resulting forces, $\sum v_0$, are the errors in statics referred to in step 2. Correction moments and forces must be added to these unbalanced forces and moments in order to obtain the final moments and forces.

Correction moments and forces are determined through the application of steps 3, 4, and 5. In applying step 3, joint A of Fig. 1 is first considered to be

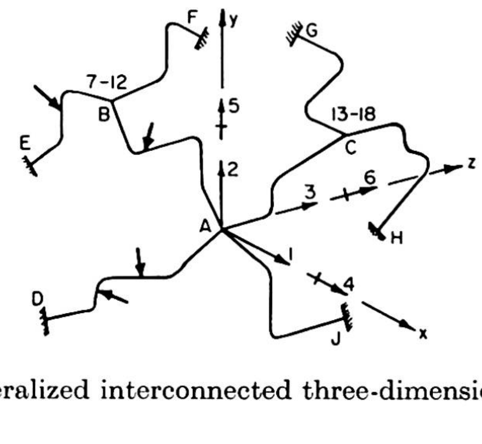

Fig. 1. Generalized interconnected three-dimensional frames.

displaced a unit distance along the positive direction of the x-axis. This is done without allowing joint A to rotate and without allowing rotation or translation at joints B and C. Each of the branches interconnected at joint A (i. e., AD, AJ, AC, AB) is analyzed separately, as explained later, to determine end moments and forces, caused by the joint movement. In general, three forces (in the x, y, and z directions) and three moments (about the x, y, and z axes) will be computed at each end of each branch inter-connected at joint A. Similarly, unit displacements of joint A in the y-direction and then in the z-direction are imposed, with rotation at A prevented and with rotation and translation prevented at B and C. The forces and moments corresponding to each of these movements are computed.

Next, a positive unit rotation about the x-axis is imposed on the structure at joint A while joints A, B, and C are prevented from translating and joints B and C are restrained against rotation. If all frames at a joint are rigidly connected, each is subjected to the imposed unit rotation. No unit rotation is imposed on frames free to rotate. Branches AD, AB, AC, and AJ are each analyzed separately for the imposed rotation, and the end moments and forces computed. Again, in general, three moments and three forces will be determined at each end of each branch. Similar analyses are performed for imposed unit rotations about the y and z axes respectively. Thus a total of six unit deformations, three rotational and three translational, are imposed at joint A with the other joints fixed.

In a similar manner, six unit deformations are imposed at joint B, with A and C fixed, and then at C, with A and B held fixed. The moments and forces thus determined are the stiffness coefficients for the individual branches. A stiffness coefficient s_{ij}^{ab} represents, for some branch ab, the force (or moment) in the direction (or about the axis) i due to a unit displacement (or rotation) in the direction (or about the axis) j. The notation used for the reference directions is the following: the x, y, and z directions at A are labeled 1, 2, and 3, and at B they are labeled 7, 8, and 9, as shown in Fig. 1. These numbers are used to identify forces and displacements. The numbers 4, 5, and 6, and the numbers 10, 11, and 12 refer to moments (or rotations) about the x, y, and z axes, respectively, at A and B. Similarly, the numbers 13 through 18 are assigned to joint C. The stiffness coefficients of branch AB can be arranged in a 12th order matrix, S^{AB}, as follows:

$$S^{AB} = \begin{bmatrix} s_{11} & s_{12} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s_{1,12} \\ s_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s_{2,12} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{12,1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s_{12,12} \end{bmatrix}. \tag{1}$$

The superscription AB has, for convenience, been ommitted from the elements of the matrix.

The matrix of Eq. (1) can be partitioned into four 6th order matrices as follows:

$$S^{AB} = \left[\begin{array}{c:c} S_{AA} & S_{AB} \\ \hdashline S_{BA} & S_{BB} \end{array}\right], \tag{2}$$

in which S_{AA} is the matrix of stiffness coefficients at the end A of branch AB due to unit displacements and rotations at end A, and S_{AB} is the matrix of stiffness coefficients at end A due to unit displacements and rotations at end B. The matrices S_{BA} and S_{BB} consist of the stiffness coefficients at the end B. For those branches such as DA, having one end at a fixed support, the stiffness matrix is of order 12×6 because no displacements or rotations are imposed at a fixed end.

