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Charge-Vortex Duality in Josephson Junction Arrays

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Abstract. In arrays of Josephson junctions vortices are important collective excitations. A Kosterlitz-Thouless-Berezinskii transition characterized by the unbinding of vortex dipoles separates a superconducting and a resistive phase. In small capacitance junctions the charging energy gains importance. In suitable arrays a KTB transition characterized by the unbinding of charge dipoles separates an insulating from a conducting phase. We present an effective description of Josephson junction arrays in the quantum regime in terms of charges and vortices. In suitable systems there exists a duality relation between both and a superconductor-insulator transition at low temperature. We present the phase diagram, investigate the effect of a magnetic field, and compare theoretical and experimental results. We also discuss further physical properties of quantum vortices, such as the vortex mass, the Aharonov-Casher effect of vortices moving around a charge, forces acting on vortices and the dissipation of the vortex motion due to quasiparticle tunneling.

1. Introduction

The configuration of the phases of classical, 2-dimensional Josephson junction arrays can be characterized by vortices and spin waves. The vortices influence the transport properties of arrays in a characteristic fashion [1, 2, 3]. They interact logarithmically, which leads to a Kosterlitz-Thouless-Berezinskii transition [4] where vortex-antivortex pairs dissociate. The transition temperature is of the order of the Josephson coupling energy $k_B T_v \approx E_J$. It separates a superconducting, low-temperature phase from a resistive, high-temperature phase.

In junctions with small capacitances the charging energy gains importance and introduces quantum dynamics into the system. As a result the vortices can move [5]; in certain limits they behave as quantum mechanical particles with a mass [6]. Charging effects lower the vortex-unbinding transition temperature. This has been studied extensively for the case where the self-capacitance C_0 of the superconducting islands dominates over the junction capacitance C [7]. A smaller number of papers dealt with the model where the junction capacitance dominates $C \gg C_0$ [8]-[11]. In this limit the charges interact logarithmically over long distances. Hence a (normal or superconducting) junction array can undergo a KTB transition where charge-dipoles (single electron or Cooper pair charges) unbind, separating an insulating from a conducting regime [11].

The charges and the phases of Josephson junctions are quantum mechanical conjugate variables. This has the consequence that charge order and phase order compete with one another [12]. Similarly in a Josephson junction array the charge order and the vortex order exclude one another. This leads to a superconductor-insulator transition at zero temperature [13]. In suitable arrays ($C \gg C_0$) there exists a nearly perfect duality between charges and vortices [13, 14, 15] which implies a universal conductivity at the superconductor-insulator transition [16, 17].

The properties of the charges and vortices, the competition between them and the phase diagram of the junction array can be discussed in a coupled-Coulomb-gas model for the charges Q_i on the islands and the vorticity v_i enclosed between them [13]. We will first review these results. Then we will study further properties of the vortices. We will derive an effective action for vortices and obtain the vortex 'mass', discuss the Aharonov-Casher effect of vortices moving around a charge and describe the forces acting on vortices. We will also investigate the tunneling of quasiparticles, which leads to a damping of the vortex motion. Finally, we comment on the properties of the charges in arrays.

2. The coupled-Coulomb-gas description

We consider a junction array with superconducting islands, ignoring all fluctuations other than those associated with the phases of the superconducting order parameters ϕ_i on the islands i. If we further ignore for the moment dissipation we can write the Hamiltonian of this system as

$$H = \frac{1}{2}\sum_{i,j}(Q_i + Q_{x,i})C_{ij}^{-1}(Q_j + Q_{x,j}) - \sum_{<i,j>} E_J \cos[\phi_{ij} - A_{ij}(\tau)] \; ; \quad Q_i = \frac{\hbar}{i}\frac{d}{d(\hbar\phi_i/2e)} \; . \tag{1}$$

The Cooper pair charge Q_i on the island i and the phase ϕ_i are quantum mechanical conjugate variables; and $\phi_{ij} = \phi_i - \phi_j$ refers to nearest neighbours. The Josephson coupling defines the energy scale E_J. The Coulomb interaction of the charges is described by a capacitance matrix C_{ij}. Electromagnetic fields are accounted for by a vector potential $A_{ij} = 2\pi/\Phi_0 \int_i^j \vec{A} \cdot d\vec{l}$ where $\Phi_0 = h/2e$. We also allowed for "offset" or "external" charges $Q_{x,i}$ on the islands, created for instance by charged impurities in the substrate, which bind a part of the total island charge. We will comment further on their origin and relevance in section 6; for the moment they are included to describe the most general model.

In the following we consider a square lattice and take into account the junction capacitance C, which dominates in fabricated arrays, and the self-capacitance of each island C_0 (the capacitance to the ground plane or to infinity), but ignore all other capacitances. Hence $C_{ii} = C_0 + 4C, C_{ij} = -C$ for i and j nearest neighbours, and $C_{ij} = 0$ otherwise. In the limit $C \gg C_0$ the inverse capacitance matrix C_{ij}^{-1} is long range, varying logarithmically with the distance $|\vec{r}_i - \vec{r}_j|$ between the islands [11]. The charges on the islands can change only by Cooper pair tunneling. In the absence of offset charges $Q_{x,i} = 0$ the total charges on the islands are integer multiples of $2e$. Hence the junction array - with its discrete charges and a logarithmic interaction - is a direct physical realization of the 2-dimensional Coulomb gas. This has attracted much attention because of its interesting phase transition [4]. However, the Coulomb gas model does not account for the Josephson coupling, which is included in (1) and which is known to yield interesting physics by itself. In the following we will study the combination of both.

