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PRECISION TESTS OF THE ELECTROWEAK THEORY AND BOUNDS ON NEW PHYSICS

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At present the attention of the high energy physics community is mainly focused on the on-going LEP experiments. Accordingly, in the following I will discuss LEP physics[1)] and its context with respect to the electroweak sector of the Standard Model.

1. STANDARD MODEL

1.1. Introduction

The main goal of LEP 1 is to perform precision tests of the standard electroweak theory[2)] at the Z peak. Theoretical predictions in the Standard Model for all relevant observables have been developed in detail[1)]. I refer the reader to my talks[3)] at the Stanford and Neutrino-90 Conferences for concise summaries and for many relevant discussions that I will not repeat here. One starts from the Standard Model Lagrangian and a conveniently chosen set of input parameters. The interesting quantities are computed in perturbation theory. The lowest-order formulae plus one-loop radiative corrections[4)], often improved by important renormalization group resummations, provide a sufficiently accurate approximation to match the precision of realistic experiments and to allow quite significant tests of the theory. For LEP physics, a self-imposing set of input parameters is given by $\alpha_s, \alpha, G_F, m_Z, m_f$ and m_H. Clearly the Fermi coupling $G_F = 1.166389(22) \times 10^{-5}$ GeV^{-2} is conceptually more complicated than $\alpha_{weak} = \frac{g_2^2}{4\pi}$ (which would more naturally accompany $\alpha = 1/137.036$ and

α_s) or $\sin^2\theta_W$ or m_W, but is preferred for practical reasons because it is known with all the desirable accuracy. Similarly, m_Z has now been measured at LEP with remarkable precision. This preliminary task of LEP in view of precision tests of the Standard Model has already been accomplished to a nearly final degree of accuracy.

The LEP results on m_Z, as summarized at the summer conferences[5], are reported in Table 1. The resulting relative precision is impressive: $\delta m_Z/m_Z = 3.4 \times 10^{-4}$.

Experiment	m_Z (GeV)
ALEPH	91.186 ± 0.013
DELPHI	91.188 ± 0.013
L3	91.161 ± 0.013
OPAL	91.174 ± 0.011
AVERAGE	91.177 ± 0.006 (Stat.) $\pm\ 0.030$ (LEP)
	$\simeq 91.177 \pm 0.031$

TABLE 1

Among the quark and lepton masses, m_f, the main unknown is the top quark mass. Our ignorance of m_t is at present a serious limitation for precise tests of the electroweak theory because the radiative corrections are relatively large for large m_t and depend quadratically on m_t [3,4]. This fact can be used to put stringent constraints on m_t from the existing electroweak measurements, in particular an upper bound on m_t, to be discussed in detail later. As for lower bounds on m_t the best results arise from the failure to observe the t quark at e^+e^- and hadron colliders. LEP and SLC lead[6] to a model-independent bound $m_t \gtrsim 45$ GeV. From CDF one learns[7] that $m_t \gtrsim 89$ GeV, provided that the t quark semi-leptonic branching ratio is as predicted by the Standard Model.

The Higgs mass m_H is largely unknown. One of the most impressive performances of LEP up to now has been the dwarfing[6] of all previous lower bounds on m_H. For the mass of the minimal Standard Model Higgs boson, OPAL was able to establish the lower limit $m_H \gtrsim 44$ GeV. Less stringent but comparable limits were also obtained by the other LEP experiments (ALEPH: $m_H \gtrsim 42$ GeV, L3: $m_H \gtrsim 41$ GeV, DELPHI: $m_H \gtrsim 41$ GeV).

For the two-doublet Higgs sector of the minimal supersymmetric extension of the Standard Model[8,9], the corresponding limit is: $m_H \gtrsim 33$ GeV. The upper limit on m_H is mainly from theoretical arguments of consistency and is not equally clear. It is well known that for $m_H \gtrsim 0.8$-1 TeV the Standard Model becomes affected by serious problems[9] (e.g., Landau singularities moving down to energies of order 1 TeV) and the perturbative framework is no more reliable (weak interactions become strong). For this reason, most computations of radiative corrections are given for $m_H < 1$ TeV. The sensitivity of the radiative corrections to variations of m_H in the range 40 GeV $< m_H <$ 1 TeV is not large. In a sense, this level of accuracy fixes the goal for precision tests of the Standard Model because the clarification of the symmetry breaking sector of the theory is the main target of present-day experiments.

Finally, for electroweak calculations involving hadrons, the value of the QCD coupling α_s must also be specified. The best value of α_s at the Z mass, obtained from experiments at energies lower than m_Z, is given by[10] $\alpha_s(m_Z) = 0.11 \pm 0.01$. The QCD corrections to processes involving quarks are typically of order $\frac{\alpha_s}{\pi}$. As a consequence the stated error on α_s leads to a few per mille relative uncertainty on the corresponding predictions.

1.2. Precision Tests of the Electroweak Theory

From the above discussion it is clear that the set of input parameters can be separated into two parts. On the one hand, $\alpha, G_F, m_Z, m_{f_{light}}$ are well known and the ambiguities associated with these quantities on the radiative corrections are quite small. We can add α_s to this class, in that, if it is true that the experimental error on α_s is relatively large, it only enters as a small correction to electroweak processes involving hadrons and is practically irrelevant for purely leptonic processes. On the other hand m_t and m_H are largely unknown. Thus, for each relevant observable, one can only express the prediction of the Standard Model as a function of m_t and m_H, obtained by using the best available calculations of radiative corrections, with $\alpha, \alpha_s, G_F, m_Z$ and $m_{f_{light}}$ fixed at their experimental values. By comparing this prediction with experiment one can check their mutual consistency and derive constraints on m_t and m_H.

Actually the sensitivity on m_H is so small that for all the measured quantities the ambiguity due to varying m_H in the range 40 GeV $< m_H <$ 1 TeV is far below the present experimental error, so that for practical purposes, at the present stage of accuracy, the relevant predictions can be plotted as functions of m_t in the form of a band of values determined by δm_H, δm_Z, ($\delta \alpha_s$) (see Figs. 1.-5.). Note that from this point of view $\sin^2\theta_W$ is not a

