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Two-dimensional spin systems in a magnetic field close to criticality

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Abstract Numercial calculations of scaling amplitudes of correlation lenghts of some $2D$ spin systems are reported. The results are compared with conformal theory.

Recently, it has been shown how to calculate, for two-dimensional statistical systems with a conformally invariant critical point, the universal scaling amplitudes. Consider a critical point conformally invariant Hamiltonian H_c with a relevant order parameter perturbation σ. The perturbed Hamiltonian is $H = H_c + h \int d^2x\, \sigma$, where h is the magnetic field conjugate to σ.

Assuming that a) the eigenstates of H can still be characterized by the eigenvalues of H_c and b) that first order terms in h are enough (since higher orders correspond to irrelevant operators at $h = 0$), Zamolodchikov [1] showed that *minimal* conformal systems can have for $h \neq 0$ non-trivial integrals of motion. This allows via a conformal bootstrap to conjecture the exact scattering matrices of the effective field theory generated by H. The $S-$ matrices are unique up to CDD factors [2].

The $2D$ Ising model (at $T = T_c$) in a magnetic field is one of the systems where this theory works. This has led to ask whether this system is in fact integrable. The $S-$ matrix contains 8 stable particles. The discrete mass spectrum is [1]

$$\frac{m_2}{m_1} = 2\cos\frac{\pi}{5} = 1.61803\ldots\,, \quad \frac{m_3}{m_1} = 2\cos\frac{\pi}{30} = 1.98904\ldots \tag{1}$$

Numerical checks can be done on inifintely long strips of finite width N. Particle masses m_i are related to correlation lengths ξ_i via $m_i \sim \xi_i^{-1}$. The technique used consists of diagonalising the transfer matrix (in the Hamiltonian limit) on finite lattices, followed by numerical extrapolation for $N \to \infty$. Define the scaling variable $\mu = hN^y$. Eq. (1) is valid for the limit $\mu \to \infty$ *taken after* the finite-size scaling limit $h \to 0, N \to \infty, \mu$ fixed. The most precise results presently available are for the Ising model [3] ($y = 15/8$)

$$\frac{m_2}{m_1} = 1.6181(5)\,, \quad \frac{m_3}{m_1} = 1.994(5) \tag{2}$$

in good agreement with (1). One has the scaling form $m_i = h^{1/y}G_i(\mu)$ and (see [4] for details) $G_i(\mu) \simeq G_i(\infty) - \rho_i \exp\left(-\lambda_i \mu^{1/y}\right)$ for $\mu \to \infty$. Calculating ρ_i gives further information on the $S-$ matrix. The numerical results [3] are completely consistent with a *minimal* $S-$ matrix and indicate the absence of CDD factors at least in the Ising model.

Besides the Ising model, there exist other $2D$ spin systems which reproduce the discrete mass spectrum (1), although *no non-trivial integrals of motion are known* and the Zamolodchikov theory is *not* applicable. The presently known examples are the tricritical Ising model ($c = 7/10, y = 77/40$) and the Ashkin-Teller model ($c = 1, y = 15/8$), both

ϵ	r_1	r_2	r_3	r_4	r_5	r_6
$-1/\sqrt{2}$	1.934(6)	2.0	-	-	-	-
-0.6	1.652(4)	2.0	-	-	-	-
-0.3	1.221(4)	1.837(6)	1.94(1)	2.0	-	-
0.0	1	1.618*	1.618	1.989*	1.989	2
0.1	1.050(3)	1.618(4)*	1.651(5)	1.99(1)*	<2.0(1)	2.0
0.25	1.125(5)	1.621(5)*	1.71(1)	1.98(2)*	<2.0(1)	2.0
0.5	1.245(5)	1.623(5)*	1.785(10)	1.99(1)*	<2.0(1)	2.0
$1/\sqrt{2}$	1.330(5)	1.615(10)*	1.79(2)	1.91(2)*	>1.95	2.0
0.875	1.370(5)	1.57(1)	1.86(1)	1.85(2)	2.00(5)	2.0
1.0	1.395(5)	1.52(5)	1.75(5)	1.8(1)	?	

Table 1: Mass ratios $r_i = m_{i+1}/m_1$ for the Ashkin-Teller model in the limit $\mu \to \infty$ for several values of ϵ covering the entire self-dual critical line. The numbers marked by * are those from the $C = +1$ sector close to (1).

perturbed by their order parameter. For the tricritical Ising model, one finds for $\mu \to \infty$ from a numerical finite-size calculation [5]

$$\frac{m_2}{m_1} = 1.62(1) \; , \; \frac{m_3}{m_1} = 1.98(2) \tag{3}$$

The results for the Ashkin-Teller model [6] are given in table 1 as a function of the four-spin coupling ϵ. The eigenstates of H_{AT} can be classified according to their charge conjugation $C = \pm 1$. For the $C = +1$ sector, the discrete mass spectrum in the interval $0 \leq \epsilon \leq 1/\sqrt{2}$ reproduces (1).

Summarizing, non-perturbative confirmation for the conjectured exact $S-$ matrix was given in the $2D$ Ising model in a magnetic field. The observation that the same discrete mass spectrum is also found in models of *different* universality classes might indicate the possibility of a considerable generalization of the present understanding of slightly off-critical systems.

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