The partition function of the system can be expressed as a path integral in imaginary times $0 \leq \tau \leq \beta = 1/k_BT$ (from now on we choose $\hbar = 1$). In a mixed representation involving the phases $\phi_i(\tau)$ and charge trajectories $q_i(\tau) \equiv Q_i(\tau)/2e = 0, \pm1, \pm2, \ldots$ it is [12]

$$Z = \prod_j \int_{q_{j0}}^{q_{j0}} Dq_j(\tau) \sum_{\{n_i\}} \prod_i \int_{\phi_{i0}}^{\phi_{i0}+2\pi n_i} D\phi_i(\tau) \exp\{-S[q,\phi]\} \tag{2}$$

which depends on the action

$$S[q,\phi] = \int_0^\beta d\tau \Big\{ \sum_{i,j} 2e^2[q_i(\tau) + q_{x,i}(\tau)] C_{ij}^{-1} [q_j(\tau) + q_{x,j}(\tau)] + i \sum_i q_i(\tau)\dot{\phi}_i(\tau) - \sum_{<i,j>} E_J \cos[\phi_{ij}(\tau) - A_{ij}(\tau)] \Big\}. \tag{3}$$

Our choice of discrete charge states ($Q_{x,i}$ plus integer multiples of 2e) implies that values of the phase which differ by 2π are equivalent, and the integral in (2) includes a summation over winding numbers $\phi_i(\beta) = \phi_i(0) + 2\pi n_i$ [12].

Vortex degrees of freedom can be introduced by the Villain transformation [18], which can be generalized [13, 19] to the present problem with charges. For this purpose we introduce a lattice in time direction, with spacing $\Delta\tau$ of the order of the inverse Josephson plasma frequency $\Delta\tau^{-1} \approx \omega_0 = \sqrt{8E_JE_C}$. But for transparency we keep in the following the continuum notation. The Villain transformation allows us to integrate out the phases at the expense of introducing at each (dual) space-time lattice point an integer-valued field, the vorticity in the plaquette i, which is $v_i(\tau) = 0, \pm1, \ldots$. Details of the derivation are given in Ref. [19]. As a result the partition function can be written as a sum over integer valued paths $q_i(\tau)$ and $v_i(\tau)$

$$Z = \sum_{\{q_i(\tau)\}} \sum_{\{v_i(\tau)\}} \exp\{-S_{CCG}[q,v]\}\,. \tag{4}$$

The action of the coupled-Coulomb-gas (CCG) is

$$S_{CCG}[q,v] = \int_0^\beta d\tau \Big\{ \sum_{i,j} \Big[2e^2[q_i(\tau) + q_{x,i}(\tau)] C_{ij}^{-1} [q_j(\tau) + q_{x,j}(\tau)] + \frac{1}{4\pi E_J} \dot{q}_i(\tau) G_{ij} \dot{q}_j(\tau) + \pi E_J [v_i(\tau) + f_i(\tau)] G_{ij} [v_j(\tau) + f_j(\tau)] + i\dot{q}_i(\tau)\Theta_{ij}[v_j(\tau) + f_j(\tau)] \Big] + i \sum_{i,\mu} \dot{q}_i(\tau) n_\mu (n\cdot\nabla)^{-1} A_{i,i+\mu} \Big\}. \tag{5}$$

Here we introduced

$$f_i(\tau) = \frac{1}{2\pi} \sum_{\mu,\nu} \epsilon_{\mu\nu} \nabla_\nu A_{i,i+\mu}(\tau) \tag{6}$$

which describes the magnetic flux through the plaquette i, measured in units of the flux quantum $\Phi_0 = h/2e$, but also an electric field. By $i+\mu$ we denote the nearest neighbor of i in μ-direction ($\mu = x, y$). Furthermore, we introduced the kernel $G_{ij} = G(\vec{r}_i - \vec{r}_j)$, which describes the interaction between vortices at sites i and j

$$G(\vec{r}) = \frac{1}{2\pi} \int d^2q \frac{1}{q^2} [\exp(i\vec{q}\cdot\vec{r}) - 1] = -\ln\{\frac{1}{2}[1 + (1 + 2\pi r)^{1/2}]\}. \tag{7}$$

The explicit result (7) is obtained for a convenient choice of the cutoff in q [6] and is defined also for small r. The kernel

$$\Theta_{ij} = \arctan(\frac{y_i - y_j}{x_i - x_j}), \tag{8}$$

where $\vec{r_i} = (x_i, y_i)$, describes the phase configuration at site i around a vortex at site j.

The first and third term in (5) represent the classical action of the electric charges and of the vortex 'Coulomb gas'. For $E_J = 0$ or $C_{ij}^{-1} = 0$ the fields are constant in time τ, and the classical Coulomb gases of charges or vortices, respectively, are recovered. In general the two different 'charges' interact via the kernel Θ_{ij}, as described by the fourth term. After a partial integration we recognize that this term involves the interaction energy of a charge q_i with the voltage $\Theta_{ij}\dot{v}_j$ at site i created by the changing vorticity at site j. The last term, also after a partial integration, describes the interaction of the charges with the line integral (represented by the operator $\vec{n}(n \cdot \nabla)^{-1}$) of the external electric field.

The action (5) shows a high degree of symmetry between the vortex and the charge degrees of freedom. If we consider the limit $C \gg C_0$ the inverse capacitance matrix becomes (for large distances)

$$e^2 C_{ij}^{-1} = \frac{E_C}{\pi} G_{ij} \qquad \text{where} \qquad E_C = e^2/2C. \tag{9}$$

In this case charges and vortices are nearly dual. The duality is broken by the term $\dot{q}_i G_{ij} \dot{q}_j$. This nonlocal kinetic contribution arises as the spin-wave contribution to the charge correlation function. The corresponding excitations in the charge gas are absent in the model defined by (2), so that an equivalent term $\dot{v}_i G_{ij} \dot{v}_j$ does not arise.

3. Phase transitions in junction arrays

<u>$f = 0$ and $Q_x = 0$:</u> We first discuss the phase transitions in the array without external fields $\vec{A} = 0$ and assuming that there exist no offset charges $Q_x = 0$. For $C_{ij}^{-1} = 0$ the action (5) reduces to the Hamiltonian of the classical Coulomb gas of vortices. The system has a KTB transition, where vortex dipoles unbind, at a temperature

$$T_v^{(0)} = \frac{\pi}{2\epsilon_v} E_J. \tag{10}$$

The dielectric constant ϵ_v is of order 1. This transition separates a superconducting from a resistive phase.