Having obtained a stiffness matrix, S, for each branch, it is possible to form a stiffness matrix, K, for the entire structure. The stiffness coefficients k_{ij}, of this matrix represent the force (or moment) at the joint in the direction (or about the axis) i due to a unit displacement (or rotation) in the direction (or about the axis) j with all other joints completely restrained. They are found as the sum of the individual stiffness coefficients, i.e.,

$$k_{ij} = \sum s_{ij}. \tag{3}$$

This summation is made at each joint where some degree of freedom exists. For the structure of Fig. 1, which has three joints, each with six degrees of freedom, K is of order 18. The matrix K may be partitioned into nine 6th order matrices as follows:

$$K = \left[\begin{array}{c:c:c} K_{AA} & K_{AB} & K_{AC} \\ \hdashline K_{BA} & K_{BB} & K_{BC} \\ \hdashline K_{CA} & K_{CB} & K_{CC} \end{array}\right], \tag{4}$$

in which $K_{AA} = \sum S_{AA}$, $K_{AB} = \sum S_{AB}$, etc. Since no branch directly connects joints B and C, $K_{BC} = K_{CB} = 0$.

In step 4, a column vector, Δ, representing the joint rotations and displacements necessary to adjust the errors in statics, is determined from the following equation of equilibrium:

$$Q + K\Delta = 0. \tag{5}$$

In this equation, Q is a column vector representing the unbalanced forces and moments at each joint. Thus,

$$Q = \begin{bmatrix} \sum v_0 \\ \sum m_0 \end{bmatrix}.$$

Eq. (5) can be readily solved by inverting the stiffness matrix to obtain

$$\Delta = -K^{-1} Q. \tag{6}$$

Physically, the inverse of the stiffness matrix represents the flexibility matrix of the actual, unrestrained structure.

In step 5, the correction forces, v_i, and the correction moments, m_i, at the ends of each branch are found from

$$\begin{bmatrix} v_i \\ m_i \end{bmatrix} = S\Delta, \tag{7}$$

in which S is the stiffness matrix of the particular branch.

In step 6, the corrections are added to the assumed values to obtain the final moments and forces. Thus,

$$\begin{bmatrix} v \\ m \end{bmatrix} = \begin{bmatrix} v_0 \\ m_0 \end{bmatrix} + \begin{bmatrix} v_i \\ m_i \end{bmatrix}. \tag{8}$$

Stiffness Coefficients and Fixed-End Moments and Forces of Branches

In making the analysis discussed in the preceding section, it is necessary to know, for each branch, fixed-end moments, fixed-end forces, and stiffness coefficients with respect to the coordinate axes. In the general case of a three-dimensional branch, three fixed-end moments and three fixed-end forces, referenced to the orthogonal axes, must be determined at each end of the branch. With respect to stiffness coefficients, a branch with six degrees of freedom at each end, such as AB in Fig. 1, involves 144 stiffness coefficients, as in Eq. (1). If one end of the branch has zero degree of freedom, 36 stiffness coefficients are required. In either case, however, the stiffness matrix is symmetrical, thus reducing the number of computations.

The analytical procedure for interconnected frames is independent of the means used to obtain stiffness coefficients, fixed-end moments, and fixed-end forces. A generalized procedure, by Baron and Michalos [1, 2], for closed-circuit structures curved in space is convenient for this purpose. This method, which has been put in matrix form by Baron [3], is numerical in the same sense as solutions of plane frames or arches analyzed by means of the column analogy [4] or the shear and torsion analogy [1, 2]. For a two-dimensional branch with loads and deformations in the plane of the branch, the generalized procedure for computing stiffness coefficients, fixed-end moments, and fixed-end forces, reduces to the column analogy. For loads and deformations normal to the branch, the procedure reduces to the shear and torsion analogy.