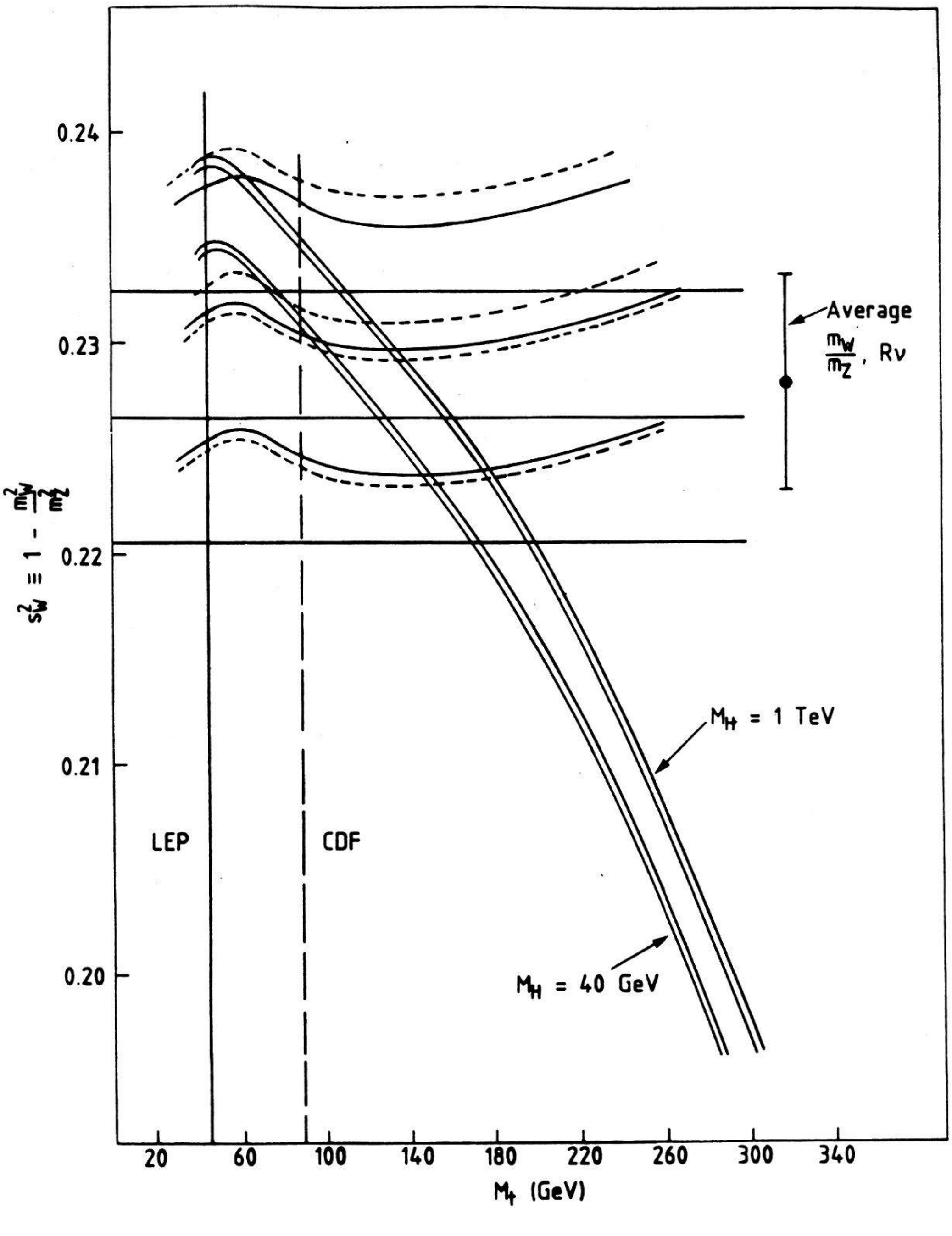


FIGURE 1

1. The value of $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, computed for m_Z given by the LEP average in Table 1 and m_H = 40-1000 GeV (the central band is determined by δm_H, while the two narrow external bounds arise from adding δm_Z linearly), compared with s_W^2 measured by CDF and UA2 (the horizontal band) and with the data on R_ν (the solid band refers to m_H = 100 GeV, while the dashed lines define the extended range according to δm_H). The combined value of s_W^2 which follows from CDF/UA2 and R_ν is also shown, together with the lower bounds on m_t from LEP (model independent) and CDF. (I am indebted with G.L. Fogli for providing the R_ν curves.)

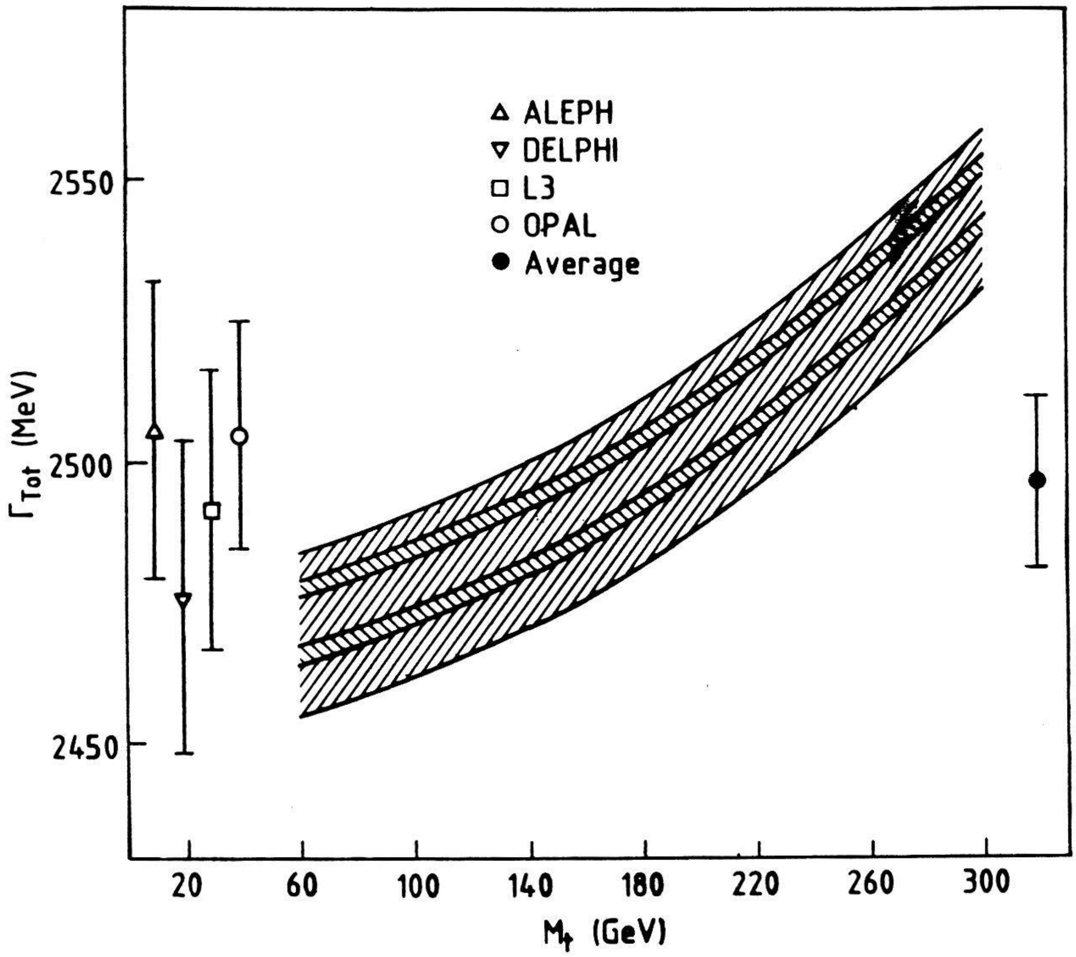


FIGURE 2

2. The prediction of the Standard Model for the total Z width Γ_T (obtained for $m_Z =$ 91.177± 0.031 GeV, $m_H =$ 40-1000 GeV, $\alpha_s = 0.12^{+0.01}_{-0.02}$) as a function of m_t is compared with the LEP results. The central band is from δm_H. The two narrow intermediate bands are from δm_Z. The external bands are from $\delta\alpha_s$. All uncertainties are added linearly.