If $E_J = 0$ the action (5) - or the Hamiltonian (1) - reduces to the Hamiltonian of the classical Coulomb gas of charges. If the junction capacitance dominates $C \gg C_0$, i.e. if the charges interact logarithmically over sufficiently long distances, also this system has a KTB transition where the dipoles, formed by a Cooper pair and a missing pair, unbind. The transition temperature is

$$T_{cs}^{(0)} = \frac{1}{4\pi\epsilon_C} \frac{(2e)^2}{2C}. \tag{11}$$

The dielectric constant ϵ_C in general differs from that of the vortex transition, but it is again of order 1. In a normal junction array a similar phase transition can be found. In this case dipoles of single electron charges unbind [11], and accordingly the transition temperature $T_{cn}^{(0)}$ is smaller by a factor 4 than $T_{cs}^{(0)}$.

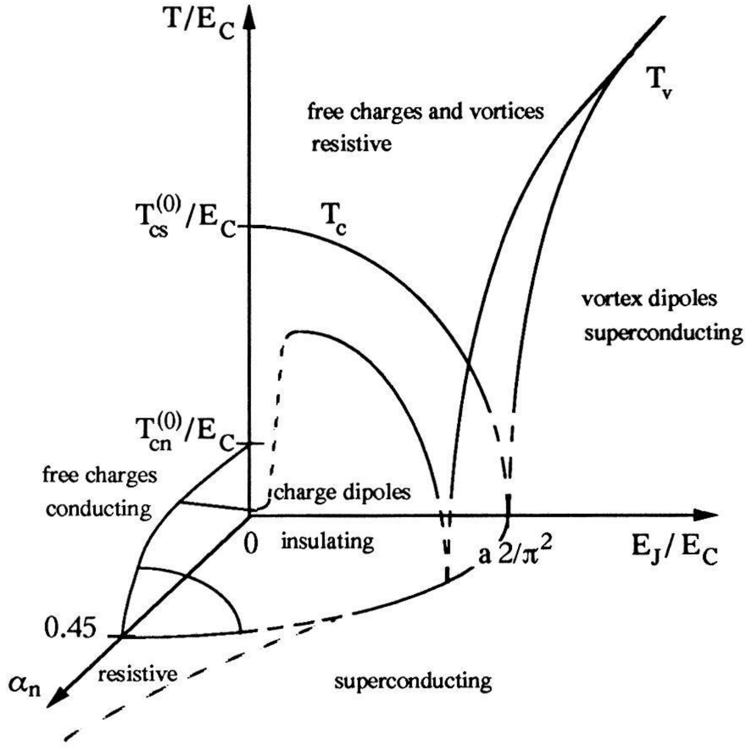

Figure 1: The zero-field phase diagram of a Josephson junction array as a function of E_J/E_C and the dimensionless normal state conductance $\alpha_n = h/(4e^2R_n)$. It is assumed that $C \gg C_0$.

At finite E_J and E_C both charges and vortices need to considered simultaneously. The charging energy provides a kinetic energy for the vortices, vice versa the Josephson coupling allows the tunneling of Cooper pairs and provides the dynamics for the charges. For definiteness we consider in the following the limit $C \gg C_0$, unless stated otherwise. If $E_C \ll E_J$ a perturbative approach shows that the transition temperature of the vortex-unbinding KTB transition is lowered below the classical value (10). Similarly, in the limit $E_J \ll E_C$ one can show [13] that the Cooper pair tunneling lowers the charge unbinding transition temperature below the value (11).

The question remains what happens for $E_J \approx E_C$. If the duality between charges and vortices would be perfect, i.e. if the duality breaking last term in (5) would be absent, the transition temperatures would be symmetric around the self-dual point

$$(E_J/E_C)_{self-dual} = 2/\pi^2. \tag{12}$$

Assuming that e.g. at $T = 0$ there exists only one transition (see below), we can immediately conclude [15] that the critical value of E_J/E_C, separating the charge- from the vortex-ordered phase, at $T = 0$ is given by (12). But the duality breaking term, even if it becomes irrelevant at the fixed point, can lead to a shift of the critical value

$$(E_J/E_C)_{cr} = 2a/\pi^2, \tag{13}$$

which differs from (12) by a factor a which is larger but of order one $a \geq 1$.

At T = 0 the system is effectively 3-dimensional and the character of the phase transitions changes. In this limit we can map the problem [13] onto a different model which had been investigated by Korshunov [20]. He concluded that the system has only one transition. Combining this information with the perturbative results we arrive at the picture shown qualitatively in Fig.1.

In Fig.1 we also show the influence of quasiparticle tunneling. Its strength is characterized by the dimensionless conductance $\alpha_n = R_q/R_n$ where R_n is the normal state resistance and $R_q = h/4e^2 = 6.45k\Omega$ the quantum resistance. See Ref. [13] for further details of the derivation. Here we only want to comment on the effect of weak tunneling: In the superconducting state at low temperature and as long as $E_C \ll \Delta$, where Δ is the superconducting gap, we have only virtual quasiparticle tunneling processes, which renormalize the junction capacitance by an amount proportional to the normal state conductance $1/R_n$ [21]

$$C \rightarrow C + 3\pi\hbar/(32\Delta R_n). \tag{14}$$

As a result the critical value of E_J/E_C is reduced below the (modified) self-dual value (13). On the other hand, in the limit $E_C \gg \Delta$, real single electron tunneling processes occur and, due to the lower activation energy, dominate over the Cooper pair tunneling. In this case a charge KTB transition involving single electrons occurs at the lower transition temperature $T_{cn}^{(0)}$.

At the self-dual point $E_J/E_C \approx 2/\pi^2$, at $T = 0$ the phase transition separates a superconducting from an insulating phase. The charges are driven by an applied voltage, and their motion produces a current. On the other hand, the vortices are driven by an applied current, and their motion produces a voltage. From the duality between charges and vortices at the superconductor-insulator transition one can conclude [16, 17] that the resistance of the array is given by the quantum resistance $R_q = h/4e^2 = 6.45k\Omega$. However this argument does not explain the origin of the dissipation.

