# Theory of 2D conductivity in almost parallel magnetic fields: the role of the third dimension

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**Abstract.** The three-dimensional conductivity tensor has been used to evaluate the magnetoresistance of 2D electron gas in tilted magnetic fields. We found that the third dimension plays decisive role in the behaviour of magnetoresistence when the parallel component of magnetic field is present.

## Introduction

A system of non-interacting electrons confined to the $x-y$ plane by the potential $V_{conf}(z)$ is considered. If the magnetic field $\mathbf{B}$ is oriented perpendicularly to the confinement plane the electron motion can be separated into an electric 'out-of-plane' contribution governed by $V_{conf}(z)$ and a magnetic 'in-plane' contribution leading to the formation of Landau levels. The separation of variables in the one-electron Hamiltonian of the system makes it possible to describe the electron transport in terms of a 2D gas, i.e. to neglect the transverse motion completely.

For any other configuration of the magnetic field this separation is not possible. The original electric and magnetic states are coupled by the parallel component of the field $\mathbf{B}$. As a result the new Landau levels are formed in a plane slightly tilted with respect to the $x-y$ plane and also the quantum states due to $V_{conf}$ are modified. The most serious consequence of the new electron structure is that the electric current mediated by the transition between novel Landau levels flows in the new tilted plane. Therefore, it contributes not only to the transport of electrons in the $x-y$ plane but has also a non-zero 'out-of-plane' $z$-component. This implies that reduction of the conductivity description to two dimensions is not correct in this case and that the full three-dimensional conductivity tensor should be used.

## Model

A harmonic confining potential $V_{conf}(z) = \frac{1}{2}m\Omega^2 z^2$ is able to accommodate a tilted magnetic field $(0, B\sin\phi, B\cos\phi)$ without mathematical difficulties [1]. New states correspond to two harmonic oscillators which oscillate perpendicularly to the $x$-axis and are tilted by an angle $\beta$ with respect to $y$ and $z$ axes. The angle $\beta$ is given by an equation $\tan 2\beta = (\omega_c^2 \sin 2\phi)/(\Omega^2 - \omega_c^2 \cos 2\phi)$ and the eigenfrequences $\omega_1$, $\omega_2$ can be determined from expressions $\omega_1\omega_2 = \Omega\,\omega_z$ and $\omega_1^2 + \omega_2^2 = \omega_c^2 + \Omega^2$. Here $\omega_c = |e|\,B/mc$, $\omega_y = \omega_c \sin\phi$ and $\omega_z = \omega_c\cos\phi$.

The linear response theory described in [2] yields the components of the 3D conductivity tensor in the form:

$$\sigma_{xx} = \frac{\cos\beta\cos(\beta+\phi)}{\cos\phi}\,\sigma_1 + \frac{\sin\beta\sin(\beta+\phi)}{\cos\phi}\,\sigma_2, \qquad \sigma_{yy} = \cos^2\beta\,\sigma_1 + \sin^2\beta\,\sigma_2,$$

$$\sigma_{zz} = \sin^2\beta\,\sigma_1 + \cos^2\beta\,\sigma_2, \qquad \sigma_{xy} = ec\frac{\partial N}{\partial B_z} - \omega_z\tau\,\sigma_{xx},$$

$$\sigma_{yz} = 0, \qquad \sigma_{zx} = ec\frac{\partial N}{\partial B_y} - \omega_y\tau\,\sigma_{zz} - \frac{\omega_y}{\omega_z}\cdot\omega_z\tau\,(\sigma_{xx} - \sigma_{yy}).$$

In these formulae $\tau$ is the relaxation time related to the mobility $\mu$ by $\mu = |e|\,\tau/m$. The conductivity in the new tilted plane is denoted by $\sigma_1$. For the case of almost parallel fields $B_z \ll B_y$ and neglecting the Shubnikov-de Haas oscillations it can be written in the quasi-classical (Drude-Zener) form $\sigma_1 = \sigma_0/(1+\omega_1^2\tau^2)$ where the zero-field conductivity is given by a standard expression $\sigma_0 = e^2/m \cdot N\tau$. The vanishingly small conductivity $\sigma_2$ is due to the transitions between the modified electric states.

## Results

Leadley et al [3] have investigated the magnetoresistance $\Delta\varrho_\perp/\varrho_0 = (\varrho_{xx}(B_z) - \varrho_{xx}(0))\,/\varrho_{xx}(0)$ in magnetic fields tilted a few degrees from the parallel configuration. $\Delta\varrho_\perp/\varrho_0$ was measured as a function of $B_z$ for several values of $B_y$. Sharp dips have been found around $B_z = 0$, with the depth proportional to $B_y$ and the half-width determined by $\omega_z\tau = 1$.

We have calculated $\Delta\varrho_\perp/\varrho_0$ using a convenient expression $\varrho_{xx} = (\sigma_{xx} + \sigma_{xy}^2/\sigma_{yy} + \sigma_{zx}^2/\sigma_{zz})^{-1}$ which can be reduced to the 2D case by neglecting the term $\sigma_{zx}^2/\sigma_{zz}$. No $B_z$ dependence was found in the 2D case. The results of full 3D calculations are presented in Fig.1 and explain qualitatively the features of $\Delta\varrho_\perp/\varrho_0$ described in [3].

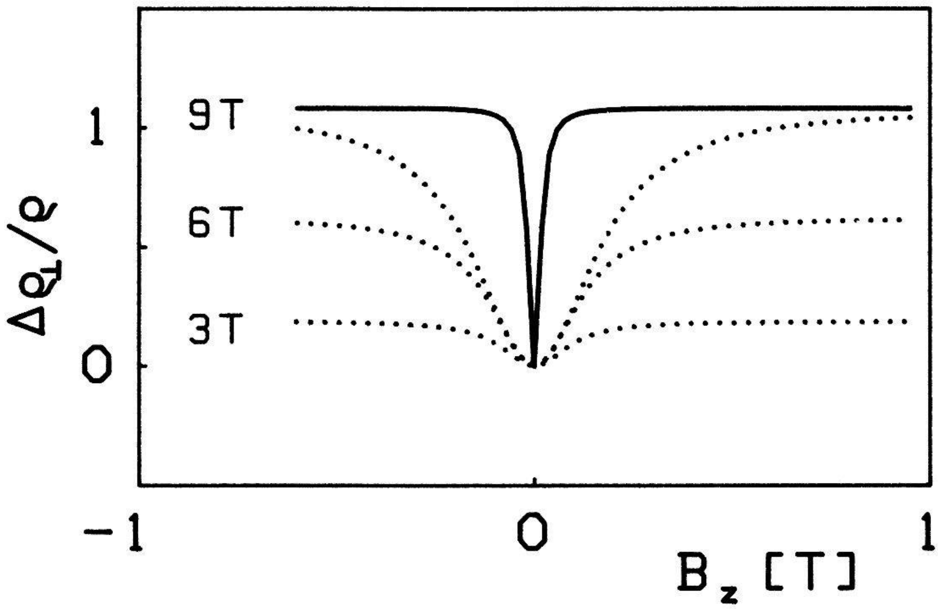


Figure 1: Magnetoresistance $\Delta\varrho_\perp/\varrho_0$ as a function of $B_z$ for three different parallel fields $B_y$. Two mobilities are considered. The dotted lines correspond to $\mu = 10m^2/Vs$ the full line to $\mu = 100m^2/Vs$. The carrier concentration is $N = 3\times10^{15}m^{-2}$, the separation of subbands is $\hbar\Omega = 20meV$.

## References

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