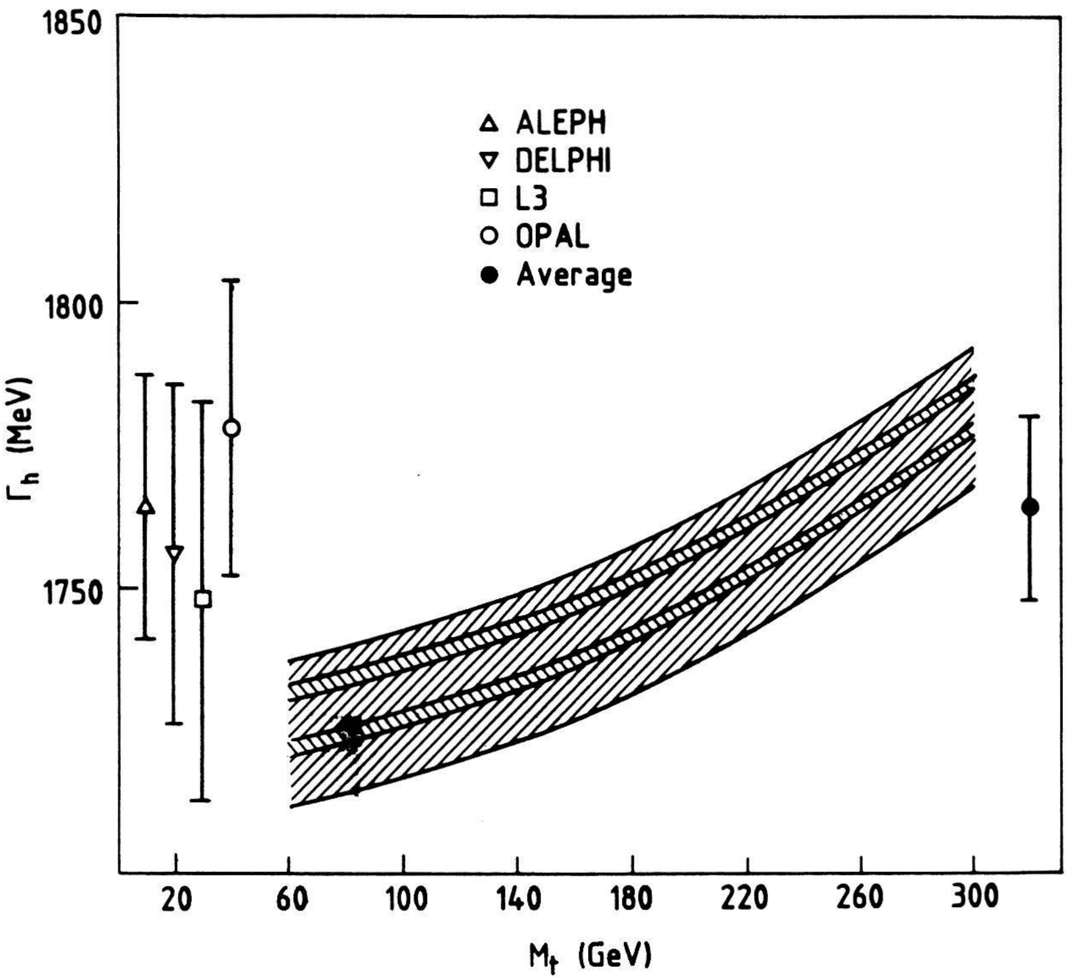


FIGURE 3

3. The hadronic width Γ_h (see Fig. 2 for a detailed explanation).

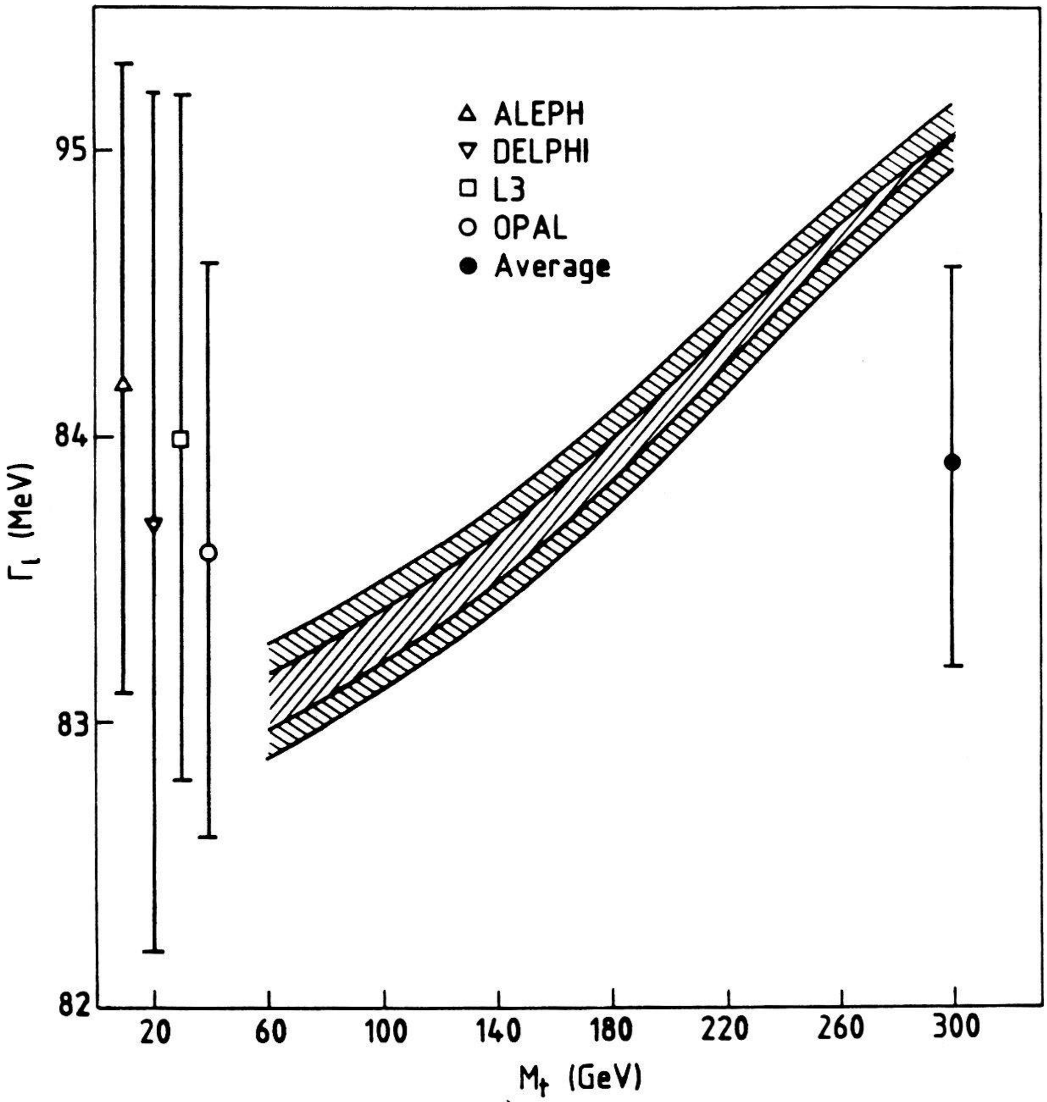


FIGURE 4

4. The leptonic width Γ_e (see Fig. 2 for a detailed explanation. Clearly Γ_ℓ is independent of α_s).

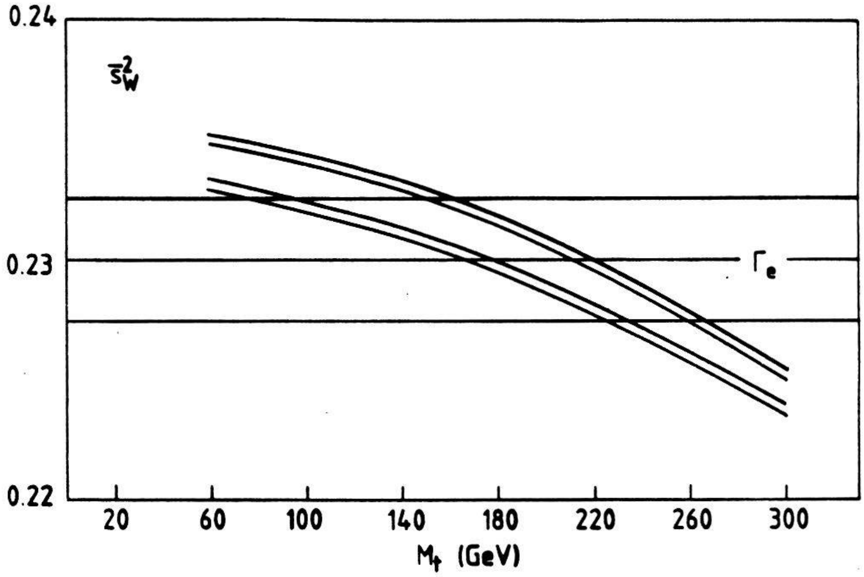


FIGURE 5

5. The effective $sin^2\theta_W$ for on-shell Z decays $\bar{s}^2_W$ predicted from $m_Z = 91.177 \pm 0.031$ GeV, $m_H =$ 40-1000 GeV[41], is compared with the experimental value obtained from Γ_ℓ.

primary quantity. It is not part of the set of input parameters. It is a derived quantity that one could even decide not to introduce at all. I stress this point in order to make clear that all disputes over which is the better definition of $\sin^2\theta_W$ beyond the tree level are completely secondary. Not only it is always true that physical results are independent of definitions. Differences in physical results obtained from a different definition of input parameters (scheme dependence) can at most occur by terms of higher order, due to the truncation of the perturbative series at a given order. But for $\sin^2\theta_W$ its precise definition is only necessary to compute it from the input parameters, but cannot matter for the prediction of observables because, with the choice specified above, $\sin^2\theta_W$ is not taken as an input parameter of the theory.

However, it certainly remains true that $\sin^2\theta_W$ is an important observable of the electroweak theory and a useful reference quantity. The results of different experiments are often compared in terms of the values and the accuracies for $\sin^2\theta_W$ that they correspond to. More important than that, with appropriate definitions of $\sin^2\theta_W$, one can write simple improved Born approximations that include the main contributions of radiative corrections (e.g., large logarithms and terms of order $G_F m_t^2$). While for precision tests the use of as complete as possible radiative corrections is mandatory, these approximate formulae are very useful for our understanding of the pattern of radiative corrections and for every-day-life estimates of rates and experimental sensitivities.

One common definition[11] of $\sin^2\theta_W$ is

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \equiv s_W^2 \tag{1}$$