$f \neq 0$ or $Q_x \neq 0$: The properties of junction arrays and their phase transitions are influenced by external magnetic fields and also by external charges. In the classical case the influence of the magnetic field has been studied extensively, and a complicated, periodic dependence on f, the flux per unit cell in units of the flux quantum, has been found (see for instance several articles in Ref. [1]). This is related to commensurabiltity properties of the vortex lattice and the underlying junction array. Also in the quantum case the phase diagram depends in a nontrivial way on f. This has been demonstrated in the limit where the self-capacitance dominates in Ref. [22]. In disordered lattices the commensurability plays no role. But also here the magnetic field leads to a 'field tuned transition'. Several scaling predictions of the theory [23] have been observed in disordered films [24]. Recently a transition with similar scaling properties, together with the expected flux periodicity has been observed in regular, fabricated junction arrays [25]. Further transitions occur near $f = 1/2$. For suitable junction parameters, such that the system is close to the superconductor-insulator transition, the critical value of f is small. In this limit the commensurability should not play an important role (a remaining weak positional disorder in the array makes it ineffective). On the other hand, the disorder is weak in these arrays and needs not to be considered explicitely. This allows us to describe the transition in a simple way. (For the transition near $f = 1/2$ a different approach would be needed.)

For the purpose of the present discussion we use the 'coarse graining' approach [26] for the Hamiltonian (1). The essence of this approach is to introduce an order parameter field ψ, whose expectation value is proportional to that of $\exp(i\phi)$. As long as ψ is small, i.e. close to the onset of phase coherence, the system is governed by an effective Ginzburg-Landau functional. Since the method has been discussed in the literature (see for instance Ref. [1]), we only quote the result

$$S[\psi] = \int d\tau \int d^2r \Big\{ -\epsilon|\psi(\vec{r},\tau)|^2 + \frac{1}{4}\Big|(\vec{\nabla} + i\frac{2e}{\hbar c}\vec{A}(\vec{r},\tau))\psi(\vec{r},\tau)\Big|^2 + c\Big|\frac{\partial\psi(\vec{r},\tau)}{\partial\tau}\Big|^2\Big\}. \tag{15}$$

We ignored higher order terms. The coefficients depend on the capacitance matrix [10]. If the juction capacitance dominates they are

$$\epsilon = 2E_J/E_C - 1 \quad ; \quad c \propto E_J/E_C^3.$$

In the limit considered we see that the effect of a magnetic field on the properties of the junction array is precisely the same as that of a field on the properties of a superconducting film. There exist an upper critical field (H_{c2} of the film), which marks a 'field-tuned' phase transition of the array as well. Its value is

$$f_{cr} = \frac{2}{\pi}\left[\frac{E_J}{E_C} - \left(\frac{E_J}{E_C}\right)_{cr}\right] \tag{16}$$

In a mean-field treatment at $T = 0$ the critical point is given by $(E_J/E_C)_{cr} = 0.5$. However, in the previous subsection we have given a better estimate of this value. If we compare the critical value of f with that of the experiments of [25] we find a rather good quantitative agreement, with values of $f_{cr} = 0.12$ or 0.2 for the two different arrays. We can also determine the critical exponent $f_{cr} \propto \epsilon^{2\nu}$. In a mean field treatment we find $\nu = 1/2$.

The coarse graining approach also yields the temperature dependence of the critical value of E_J/E_C. The charge fluctuations determine the correlation functions of ψ. Since they are supressed exponentially at low T their effect is weak, and $(E_J/E_C)_{cr}$ depends only weakly on T. In other words the critical temperature depends logarithmically on the distance from the critical point [8]

$$T_v \propto -1/\ln(\epsilon).$$

External charges can also be discussed in the coarse graining approach. Their effect is to modify the coefficients in (15) $\epsilon_{q_x} = \epsilon[1 + g_1(q_x)]$ and to add a term

$$\int d\tau \int d^2r g_2(q_x)\psi^*\partial_\tau\psi,$$

where $g_1 q_x$ and $g_2 q_x$ are even and odd functions of q_x, respectively. As a result the critical value $(E_J/E_C)_{cr}$ first decreases with increasing q_x, favouring the superconducting phase. For larger q_x we find a periodic dependence. In contrast, the frustration f shifts the critical value $(E_J/E_C)_{cr}$ to larger values, favouring the insulating phase. If $f = q_x$ duality implies that the transition is not shifted.

4. Experiments on Josephson junction arrays

In Fig. 2, we present the zero-field phase diagramm obtained experimentally for square arrays of all-aluminium Josephson tunnel junctions. Each square in this plot is obtained from measurements of one individual array (except the open and solid square at $E_J/E_C = 0.6$, which are from the same array, see below). The junction capacitances vary between $C = 1.1fF$ (small E_J/E_C) and $160fF$ (large E_J/E_C). We assume that the Josephson coupling energy E_J is given by the Ambegaokar-Baratoff relation, proportional to normal-state junction conductance $1/R_n$ and proportional to the measured critical temperature, which varies between 1.25 and 1.35 K in different samples. For the $1.1fF$ samples, R_n varies from 50 $k\Omega$ (open square at the left hand side) to 1 $k\Omega$. For the 160 fF junctions, R_n varies between 20.6 and 3.6 $k\Omega$. The samples with $1.1fF$ are 190 cells long and 60 cells wide, whereas the samples with $160fF$ junctions are 300 cells long and 100 wide.

The solid squares indicate a vortex-KTB transition. The transition temperatures were obtained from square-root cusp fits of the linear resistance measured above the transition. Below the transition temperature the I-V characteristica are nonlinear, showing a supercurrent before a voltage sets

