Multifractality and Scaling in Disordered 2-D Landau Levels

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Abstract. The structure of the eigenstates of disordered 2-D systems of size $M \times M$ in strong perpendicular magnetic fields is investigated. A multifractal behaviour is found in the centres of the Landau bands, which is described in terms of the $f(\alpha(q))$ vs. $\alpha(q)$ spectrum. The eigenstates $\psi_{E,M}(\boldsymbol{r})$ and the equilibrium current density $\boldsymbol{j}_{E,M}(\boldsymbol{r})$ show similar $f(\alpha(q))$ spectra which supports the view of a universal one-parameter scaling behaviour.

Introduction

Electronic states in disordered two-dimensional systems, subject to a strong perpendicular magnetic field, exhibit complex self similar structures on length scales l, which are larger than both the magnetic length and the correlation length of the potential , but are smaller than the system size M [1, 2, 3]. These states appear at energies near the centres E_n of the Landau bands, where the localization length $\xi(E)$ diverges as $|E - E_n|^{-\nu}$, with $\nu = 2.35$ [4, 5, 6]. To characterize the multifractal states, the $f(\alpha(q))$ vs. $\alpha(q)$ spectrum has been calculated [3, 6], which can be fitted to a parabola, $f(\alpha) = D_0 - (\alpha - \alpha_0)^2/(4(\alpha_0 - D_0))$. This reflects the log-normal distribution of the box-probabilities $p_i(l, E, M) = \sum_{\boldsymbol{r}\in\Omega_i(l)} |\psi_{E,M}(\boldsymbol{r})|^2$. With $D_0 = 2$, we find for the lattice model $\alpha_0 = 2.29 \pm 0.02$, which is in accordance with the result for the continuum model [3].
In this contribution a multifractal analysis of the equilibrium current density $\boldsymbol{j}_{E,M}(\boldsymbol{r})$ is presented. This quantity shows a similar behaviour as found for the eigenstates. Thus, a universal $f(\alpha(q))$-spectrum for a variety of physical observables seems likely.

Model and Method

A one band tight binding model on a simple cubic lattice with periodic boundary conditions is considered to describe non-interacting electrons in disordered 2-D systems in strong perpendicular magnetic fields. The site energies are independent random numbers distributed with probability $1/W$ over the interval $[-W/2, W/2]$ and the magnetic field B enters the phase factors of the transfer matrix elements. A Lanczos algorithm is applied to calculate the eigenvalues and eigenvectors of the corresponding sparse hermitian matrices.
The $f(\alpha)$-spectrum is calculated according to [7] from

$$\alpha(q) = \lim_{\lambda\to 0} \sum_i \mu_i(q,\lambda) \ln P_i(\lambda) \Big/ \ln(\lambda) \tag{1}$$

and

$$f(q) = \lim_{\lambda\to 0} \sum_i \mu_i(q,\lambda) \ln \mu_i(q,\lambda) \Big/ \ln(\lambda), \tag{2}$$

with $\mu_i(q,\lambda) = P_i^q(\lambda)/\sum_j P_j^q(\lambda)$ and $\lambda = l/M$.
For the investigation of the equilibrium current density of an eigenstate with energy E, $\boldsymbol{j}_{E,M}(\boldsymbol{r}) = \sum_{\boldsymbol{r}'} \langle E \,|\, \boldsymbol{r}' \rangle \langle \boldsymbol{r}' \,|\, \hat{\boldsymbol{j}} \,|\, \boldsymbol{r} \rangle \langle \boldsymbol{r} \,|\, E \rangle$, a normalized box-probability was defined as $P_i(\lambda) = |\boldsymbol{j}_i(\lambda)| / \sum_k |\boldsymbol{j}_k(\lambda)|$ [3], because the current density is, in contrast to the eigenstates, not normalized, and $\boldsymbol{j}_i(\lambda) = \sum_{\boldsymbol{r}\in\Omega_i(\lambda)} \boldsymbol{j}_{E,M}(\boldsymbol{r})$.