to all orders in the electroweak couplings. Clearly in this case the observables s_W^2 and m_W are directly equivalent given that m_Z is among the input parameters. In the Standard Model, s_W^2 can be computed from the input parameters by the relation

$$s_W^2 c_W^2 \equiv \left(1 - \frac{m_W^2}{m_Z^2}\right)\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F}\,\frac{1}{m_Z^2}\,\frac{1}{1-\Delta r} \tag{2}$$

where $c_W^2 = 1 - s_W^2$ and $\Delta r \equiv \Delta r(\alpha, \alpha_s, G_F, m_Z, m_f, m_H)$ is the effect of radiative corrections. The quantity Δr as a function of the input parameters has been studied in great detail[12]. The result for $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, obtained starting from the average LEP value for m_Z (see Table 1), as a function of m_t, is plotted in Fig. 1, where the uncertainties for 40 GeV

$< m_H < 1$ TeV and $\delta m_Z = \pm$ 31 MeV are also visible. We see that m_t is the main unknown in the calculation of $\frac{m_W}{m_Z}$ from m_Z, followed in importance by the ambiguity from varying the Higgs mass in the above range, while the remaining uncertainty from the experimental error on m_Z is very small.

When the available direct experimental information on $\frac{m_W}{m_Z}$ is added, the sensitivity of s_W^2 to m_t provides the best constraint that we have on m_t. $\frac{m_W}{m_Z}$ is directly measured at hadron colliders and can also be obtained (assuming the validity of the Standard Model) from the ratio $R_\nu = \sigma^{NC}/\sigma^{CC}$ of neutral current (NC) to charged current (CC) cross-sections in neutrino-nucleus deep inelastic scattering.

The value of $\frac{m_W}{m_Z}$ has been measured at hadron colliders[13]. From CDF and UA2 we have the results reported in Table 2.

Experiment	$\frac{m_W}{m_Z}$	$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$
CDF	0.8768 ± 0.0046	0.231 ± 0.008
UA2	0.8831 ± 0.0055	0.220 ± 0.010
AVERAGE	0.8794 ± 0.0035	0.2265 ± 0.006

TABLE 2

By combining $\frac{m_W}{m_Z}$ with the LEP value for m_Z one obtains $m_W = 80.19 \pm 0.32$. The corresponding average value of s_W^2 is also shown in Fig. 1 as horizontal band, obviously independent of m_t, in the $s_W^2 - m_t$ plane.

As is well known, the value of s_W^2 extracted from R_ν is also nearly independent of m_t in the interesting range of values for the top mass. This fact arises from a largely accidental cancellation[14], specific to this process and to the Standard Model, between two different sources of m_t dependence, as discussed in the following.