If a planar branch is skewed with respect to the coordinate axes, the fixed-end moments and forces can be resolved into components parallel to the coordinate system. As regards the stiffness coefficients, they can be determined first with respect to the plane of the branch by imposing unit deformations both in and normal to the plane. Next determine the projections in the plane and normal to the plane that result from a unit displacement along one coordinate axis. Then multiply the forces (and moments) due to unit displacements in the plane of the branch by the corresponding projected displacements, and, finally, combine the resulting forces (and moments) and resolve them into components parallel to the coordinate system. These components are the desired stiffness coefficients for the particular unit displacement.

Analysis through Use of Moment Distribution

Stiffness coefficients of individual branches represent, in the terminology of Hardy Cross [5], stiffness and carry-over factors, and, if desired, an analysis of interconnected space frames can be made by an extension of the method of moment distribution. Such a method has been used previously to analyze space frames consisting of straight members [2, 6] and frames with curved girders [2, 7].

In applying the method of moment distribution, it is assumed that all common joints are restrained from translating. Moments are successively distributed and carried over with respect to each of the axes of the chosen orthogonal reference system. A moment distributed about one axis at a joint results in a carry-over moment about each of the other two axes at the joint and about each of three axes at the far end of each branch framing into the joint.

With the moments known at the end of each member, the unbalanced force in each orthogonal direction at each common joint can be determined by statics. Corrections are then made for the displacements which must take place along each axis if the joints are actually free to move. This requires the solution of a set of simultaneous equations involving displacements only. Thus, the use of moment distribution reduces the number of equations by up to three at each joint.

Application

In this section the method of analysis is illustrated in connection with Fig. 2. This structural system is composed of three frames interconnected at joint C. Frames AC and CG lie in a single plane, whereas a portion of frame CH is skewed with respect to all three orthogonal coordinate planes. A concentrated load of 5000 lb. acting in the negative y direction is applied midway between B and C and a load of 1000 lb. acting in the z direction is applied at F.

The framework shown in Fig. 2, with one common joint, has been chosen without regard to any practical considerations. It is used merely to present the method of analysis in as straightforward a manner as possible. A hollow circular cross section is used to simplify the presentation further. It should be emphasized that the method of analysis, being perfectly general, is applicable to bending of non-circular sections about non-principal axes, and that any number of common joints could be introduced.

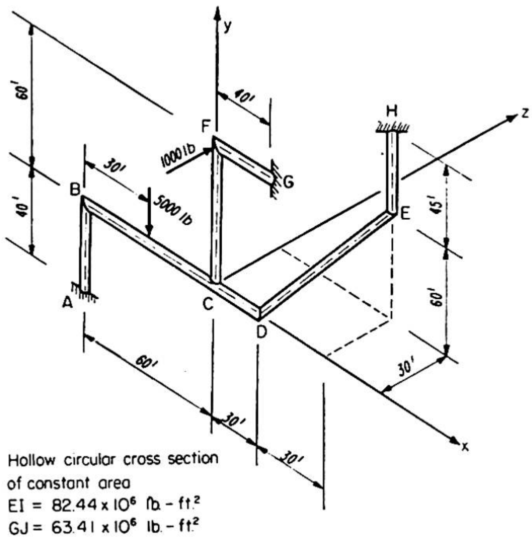

Fig. 2. Structure for numerical example.

The structure is partitioned into three branches, AC, CG, and CH. In step 1 joint C is assumed to be restrained against translation and rotation. Branch AC is then analyzed for the fixed-end condition by the column analogy and branch CG is analyzed for the fixed end condition by the shear and torsion analogy. The resulting moments and forces at joint C are shown in Table I. Branch CH is not loaded and, therefore, has no fixed-end moments or forces. Had it been loaded, the general procedure of BARON and MICHALOS could have been used to obtain fixed-end moments and forces.

The resulting errors in statics are found by adding the fixed-end moments and forces at joint C.