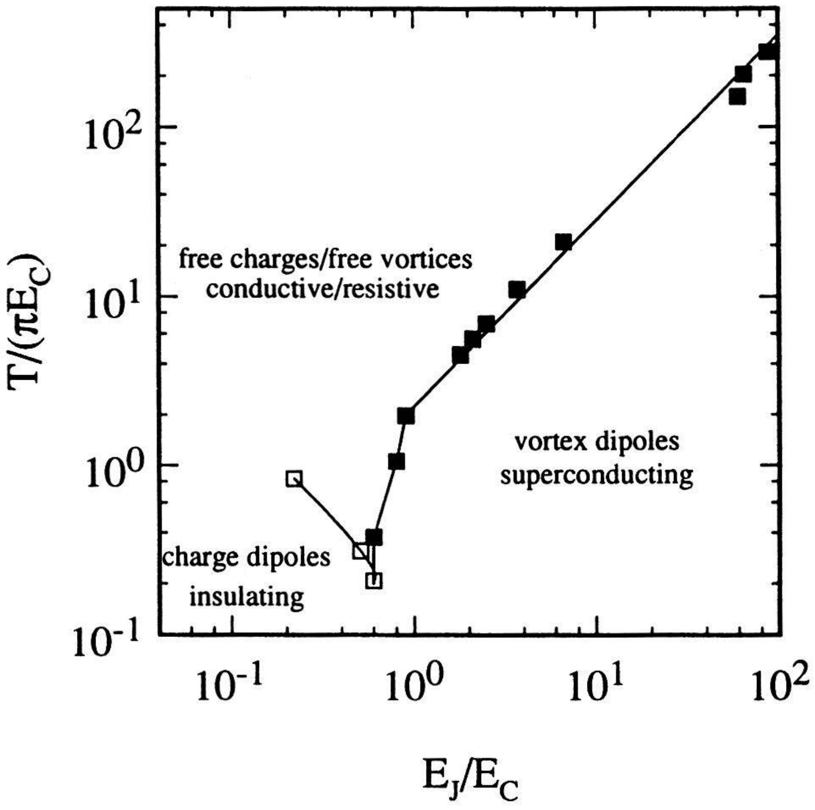

Figure 2: Phase diagram of fabricated Josepshon junction arrays. The solid squares marks a transition between superconducting and conducting, the open squares between insulating and conducting.

in. For large values of E_J/E_C the transition temperature is $T_v^{(0)} \propto E_J$, in good agreement with the theoretical predictions. Deviations are found close to the superconducting-insulating transition [27] in the range $0.6 < E_J/E_C < 1$. The reason is that in a finite system even below the KTB transition temperature single vortices can cross the sample thermally activated, leading to an exponentional decrease of the resistance. In the 'superconductiong' region the resistance is at least three orders of magnitude smaller than in the normal state.

For the two samples with E_J/E_C ratios smaller than 0.6 (open squares), the resistance always increases when lowering the temperature. The samples become insulating, showing Coulomb blockade of Cooper pair tunneling. For these samples we have defined the transition temperature as the point where the resistance has increased by three orders of magnitude compared to the normal state. The transition temperature is close to the value where it is predicted by the charge-KTB theory.

In our junction arrays the superconductor-insulator transition occurs at a value of $E_J/E_C \approx 0.6$. This has again the right order of magnitude, as expected from the charge-vortex duality. The array with $E_J/E_C = 0.6$ shows a remarkable temperature dependence. When cooling down, the resistance first decreases by more than two orders of magnitude. Around $T = 120mK$ the resistance starts to increase and a small Coulomb gap appears in the current-voltage characteristic. At $40mK$ the resistance has gone up by more than three orders of magnitude. Below $40mK$ it begins to decrease again, when the temperature is lowered further. The reentrant behavior at $120mK$ has been predicted by Zaikin [28], who attributes this to the finite size of arrays.

5. Vortex dynamics

The coupled-Coulomb-gas representation (5) displays the coupling between the charges and the vortices. In the extreme limits, $E_J = 0$ or $E_C = 0$, only one or the other needs to be considered. In the limit $E_J \gg E_C$, we still obtain an effective action for the vortices only, but they are influenced by the charging effects. In the considered limit the charges are fluctuating strongly and can be treated as continuous variable. Hence they can be integrated out from the partition function. We first ignore external fields $f = 0$ and offset charges $Q_x = 0$. In the limit where the junction capacitance dominates $C \gg C_0$ the result is

$$S[v] = \int_0^\beta d\tau \sum_{i,j} \Big[\frac{\pi}{8E_C}\dot{v}_i(\tau)G_{ij}\dot{v}_j(\tau) + \pi E_J v_i(\tau)G_{ij}v_j(\tau)\Big]. \tag{17}$$

The vorticity at each space-time point can take the values 0 or ± 1. It changes in discrete steps. In particular vortices can be created and annihilated in pairs. Under certain conditions, however, we need to consider only vortices which move in a continuous fashion. If we label the vortices by their centre coordinate $\vec{r}_n(\tau)$ and the sign of the vorticity $v_n = \pm 1$ the vortex density at site $\vec{r}_i$ can be represented as

$$v_i(\tau) = \sum_n v_n \delta[\vec{r}_i - \vec{r}_n(\tau)]. \tag{18}$$

Substituting this expression into (17) we obtain the effective action

$$S[r] = \int_0^\beta d\tau \Big\{\frac{1}{2}\sum_n \frac{\pi^2}{4E_C}\dot{r}_n^2 + \sum_{n,m} v_n v_m \pi E_J G[\vec{r}_n(\tau) - \vec{r}_m(\tau)]\Big\}, \tag{19}$$

which demonstrates that in the limit where the junction capacitance dominates the vortices can be viewed as particles with a mass

$$M_v^0 = \frac{\pi^2}{4E_C}\frac{\hbar^2}{p^2}. \tag{20}$$

In order to get a feeling for the magnitude we can compare the mass to that of an electron and the lattice spacing p to the Bohr radius a_0. For $E_C \approx 0.1K$ and a cell size of $5\mu m^2$ the mass of the vortex is smaller than the electron mass by a factor 0.004, and we can expect strong quantum mechnical effects.

The idea of a 'vortex mass' and the expression for the bare mass (20) have been developed before by Eckern and Schmid [6]. They assumed that the phase configuration of a classical vortex is not affected by the potential barriers as the vortex moves through the lattice. This is the case only if the effective potential barriers are small, which according to Lobb et al. [2] have a height $0.2E_J$. Hence the bare vortex mass can be found only if the kinetic energy scale exceeds the barrier $E_C \geq 0.2E_J$. On the other hand, one can account at least partially for the complexity of the vortex motion, by reintroducing a potential

$$S_{eff}[r] = S[r] + \int_0^\beta d\tau \sum_n V(\vec{r}_n). \tag{21}$$

Here $S[r]$ is given by (19) and $V(\vec{r}_n)$ is the periodic potential with a modulation amplitude $0.1E_J$ as found in Ref. [2]. Using the simple picture, a vortex with bare band mass (20) moving in a periodic potential yields in the tight binding limit $E_J \gg E_C$ the band mass

$$M_v \simeq \frac{1}{\sqrt{8E_J E_C}} \exp\Big[\sqrt{0.41 E_J/E_C}\Big]. \tag{22}$$