In general, at tree level, the four-fermion interaction from Z exchange is given by

$$M_{if} = \frac{\sqrt{2} G_F m_Z^2}{D(s)} \rho_{tree} \qquad (3)$$
$$(J_3^i - 2\sin^2\theta_W \; J_{em}^i) \cdot (J_3^f - 2\sin^2\theta_W \; J_{em}^f)$$

where $D(s)$ is the Z propagator and $J_3^{i,f}, J_{em}^{i,f}$ are the weak isospin and electromagnetic currents for the fermion i or f. Excluding pure QED corrections, electroweak radiative corrections[4] modify M_{if} according to

$$M_{if} = \frac{\sqrt{2} G_F m_Z^2}{D(s)} \rho_{i,f} \qquad (4)$$
$$(J_3^i - 2k_i \sin^2\theta_W \ J_{em}^i) \cdot (J_3^f - 2k_f \sin^2\theta_W \ J_{em}^f) + \dots$$

where $\rho_{i,f} = \rho_{tree}(1 + \delta\rho_{i,f}), k_a = 1 + \delta k_a \ (a = i, f)$ are different for different fermions and depend on the scheme adopted (for example δk_i depend on the definition of $\sin^2\theta_W$). The ellipses indicate possible additional non-factorizable terms (for example from box diagrams). Let us call "large" radiative corrections those terms containing large logarithms, i.e., $\frac{\alpha}{\pi} \ln \frac{m_Z^2}{m_{f_{light}}^2}$ or quadratic dependences on m_t, i.e., $\sim G_F m_t^2$. For large enough m_t, the bulk of the contribution of electroweak radiative corrections arises from these terms[3,4]. The "large" contributions to $\delta\rho_{i,f}$ and δk_f in Eqs. (4) are universal, i.e., they are the same at fixed q^2 for all i and f (except for b quarks). If for $\sin^2\theta_W$ one adopts the definition $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$ one obtains[2,4]:

$$1 - \Delta r = (1 - \Delta\alpha)(1 + \frac{c_W^2}{s_W^2}\delta\rho + \text{“small”}) \qquad (5)$$

$$\rho \cong 1 + \delta\rho = 1 + \frac{3 G_F m_t^2}{8\pi^2\sqrt{2}} + \text{“small”} \qquad (6)$$

$$k \cong 1 + \delta k = 1 + \frac{c_W^2}{s_W^2}\delta\rho + \text{“small”} \qquad (7)$$

(for b quarks there are additional large terms) where Δr is defined in Eq. (2) and $\Delta\alpha$ arises from the running of the QED coupling:

$$\alpha(m_Z) = \frac{\alpha}{1 - \Delta\alpha} \qquad (8)$$

$\Delta\alpha$ is dominated by large logs and its value is given by[4]

$$\Delta\alpha \simeq 0.0601 \pm 0.0009 \qquad (9)$$

(or $\alpha^{-1}/1.3) \simeq 128.8 \pm 0.1$). Note that both δk and Δr contain the large term $\delta\rho$ enhanced by the factor $\frac{c_W^2}{s_W^2}$. Logarithmic scale violations of order $\frac{\alpha}{\pi} \ln q^2/m_Z^2$ are included in the "small" terms (which is only appropriate for $q^2 \gg m_{f_{light}}^2$).

The ratio $R_\nu = \frac{\sigma^{NC}}{\sigma^{CC}}$ for $\nu - N$ scattering is given in terms of s_W^2 by:

$$R_\nu = \rho_{\nu N}^2(\frac{1}{2} - k_{\nu N}s_W^2 + \frac{5}{9}(k_{\nu N}s_W^2)^2(1+r)) + \dots \tag{10}$$

where $r = (\sigma^{\bar{\nu}}/\sigma^{\nu})_{CC}$ is also measured. The tree approximation (with $\rho_{tree} = 1$) is recovered for $\rho_{\nu N} = k_{\nu N} = 1$. Some large logarithms from the radiative corrections to σ^{CC} are also included in $\rho_{\nu N}$. But for the sake of this argument we are only considering the $G_F m_t^2$ terms. For fixed $R_\nu \approx$ (the experimental value) and $s_W^2 \sim 0.23$ there is a strong cancellation in the Standard Model between the m_t dependence of $\rho_{\nu N} \simeq 1 + \delta\rho$ and of $k_{\nu N} \simeq 1 + \frac{c_W^2}{s_W^2}\delta\rho$, so that as a result $\delta s_W^2 \simeq 0.2\delta\rho$, where $\delta\rho$ is given in Eq. (6). For realistic values of m_t the resulting contribution of the quadratic m_t terms is no more dominant.

The most precise experimental results on R_ν were obtained by the CHARM[15] and CDHS[16] collaborations at CERN. The original results on $\sin_W^2$ were given for fixed m_t and m_H. CHARM obtained $s_W^2 = 0.236 \pm 0.005$ (exp) $\pm$ 0.005 (th) for m_t = 45 GeV and m_H = 100 GeV, while the CDHS result was $s_W^2 = 0.2275 \pm$ 0.005 (exp) $\pm$ 0.005 (th) for m_t = 60 GeV and m_H = 100 GeV. The theoretical error arises from hadronic uncertainties and the effect of the charm threshold. An average at m_t = 60 GeV and m_H = 100 GeV gives $s_W^2 = 0.232 \pm 0.006$ (where the error 6×10^{-3} is obtained as $6 \times 10^{-3} = \sqrt{(\frac{5}{\sqrt{2}})^2 + 5^2} \times 10^{-3}$). The corresponding combined result at different values of m_t and m_H can also be obtained from the known form of the radiative corrections. The result is shown[17] in Fig. 1.

There are many more less precise experimental results on s_W^2 from neutral current data most of them being well known[3,18,19]. The new CHARM II result[20] on $\overset{(-)}{\nu}_\mu e$ scattering will be discussed in Section 1.4. These additional data are all consistent among them and with the data in Fig. 1. But the resulting values of m_t and s_W^2 are essentially determined by the data in Fig. 1. From those data I obtain the results:

$$m_t = 140 \pm 45 \text{ GeV} \tag{11}$$

$$s_W^2 = 0.228 \pm 0.005 \tag{12}$$

These values are in agreement with other good analyses[19,21–23] of the data on the electroweak theory. The quoted errors in Eqs. (11), (12) include **all** mentioned experimental and theoretical errors (on which I tend to be more conservative than others) and the effect of varying m_H in the whole range 40 GeV $< m_H <$ 1 TeV.