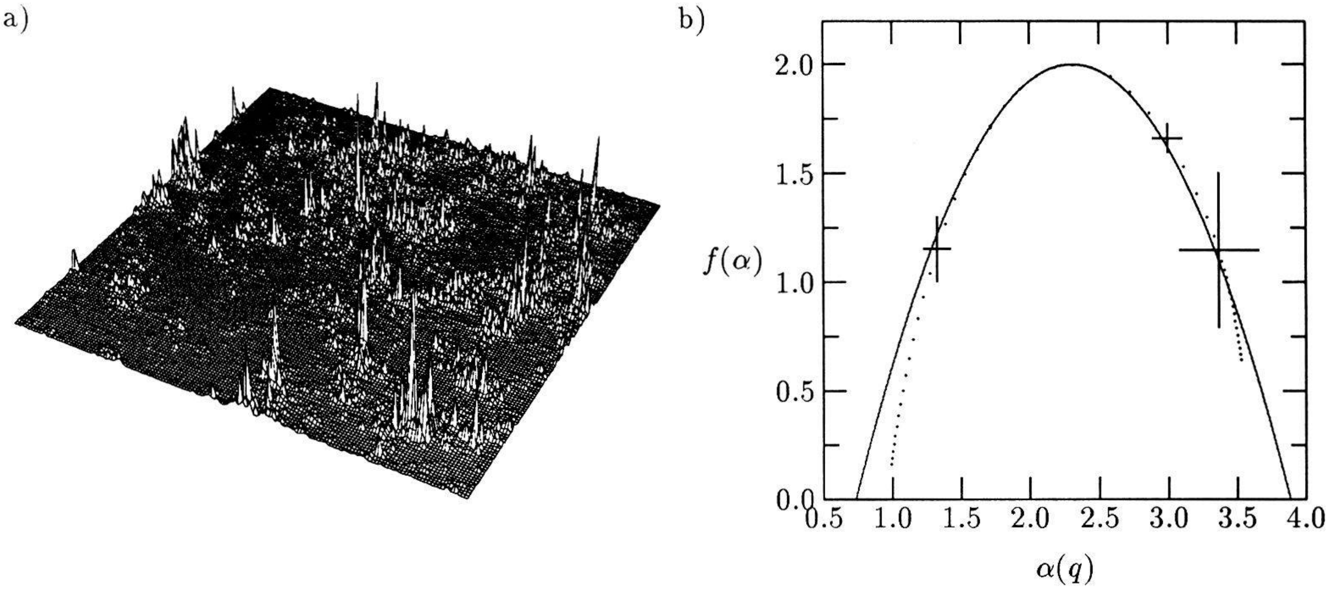

Fig. 1: a) The density $|\psi_{E,M}(\boldsymbol{r})|^2$ of the most extended state in the centre of the lowest Landau band. b) $f(\alpha)$-spectrum of the current density of the state shown on the left. Crosses give the errors in $f(q)$ and $\alpha(q)$ for $q = 2, 1, -1, -2$ (from left to right).

Results and Discussion

In fig. 1a the modulus of the most extended state, $|\psi_{E,M}(\boldsymbol{r})|^2$, of the lowest Landau band for a system of size $M = 150\,a$, is shown as a function of the position. The magnetic field strength is equivalent to 1/5 flux-quanta h/e per unit lattice cell a^2. A strongly peaked structure of the wavefunction is observed. However, this must not be regarded as a sign of localization, but reflects the fact that the logarithm of the density $|\psi_{E,M}(\boldsymbol{r})|^2$ is normally distributed [3]. The $f(\alpha(q))$-spectrum of the corresponding equilibrium current density $\boldsymbol{j}_{E,M}(\boldsymbol{r})$ is plotted in fig. 1b together with a parabolic fit from which a value of $\alpha_0 = 2.31 \pm 0.03$ is obtained. Therefore, the $f(\alpha(q))$-spectrum of the current density is, within the current uncertainty, the same as for the eigenstates [6].

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