Alternatively, in the limit $E_J \gg E_C$ we can use the action (17) without further approximations. The instanton action corresponding to a vortex moving in a time step $\Delta\tau$ from one site j to the neighbouring one $j+1$ becomes [29, 19]

$$S_{inst} = -\frac{2\pi}{8E_C\Delta\tau}G(1) \simeq \sqrt{3.04E_J/E_C}. \tag{23}$$

There remains some ambiguity (factors of order one) in the numerical coefficients in (23) due to uncertainties in the precises value of the time step $\Delta\tau$ and of $G(1)$. The explicit result given above is chosen to coincide with a result of a direct instanton calculation, based on the original action in the ϕ-representation [5, 29]. From the instanton action we obtain the 'band mass' of a vortex which coincides with (22), except that the coefficient 0.41 is replaced by 3.04. This difference, which is larger than the ambiguity in the numerical coefficients demonstrates that the vortex is not moving as a rigid object but adjusts during its motion its internal degrees of freedom.

At finite temperatures the vortices can move thermally activated from one site to another with a rate which depends on the barrier height. At low temperatures this process is still possible due to quantum mechanical tunneling. The tunneling rate is given by the instanton action (23)

$$\Gamma_q \propto \exp[-S_{inst}], \tag{24}$$

and provides a measure for the vortex mass. If the vortices have a mass one can also expect them to move ballistically under suitable circumstances. This has recently been demonstrated in the experiments of van der Zant et al. [30].

A non-vanishing self-capacitance $C_0 \neq 0$ provides a damping mechanism for the vortex dynamics [6]. (In this case the duality breaking term in (5) becomes important.) The difference between the effect of a self-capacitance and a junction capacitance can be understood by comparing the dispersion relations of the spin waves in the two limits. If $C \gg C_0$ the spin waves have only an optical branch, whereas for $C_0 \neq 0$ they have an accustic branch, which allows the generation of low energy spin waves, providing a mechanism for dissipation. The difference beteen these properties can be traced further back to the property of the system under Galilei transformations. It is invariant in the former limit, whereas the self-capacitance provides a frame of reference [6].

6. Dissipation by quasiparticle tunneling

The vortex motion is limited by dissipation. It is, therefore, essential to include the dissipation in our description. In an ideal array the most important source of dissipation is the tunneling of quasiparticles. This can be described in a compact form by an effective action [21]

$$S_{eff}[\phi] = S[\phi] + \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{<i,j>} \alpha(\tau-\tau')\left\{1 - \cos\frac{\phi_{ij}(\tau)-\phi_{ij}(\tau')}{2}\right\}. \tag{25}$$

The first part $S[\phi]$ describes the charging and the Josephson coupling, the second part, describing the tunneling of quasiparticles, involves the kernel $\alpha(\tau)$. In the normal state it is given by

$$\alpha(\tau) = \alpha_n \frac{(1/\beta)^2}{\sin^2(\pi\tau/\beta)} \quad , \quad \alpha_n = \frac{R_q}{R_n} \tag{26}$$

where R_n is the normal state tunneling resistance. In the superconducting state $\alpha(\tau)$ depends on the superconducting gap. At $T = 0$ in ideal junctions it is short range, and for small frequencies the second term in (25) reduces to a renormalization of the nearest neighbour capacitance (14). A finite subgap conductance $1/R_{qp}$ can be accounted for by a kernel of the form (26), but α_n is replaced by $\alpha_{qp} = R_q/R_{qp}$.

Also for the more general model (25) we can proceed along the lines described above. We introduce the charges on the islands and express the phase configuration in terms of spin waves and vortices. The details of the derivation will be presented elsewhere [31]. We find again a coupled-Coulomb-gas description for the charges and the vortices; however, we have to sum over all the events where a single electron tunnels. In the limit $E_J \gg E_C$ we can eliminate again the charges and find an effective action for the vortices only. In leading order in the frequencies it is

$$S_{eff}[r] = S[r] + \int_0^\beta d\tau \sum_n V[\vec{r}_n(\tau)] - \int_0^\beta d\tau \int_0^\beta d\tau' \int d^2r \sum_\mu \alpha(\tau-\tau')$$
$$\times \cos\Big\{\frac{1}{2}\sum_n v_n \frac{\partial}{\partial r_\mu}[\Theta(\vec{r}-\vec{r}_n(\tau)) - \Theta(\vec{r}-\vec{r}_n(\tau'))]\Big\}. \quad (27)$$

The first term accounts for the kinetic energy and interaction of vortices and is given by (19). We added the periodic potential (as given e.g. by Lobb et al. [2]) in order to account for the fact that the vortex motion is not necessarily smooth (see section 4). The dissipative term involves the phase configuration $\Theta(\vec{r}-\vec{r}_n(\tau))$ at position $\vec{r}$ due to a vortex at site $\vec{r}_n(\tau)$. Assuming that the vortices move slowly we can expand this nonlinear term and obtain

$$S_{eff}[r] = S[r] + \int_0^\beta d\tau \sum_n V[\vec{r}_n(\tau)] - \frac{\pi}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \sum_{n,m} v_n v_m \alpha(\tau-\tau') G[\vec{r}_n(\tau) - \vec{r}_m(\tau')]. \quad (28)$$

This expression shows that each vortex, apart from the logarithmic interaction with the other vortices, has properties which coincides with that of a Fröhlich polaron in two dimensions [32, 6].