Given m_Z and s_W^2 from Eq. (12) (which also includes the information from R_ν) one immediately derives the corresponding value of m_W:

$$m_W = 80.1 \pm 0.3 \text{ GeV} \tag{13}$$

We shall see later that the LEP measurement of the Z partial widths adds little to the limits on m_t (so far at least). But LEP gives an important element for determining m_t by fixing m_Z. From Eq. (11) one obtains $m_t \lesssim 200$ GeV (90 % c.ℓ.). The upper limit on m_t is only slowly moving with time. A few years ago when m_W was better known than m_Z the data on $\nu - N$ combined with m_W favoured relatively large values of s_W^2. Now that m_Z is precisely measured the upper limit on m_t has not improved by much because the recent data on $\frac{m_W}{m_Z}$ from hadron colliders[13] favour smaller values of s_W^2.

In the minimal standard model, Δr and ρ are computable (given m_t and m_H). More in general by using Eq. (2) as a general definition of Δr one can obtain Δr from the data on m_Z and $\frac{m_W}{m_Z}$. From Δr one can then derive a value for $\delta\rho$ from Eq. (5). By using the data on $\frac{m_W^2}{m_Z^2}$ from both CDF/UA2 (Table 2) and $\nu - N$ scattering, one finds

$$\Delta r = 0.050 \pm 0.015 \tag{14}$$

From the approximate relation (Eq. (5))

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\delta\rho \tag{15}$$

the previous result corresponds to

$$\delta\rho = 0.0030 \pm 0.0045 \tag{16}$$

Note that the derivation of $\delta\rho$ from an approximate relation (obtained by neglecting "small" terms) is adequate because a universal $\delta\rho$ (i.e., process independent) is only appropriate when "small" terms are neglected. It is perhaps safer to take $\frac{m_W^2}{m_Z^2}$ from only CDF/UA2. In fact the indirect extraction of $\frac{m_W^2}{m_W^2}$ from $\nu - N$ data could be modified by some new physics, for example a new heavy Z' contribution. This more model-independent derivation of Δr leads to the results:

$$\Delta r = 0.045 \pm 0.018 \tag{17}$$

$$\Delta \rho = 0.0044 \pm 0.0056 \tag{18}$$

Note that the corresponding limit on $\delta\rho$:

$$\delta\rho \lesssim 0.014 \qquad (95\% c.l.) \tag{19}$$

is a powerful constraint on all forms of non-standard physics which keep the relation (15) between Δr and $\delta\rho$:

$$\begin{aligned} \delta\rho = {} & \delta\rho_{standard} + \delta\rho_{heavy\ loops} + \delta\rho_{non\ doublet\ Higgs} \\ & + \delta\rho_{Z'} + \ldots \end{aligned} \tag{20}$$

In particular one can address the question: how solid is the limit $m_t \lesssim 200$ GeV? I think it is quite reliable. It is true that one assumes no large cancellations of the m_t term in $\delta\rho$ with some other new physics contribution. But it is also true that $\delta\rho_{heavy\ loops}$ tends to be positive in all quantitative enough models, for example in the minimal supersymmetric extension of the Standard Model[24)] (even if we only include the effect of the two Higgs doublets and no contribution from s-particles). Similarly $\delta\rho_{Z'} > 0$ in models with an extra $U(1)$[25–27)] (see Section 1.5.). $\delta\rho$ from heavy gauge bosons can only be negative if there are extra charged W' with sufficiently low mass[28)] (e.g., $m_{W'} < m_{Z'}$). But at low masses W' are more unlikely than Z'. For example, $m_{W'} \gtrsim 2$ TeV in left-right models[29)] (with equal or complex conjugate CKM mixing matrix for left- and right-handed quarks). $\delta\rho_{non-doublet\ Higgs}$ could in fact be negative already at tree level. Note that we have always assumed $\rho_{tree} = 1$, so that possible deviations from this relation are included in $\delta\rho$. The general form of ρ_{tree} is given by[30)]:

$$\rho_{tree} = \frac{\sum_i v_i^2 [I_i(I_i+1) - I_{3i}^2]}{\sum_i v_i^2 2 I_{3i}^2} \tag{21}$$

where v_i, I_i and I_{3i} are the vacuum expectation value, the total weak isospin and its third component for the Higgs multiplet i. For a doublet plus an additional non-doublet multiplet

X one obtains:

$$\rho_{tree} = \frac{1 + \frac{v_x^2}{v_{1/2}^2} 2[I_x(I_x+1) - I_{3x}^2]}{1 + \frac{v_x^2}{v_{1/2}^2}\, 4\, I_{3x}^2} \tag{22}$$

We see that in order to obtain $\rho_{tree} < 1$ one needs I_{3x} to be large. This in turn implies charged Higgses with charge two or more. For example, for triplet Higgses, I_{3x} must be ± 1. But recall that this is the I_3 of the neutral Higgs. By displacing I_{3x} by two units one then obtains the weak isospin of a doubly charged Higgs. In conclusion, $\delta\rho$ non-doublet Higgs can in principle be negative, but this possibility is actually associated with a somewhat baroque Higgs sector not really plausible. More in general the possibility of conspicuously evading the m_t upper limit by a cancellation of terms in $\delta\rho$, while in principle not excluded, is in practice difficult to implement.

Also note that in deriving the limit on m_t one always assumes $m_H < 1$ TeV. Formally if m_H is increased the upper limit on m_t is also increased. For $m_H \sim$ several TeV the perturbative expansion for $\delta\rho$ breaks down[31] and in principle the radiative corrections become uncalculable. However, it is difficult to imagine that the m_t limit can be sizeably modified by this effect without at the same time observing other conspicuous deviations from the perturbative predictions.

1.3. The Z Line Shape

We now consider the implications for the standard electroweak theory of the LEP results on the Z partial widths. The relevant results are collected in Table 3[5].

In Figs. 2-4, we compare the data on the Z widths with the predictions of the Standard Model, obtained by the programme ZSHAPE[32] which includes a state of the art set of electroweak radiative corrections. Totally equivalent predictions are obtained by other complete calculations of the line shape[33,34]. The predicted widths are plotted as functions of m_t. In all figures the uncertainties due to the errors on m_Z ($m_Z = 91.177 \pm 0.031$ GeV), on m_H ($m_H =$ 40-1000 GeV) and α_s ($\alpha_s = 0.12^{+0.01}_{-0.02}$) are linearly summed. At the centre, the Higgs uncertainty for $m_Z = 91.177$ GeV and $\alpha_s = 0.12$ is shown. Then the effect of varying m_Z by $\pm 1\sigma$ is linearly added to enlarge the previous band and finally the same is done for α_s. Note that the range adopted here for α_s is different than the best value from all low energy experiments ($\alpha_s = 0.11 \pm 0.01$). The choice of α_s in Figs. 2-4 is more conservative and more or less corresponds[10] to only taking PEP, PETRA, TRISTAN experiments into account (thereby comparing e^+e^-