Thus,

$$
\begin{aligned}
Q_1 &= \sum v_{0x} = -845 \text{ lb.}\\
Q_2 &= \sum v_{0y} = 2{,}875 \text{ lb.}\\
Q_3 &= \sum v_{0z} = -270 \text{ lb.}\\
Q_4 &= \sum m_{0x} = 11{,}852 \text{ ft.-lb.}\\
Q_5 &= \sum m_{0y} = 4{,}951 \text{ ft.-lb.}\\
Q_6 &= \sum m_{0z} = 44{,}995 \text{ ft.-lb.}
\end{aligned}
$$

As explained in connection with Eq. (2), the stiffness matrices, S^{AC}, S^{CG}, S^{CH}, for the individual branches are of order 12×6 because each branch has one end fixed. The elements of the matrices for branches AC and CG were obtained through application of the column analogy and the shear and torsion analogy to each of the two branches. The elements of the matrix for branch

CH were obtained through use of the general procedure for non-planar branches. These matrices are not included because of space limitations.

The stiffness matrix, K, for the entire structure was determined as per Eq. (3) by combining individual stiffness coefficients at C obtained from the elements of the stiffness matrices for the individual branches. The matrix K, of order 6×6, becomes the following:

$$K = \begin{bmatrix} 13{,}468 & 9{,}112 & 1{,}069 & 5{,}427 & 16{,}356 & 114{,}800 \\ 9{,}112 & 18{,}142 & 2{,}212 & 17{,}192 & 47{,}057 & 7{,}914 \\ 1{,}069 & 2{,}212 & 3{,}925 & -59{,}913 & -20{,}172 & -53{,}584 \\ 5{,}427 & 17{,}192 & -59{,}913 & 5{,}274{,}300 & -1{,}312{,}200 & -545{,}400 \\ 16{,}356 & 47{,}057 & -20{,}172 & -1{,}312{,}200 & 5{,}803{,}900 & -1{,}414{,}900 \\ 114{,}800 & 7{,}914 & -53{,}584 & -545{,}400 & -1{,}414{,}900 & 15{,}293{,}700 \end{bmatrix}.$$

Rotations and displacements at C were obtained from Eq. (6) by multiplying the inverse of K by the column vector consisting of values Q_1 through Q_6. Correction moments and forces at the ends of the individual branches were obtained as per Eq. (7), and final moments and forces were obtained as per Eq. (8). These values are shown in Table I.

Table I. Moments and Forces at Joint C

Branch		Forces			Moments		
		x	y	z	x	y	z
AC	fixed-end	–845	2,875	0	0	0	44,995
	correction	1,400	–600	115	–117	–5,337	–40,512
	final	555	2,275	115	–117	–5,337	4,483
CG	fixed-end	0	0	–270	11,852	4,951	0
	correction	–524	–2,068	180	–8,829	952	–9,592
	final	–524	–2,068	–90	3,023	5,903	–9,592
CH	fixed-end	0	0	0	0	0	0
	correction	–31	–207	–25	–2,906	–566	5,109
	final	–31	–207	–25	–2,906	–566	5,109

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Summary

A method for the analysis of any system of interconnected frames subjected to a general system of loads or deformations is presented. The frames, consisting of straight or curved members, may occupy any position in space and the common joints may be free to move in any direction. The method is illustrated by a numerical example.

Résumé

Les auteurs présentent une méthode pour le calcul d'un système hyperstatique quelconque, soumis à un système général de charges ou de déformations. Le système, constitué par des éléments droits ou courbes, peut occuper n'importe quelle position dans l'espace et les nœuds peuvent se déplacer dans toutes les directions. Un exemple numérique illustre la méthode.

Zusammenfassung

Die Autoren beschreiben eine Methode zur Berechnung eines beliebigen, statisch unbestimmten Systems, welches einem beliebigen System von Belastungen oder Verformungen unterworfen ist. Das aus geraden oder krummen Elementen zusammengesetzte System kann eine beliebige Stellung im Raum einnehmen und die Knoten sind nach allen Richtungen verschieblich. Die Anwendung der Methode wird anhand eines numerischen Beispiels dargestellt.

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