As is obvious from (25) the effect of dissipation on the phase dynamics of a Josephson junction is characterized by the dimensionless quasiparticle conductance $\alpha_n \equiv R_q/R_n$ (which in general depends strongly on the voltage and hence on the frequency scale). Similarly the last term in (28) describes the dissipation of the vortex motion. We can estimate its strength by expanding the function $G(r)$. Then the action of one vortex with trajectory $\vec{r}(\tau)$ becomes

$$S_{eff}[\vec{r}(\tau)] = \int_0^\beta d\tau \Big[\frac{M_v}{2}\dot{r}^2 + V(\vec{r})\Big] + \frac{\pi^2}{4}\int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau-\tau')[\vec{r}(\tau) - \vec{r}(\tau')]^2. \quad (29)$$

If we compare (29) with the expansion of (25), we realize that the vortex coordinate is influenced by dissipation in the same way as the phase of a single junction. However, the dissipation of the vortex motion (the effective α) is weaker by a factor 2 than that of the phase dynamics of a single junction. This result also been found by Orlando et al. [33] on a more phenomenological level. In this argumentation we have to keep in mind that the effective resistance of good quality Josephson junctions - and hence also of arrays of such junctions - is strongly voltage dependent. If the conductance of the junctions is finite (say $1/R_{qp}$) we can parameterize the vortex dissipation by the parameter $\alpha_{eff} = R_q/2R_{qp}$. In general α_{eff} depends strongly on the velocity of the vortices [6].

One consequence of the dissipation is the possibility of a phase transition of individual vortices, similar to the transition discussed by Schmid [34] for the phase of a single Josephson junction. This transition may occur in arrays at low temperatures where by a magnetic field excess vortices with a low density have been produced, such that their interaction can be ignored. The phase transition separates a localized and a delocalized phase. It occurs at a critical strength of the dissipation

$\alpha_{eff,c} = 1$. For strong dissipation $R_n < R_q/2$ the individual vortices are localized in the minima of the potential and the system is superconducting, whereas for $R_n > R_q/2$ the vortices are free and the system is resistive.

7. The Aharonov-Casher effect

The analogy between a vortex and a quantum mechanical particle goes even further. It has been suggested [35, 36] that charges act as a gauge field on vortices in the same way as a magnetic flux acts on a charged particle. This can be demonstrated from the coupled-Coulomb-gas description if we include the effect of the offset charges Q_x. These offset or external charges $Q_{x,i}$ on the islands are fixed by external constraints and do not fluctuate. They can arise for example by coupling islands by means of the capacitance C_0 to external voltage sources. This binds a part of the charge $Q_{x,i}$ on the island i at the capacitance C_0. If $C_0 \ll C$ the external charge remains approximately fixed even if the total charge on the island changes by tunneling. We can also imagine to couple some of the external islands of the array via high-Ohmic resistors to the external circuit. In this case the external charge changes due to the externally imposed current. Finally we mention that charged impurities, e.g. in the substrate under the array, can create random external charges.

The coupled-Coulomb-gas action (5) describes the effect of the external charges. In the limit $E_J \geq E_C$ we can again integrate out the charges. This yields (for $f = 0$)

$$S[v; q_x] = S[v; 0] + i \int_0^\beta d\tau \sum_{i,j} q_{x,i}(\tau)\Theta_{ij}\dot{v}_j(\tau) , \tag{30}$$

where the vortex action without external charge $S[v; 0]$ is given by (17). Again we introduce continuous vortex trajecories $\vec{r}_n(\tau)$. Then the effective action becomes

$$S[r; q_x] = \int_0^\beta d\tau \Big\{ \sum_n \Big[\frac{M_v}{2}\dot{r}_n^2 - iv_n \vec{A}_v(\vec{r}_n) \cdot \dot{\vec{r}}_n \Big] + \sum_{n,m} v_n v_m \pi E_J G(\vec{r}_n - \vec{r}_m) \Big\} . \tag{31}$$

Here $\vec{A}_v(\vec{r}_n) = \sum_i q_{x,i}\vec{a}_v(\vec{r}_n - \vec{r}_i)$ is a fictitious 'vector potential' seen by the moving vortex at the position $\vec{r}_n(\tau)$ which is created by the external charges $q_{x,i}$ at the sites $\vec{r}_i$. The 'vector potential' of one unit charge is

$$\vec{a}_v(\vec{r}) = \vec{\nabla}\Theta(\vec{r}) = \hat{z} \times \vec{r}/r^2. \tag{32}$$

In the action (31) we see that the charge creates a gauge field for a moving vortex in the same way as an ordinary vector potential influences a charged particle. Such an influence of a charge on a magnetic particle was first studied by Aharonov and Casher [35]. Later van Wees [36] pointed out that the same effect applies for a vortex in a Josephson array. By duality the charges can be seen as flux tubes and the vortices as particles with quantum mechanical properties which depend on the enclosed charge. At the sites of the charges there exists also a fictitious 'magnetic field' $\vec{B}_v = \vec{\nabla} \times \vec{A}_v$. This leads to a Magnus force on vortices which will be discussed in the next section.

We can study the consequences of the Aharonov-Casher effect by considering a ring-shaped array as shown in Fig. 3 [36, 29] . In this case we can fix the net number of vortices in the array by controlling the supercurrents in the thick inner and the outer ring-electrodes. We impose a phase gradient on the outer electrode and chose this phase as time independent. Then the value of the phase of the inner electrode $j = 1$ in the classical limit determines the azimuthal position of the vortex core. Thus the dependence on the band structure $E_n(\{Q_x\})$ on $Q_{x,1}$ reflects the quantum

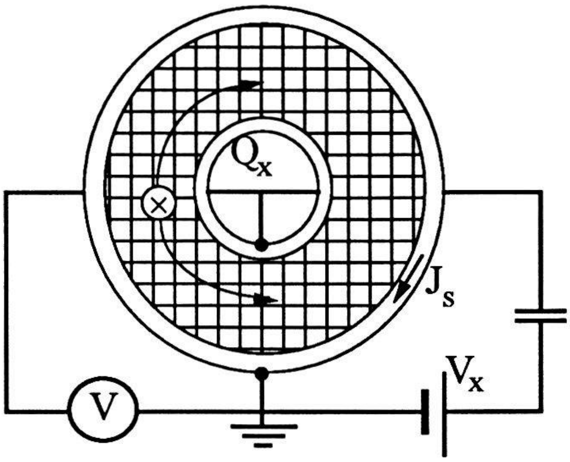

Figure 3: Geometry for the Aharonov-Casher effect. A vortex is enclosed between the thick outer and inner electrodes and moves around a charge Q_x induced on the inner electrode.

motion around the ring. The band mass is given by the curvature of the ground state energy $[\partial^2 E_0/\partial Q_x^2]^{-1}$ and (up to factors) is equal to the effective capacitance. By measuring the voltage difference V between the inner and outer electrode (in the ground state $V = \partial E(Q_{x,1})/\partial Q_{x,1}$) one can obtain information about the band structure.