	ALEPH	DELPHI	L3	OPAL	Average
Γ_Z MeV	2506. ± 26	2476. ± 28	2492. ± 25	2505. ± 20	2497. ± 15
Γ_ℓ MeV	84.2 ± 1.1	83.7 ± 1.5	84.0 ± 1.2	83.6 ± 1.0	83.9 ± 0.7
Γ_{had} MeV	1764. ± 23.	1756. ± 30.	1748. ± 35.	1778. ± 26.	1764. ± 16.
Γ_{inv} MeV	489. ± 22.	469. ± 29.	494. ± 32.	476. ± 25.	482. ± 16.
$R = \frac{\Gamma_{had}}{\Gamma_\ell}$	20.95 ± 0.31	21.00 ± 0.48	21.02 ± 0.62	21.26 ± 0.32	21.08 ± 0.20
σ^0_{had} (nb)	41.78 ± 0.63	42.38 ± 1.02	41.38 ± 0.71	41.88 ± 0.74	41.78 ± 0.53
Γ_e MeV	84.9 ± 1.4	82.0 ± 1.9	84.3 ± 1.6	82.7 ± 1.3	83.6 ± 0.9
Γ_μ MeV	80.7 ± 2.2	87.2 ± 3.5	82.3 ± 2.9	85.9 ± 2.0	83.8 ± 1.2
Γ_{tau} MeV	81.8 ± 2.2	86.0 ± 4.1	83.5 ± 3.7	83.9 ± 2.3	83.3 ± 1.4

TABLE 3

Results from LEP. The average also includes systematic errors as given by E. Fernandez[5]. The average value of Γ_{inv} corresponds to $N_\nu = 2.89 \pm 0.11$ which is the best current determination of the number of light neutrinos from LEP.

at LEP with e^+e^- at lower energy). Figures 2-4 contain all the information on the relation of the experimental values for the widths with the Standard Model predictions. Each width is predicted as a function of m_t given the input parameters $\alpha, G_F, m_Z, m_H, m_{f_{light}}$ with their present error, by using a full-fledged set of radiative corrections. Two main conclusions are immediately derived. First, the observed widths are in perfect agreement with the Standard Model for m_t in the range indicated by previous experiments. Second, the additional information on m_t provided by the widths, does not very much improve the upper limit on m_t (the difference being of a few GeV)[35]. The precision and sensitivity would be adequate but the central values are somewhat displaced toward the large m_t side.

For processes at the Z mass one can define an effective value of $\sin^2\theta_W$ that makes improved Born approximations[36,37] particularly simple. If $\sin^2\theta_W$ is defined as $\sin^2\theta_W \equiv s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, then an approximation for the Z widths that takes all "large" terms into account, can be written down in the following form ($f \neq b$):

$$\Gamma(Z \to f\bar{f}) = N_c \frac{G_F m_Z^3 (1+\delta\rho)}{24\pi\sqrt{2}} \left[1 + (1 - 4|Q_f|(1+\delta k)s_W^2)^2\right] \tag{23}$$

where

$$\begin{aligned} N_c =& 1 \quad \text{leptons} \\ =& 3\ [1 + \frac{\alpha_s(m_Z)}{\pi} + \ldots] \quad \text{quarks} \end{aligned} \tag{24}$$

and $\delta\rho$ and δk, given by Eqs. (6), (7), contain all "large" terms. We mentioned that the combination $(1+\delta k)\ s_W^2$ will always appear in all neutral current processes when only "large" terms are included (and logarithmic scale violations are neglected). One is then naturally led to redefine $\sin^2\theta_W$ in the following way[4]:

$$\begin{aligned} \bar{s}_W^2 &\cong (1+\delta k)\ s_W^2 \\ &\cong s_W^2 + c_W^2 \delta\rho + \text{"}small\text{"} \end{aligned} \tag{25}$$

Note that this relation is equivalent to

$$\bar{s}_W^2 \cong 1 - \frac{m_W^2}{\rho m_Z^2} \tag{26}$$

to first order in $\delta\rho$ with $\rho = 1 + \delta\rho +$ "small". Similarly we can go back to Eq. (2) and find how Δr is modified in the present case. We easily obtain the relations:

$$\bar{s}^2\bar{c}^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F}\frac{1}{m_Z^2\rho} + \text{"}small\text{"} \tag{27}$$

or

$$\bar{s}^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F}\frac{1}{m_W^2} + \text{"small"} \tag{28}$$

To within "small" corrections a whole class of $\sin^2\theta_W$ coincides with $\bar{s}_W^2$: $\sin^2\theta_{\overline{MS}}(m_Z^2)$[38] (computations of $\sin^2\theta_W$ from grand-unified theories usually end up with a prediction for this quantity[18,19]), $\sin^2\theta^*(m_Z^2)$[37], $\sin^2\theta_{on-shell}$ [4] and so on. They are all equivalent for the present purposes in the sense that they lead to the same improved Born approximations valid when "small" terms are neglected.

In terms of $\bar{s}_W^2$, the improved Born approximation for the width can be written in the form

$$\Gamma(Z \to f\bar{f}) = N_c \frac{G_F m_Z^3 \rho}{24\pi\sqrt{2}}(1 + (1 - 4|Q_f|\bar{s}_W^2)^2) \tag{29}$$

For $\Gamma(Z \to b\bar{b})$ replace[4,39,40] ρ by $\rho_b = \rho(1 - \frac{4}{3}\delta\rho)$ and $\bar{s}_W^2$ by $\bar{s}_W^2(1 + \frac{2}{3}\delta\rho)$. Note that by using Eq. (27) one can cast the previous formula into the form

$$\Gamma(Z \to f\bar{f}) = N_c \frac{\alpha_s(m_Z)\,\mathrm{m}_Z}{48\bar{s}_W^2\bar{c}_W^2}\left[1 + (1 - 4|Q_f|\bar{s}_W^2)^2\right] \tag{30}$$

This relation, valid up to "small" terms in the Standard Model, is less general than the previous one, where the effects of ρ and $\bar{s}_W^2$ are kept separate. (In general, beyond the Standard Model, $\Delta r, \delta\rho$ and δk should be taken as independent parameters - see Section 2.1.) Equation (30) is interesting because it shows that a value of $\bar{s}_W^2$ can be directly derived from the measured widths independent of m_t. This does not of course mean that the predictions for the widths do not depend on m_t. The dependence on m_t is hidden in $\bar{s}_W^2$ when computed from the input parameters. In fact, it is practical for LEP experiments to define $\bar{s}_W^2$ from a given simple Z process (for example $\Gamma(Z \to \ell^+\ell^-)$ with $\ell = e, \mu, \tau$) as given by Eq. (30), taken as exact (with $\alpha_s(m_Z)^{-1} = 128.8$). From the LEP average $\Gamma_{\ell\ell} = 83.9 \pm 0.7$ MeV one obtains

$$\bar{s}_W^2 = 0.230 \pm 0.0025 \tag{31}$$

This value is to be compared (apart from "small" terms) with the result

$$\bar{s}_W^2 = 0.232 \pm 0.002 \tag{32}$$

which is obtained[41] (Fig. 5) from the input parameters $\alpha, G_F, m_{f_{light}}, m_Z = 91.177 \pm\ 0.031$ GeV, m_H = 40-1000 GeV and $m_t = 140\pm$ 45 GeV (see Eq. (11)). Equation (32) is the analogous of Eq. (12) which refers to s_W^2. Both describe the conclusions of taking the LEP value for m_Z (and the bound on m_H) together with the whole of non-LEP results on neutral current processes and on $\frac{m_W}{m_Z}$. Note that the error on $\bar{s}_W^2$ in Eq. (32) ($\pm$ 0.002) is much smaller than that on s_W^2 which appears in Eq. (12) ($\pm$ 0.005). This difference reflects the markedly milder dependence of $\bar{s}_W^2$ on m_t with respect to s_W^2.