8. Forces on vortices and the Hall angle

It is known that an imposed external current creates a force on the vortices. Whether vortices experience a Magnus force has remained a controversial question and appears to depend sensitively on the particular model [37]. We can derive these forces within our model. An external current is included by adding to the Hamiltonian (1) the term

$$\int_0^\beta d\tau \sum_i \frac{1}{2e} \vec{I}_i(\tau) \cdot \vec{\nabla}\phi_i(\tau). \tag{33}$$

We can proceed using the same transformations as before (for more details see Ref. [19]). Then we find a contribution to the action of the form

$$-\int_0^\beta d\tau \sum_{i,j} \frac{1}{2e} \vec{I}_i(\tau) \cdot \vec{\nabla}\Theta_{ij} v_j(\tau) \tag{34}$$

The resulting force due to a uniform current on a vortex with vorticity v_n is

$$\vec{F}_I = -\frac{2\pi}{2e} v_n \hat{z} \times \vec{I}. \tag{35}$$

It is perpendicular to the current. Since it involves a cross-product of a flux (the vorticity) and an electron velocity (the current) it has been denoted as Lorentz force. A vortex motion perpendicular to the current produces a voltage drop in the direction of the current.

A naive analysis of the action (31) leads to a fictitious 'Lorentz force' perpendicular to the vortex motion due to the fictitious 'magnetic field' $\vec{B}_v$, namely

$$\vec{F}_M(\vec{r}) = v_n \frac{Q_x(\vec{r})}{2e} \hat{z} \times \dot{\vec{r}}. \tag{36}$$

This force can be called a Magnus force.

Summarizing our results we find the following equation of motion for a vortex with vorticity v_n and coordinate $\vec{r}(\tau)$ (in units of the lattice spacing)

$$M_v\ddot{\vec{r}} + \alpha_{eff}\dot{\vec{r}} = -\frac{2\pi}{2e}v_n\hat{z}\times\vec{I} + \vec{F}_M(\vec{r}) - \nabla_{\vec{r}}V(\vec{r}) \,, \tag{37}$$

As a result of the Magnus force the vortex velocity gets a component parallel to the direction of the current. This in turn implies a Hall voltage. The ratio of the Hall voltage and the transport voltage defines a Hall angle.

$$\tan\theta = \frac{Q_x v_n}{2e\alpha_{eff}}$$

Notice that the sign of the Hall voltage depends on the charge profile. The result (36) coincides with that obtained recently by Fisher [38] for continuous films. In the lattice problem we find an additional periodicity. The properties of the system are invariant if we change the charge on an island by multiples of the Cooper pair charge. Hence also the Magnus force depends 2e-periodically on the local charges. This implies that it depends only on the offset charges Q_x and not on the charges created by the Cooper pair tunneling. In a classical array of junctions the effect of Q_x is negligible and the Hall angle should vanish.

A more careful analysis of our lattice model raises further questions. The external charges are sitting on the islands and also the field $B_v(\vec{r})$ is nonzero only on the islands. On the other hand, the vorticity is defined in the plaquettes between the islands. We, therefore, had concluded in the past [19] that the vortices experience no force due to this field. However, we have to be careful when using the approach presented here to study such short distance details, since in most of our expressions we concentrated on distances larger than a lattice spacing. In order to obtain a conclusive answer we repeated the analysis in the style of Eckern and Schmid [6], inserting the phase configuration around a vortex into the original action (1) in the presence of external charges. From this analysis one can conclude that the Magnus force is indeed given by (36) [39].

9. Charge dynamics

In junction arrays where the charging energy dominates the most important degrees of freedom are the charges on the islands. In the limit $E_C \gg E_J$ we can disregard the discrete nature of the vortices and integrate them out, in analogy to what we did with the charges in section 5. Here we also include the effect of electromagnetic fields. The result is an effective action for the charges

$$S[q] = \int_0^\beta d\tau \sum_{i,j}\Big[\frac{1}{2\pi E_J}\dot{q}_i(\tau)G_{ij}\dot{q}_j(\tau) + \frac{2E_C}{\pi}q_i(\tau)G_{ij}q_j(\tau) + if_i(\tau)\Theta_{ij}\dot{q}_j(\tau)\Big], \tag{38}$$

This result shows that the 'mass' of the charge is $M_q = 1/E_J$. It is the band mass of a particle moving in a lattice with the matrix element of strength E_J [15]. (Technically the kinetic energy term in (38) arises as a sum of two terms, one from the integration, the other from the duality breaking term explicit in (5).)

The third term in (38) describes the Aharonov-Bohm effect. It is dual to the last term in (30), i.e. the magnetic frustration influences charges in the same way as the 'charge frustration' influences the vortices. In contrast to the fictitious flux associated with charges (section 6) the real magnetic flux is not confined to flux tubes. Hence it leads to a force on the charges

$$\vec{F} = q\vec{E} + q\dot{\vec{r}}_q \times \vec{B}, \tag{39}$$

where $\dot{\vec{r}}_q$ is the velocity of a charge and $\vec{B}$ is the static field related to f. The electric field describes a force on charges due to an applied voltage along the array, dual to the force (35) on vortices excerted by an imposed current. The magnetic field produces a Lorentz force on the Cooper pair charges moving in a Josephson junction array which is similar as for ordinary free charges. Hence we expect a Hall effect for the Cooper pairs in these arrays. There exist a difference between the lattice problem considered here and free charges. The present system is periodic in the external field, with a periodicity corresponding to one flux quantum per unit cell. Hence also the Lorentz force must have this periodicity. This also implies that the charges do not experience a Lorentz force due to the vortices created in the array.

Quasiparticle tunneling again provides a source of dissipation. In view of the analogy to the dynamics of a single junction [12] we expect that the strength of the dissipation for the charge dynamics is governed by a parameter $\bar{\alpha} \approx R_n/R_q$, i.e. the inverse of the parameter describing the dissipation of the vortex motion.

The analogy of the action for the charges (38) and that for the vortices (19) suggests further physical effects. At this stage we can speculate for instance about 'ballistic charge motion' in arrays.

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