1.4. Neutrino-Electron Scattering

The CHARM II collaboration has recently presented[20] new results on $\sin^2\theta_W$ measured from the ratio $R_\nu = \frac{\sigma\nu e}{\sigma\bar{\nu}e}$ in $\nu_\mu - e$ scattering. The resulting value of $\sin^2\theta_W$ is

$$\sin^2\theta_W = 0.240 \pm 0.012 \quad \text{CHARM II} \tag{33}$$

The corresponding accuracy is far better than that of previous experiments[20]:

$$\sin^2\theta_W = 0.195 \pm 0.022 \quad \text{BNL} \tag{34}$$

$$\sin^2\theta_W = 0.211 \pm 0.037 \quad \text{CHARM I} \tag{35}$$

The present average value is thus:

$$\sin^2\theta_W = 0.228 \pm 0.010 \quad \text{AVERAGE} \tag{36}$$

What $\sin^2\theta_W$ is this one? The reported values are obtained from the Born expression for R_ν without non-QED corrections. As R_ν is in this case given by a pure Z exchange process, it is clear that the measured value of $\sin^2\theta_W$ refers to $\bar{s}^2_W$ measured at q^2 small. The Z exchange diagram for $\nu_\mu - e$ scattering is just the crossed one of the LEP process $e^+e^- \to \nu_\mu\bar{\nu}_\mu$ via Z exchange. But the LEP widths measure the effective $\sin^2\theta_W$ entering in the Z couplings at $q^2 = m_Z^2$. Thus the effective $\sin^2\theta_W$ of LEP and of $\nu_\mu e$ are at different scales. The running of the effective $\sin^2\theta_W$ between q^2 small and $q^2 \simeq m_Z^2$ can be accurately computed[42]. The leading logarithmic approximation[43,44] is not good in this channel. In this approximation the effect of the change of scale in $\sin^2\theta_W$ arises from a combination of the running of α and α_W plus the induced effects of charged $\nu_\mu - \mu$ currents via the relevant penguin diagram. While individual terms are large, there are strong cancellations among the different contributions. The resulting scale dependence[42] for $q^2 \sim 0$ is small in comparison to the experimental errors in Eqs. (33)-(36)

$$\sin^2\theta_W(q^2) - \sin^2\theta_W(m_Z^2) \simeq +0.002 \tag{37}$$

In conclusion the result given in Eq. (36) for $\sin^2\theta_W$ measured from R_ν in $\nu_\mu - e$ scattering is in good agreement with the values determined from LEP and from low energy neutral current processes.

2. BEYOND THE STANDARD MODEL

As no new particles have been found so far the search for possible effects of new physics at LEP1 is limited to in depth probing the Z couplings to ordinary particles or, in other words, the effective Lagrangian for Z exchange in $e^+e^- \to f\bar{f}$, with f being any of the light fermions. The predictions of the Standard Model for processes involving light fermions could be violated already at the tree level (e.g., by non-doublet Higgses, leading to $\rho_{tree} \neq 1$ or by a new Z' which, by mixing, modifies the couplings of the observed Z' and shifts the measured mass, effectively leading to $\delta\rho_{Z'} > 0$) or by virtual loop effects (vacuum polarization[45−52] and vertex[53] corrections). The vacuum polarization corrections, also called oblique corrections[45], are especially interesting because of their universality. Recently a number of papers[46−52] have been devoted to the limits on vacuum polarization effects from LEP data and their impact on different models of new physics. In the following section, we shall briefly describe these developments. Then, in Section 2.2, we will discuss extended gauge models.

2.1. Vacuum Polarization Effects

Assume that there is new physics at a scale Λ and that the effects of this new physics on low energy experiments up to m_Z are concentrated in vacuum polarization amplitudes (so that the corresponding terms must be well defined and observable). In general we have

$$\Pi^{ij}_{\mu\nu}(q^2) = -ig_{\mu\nu}[A^{ij} + q^2 F^{ij}(q^2)] + q_\mu q_\nu \quad \text{terms} \tag{38}$$

where i, j stand for W, Z, γ. We now make an expansion in q^2 and keep only $F^{ij}(0) \equiv F^{ij}$. Clearly for $m_Z^2 \lesssim q^2 \ll \Lambda^2$ higher order terms in the expansion are suppressed by powers of q^2/Λ^2. Taking into account that in physical gauges, $\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0$, one is left with a total of six independent constants: A_W, F_{WW}, A_{ZZ}, F_{ZZ}, $F_{\gamma Z}$ and $F_{\gamma\gamma}$. These constants are real numbers because there are no thresholds associated with the new physics at $q^2 \lesssim m_Z^2$. They are defined in the unrenormalized theory with a cut-off. In the renormalization procedure three combinations of these constants are reabsorbed in the definitions of α, G_F, m_Z. It is in fact simple to derive the relations[50]:

$$\begin{aligned}\frac{\delta\alpha}{\alpha} &= -F_{\gamma\gamma} \\ \frac{\delta G_F}{G_F} &= +A_{WW}/m_W^2 \\ \frac{\delta m_Z^2}{m_Z^2} &= -A_{ZZ}/m_Z^2 - F_{ZZ}\end{aligned} \tag{39}$$

The remaining independent combinations can be conveniently regrouped[50] as:

$$\begin{aligned}\epsilon_1 &= \frac{A_{ZZ}}{m_Z^2} - \frac{A_{WW}}{m_W^2} = \frac{A_{33} - A_{WW}}{m_W^2} \\ \epsilon_2 &= F_{WW} - F_{33} \\ \epsilon_3 &= F_{3\gamma/S} - F_{33} = \frac{c}{s}F_{30}\end{aligned} \tag{40}$$

where $s = \sin\theta_W$, $c^2 = 1 - s^2$ (no precise definition of s^2 has to be specified here, because the ϵ_i are small corrections) and the indices 3, 0 refer to $W_3 = s\gamma + cZ$, $W_0 = c\gamma - sZ$ with W_3 and W_0 being the partner in $SU(2)$ of $W^{\pm}$ and the $U(1)$ gauge vector, respectively. In terms of directly observable quantities one finds[50]:

$$\begin{aligned}\delta\rho &= \epsilon_1 \\ \delta k' &= (-c^2\epsilon_1 + \epsilon_3)/(c^2 - s^2) \\ \delta r_W &= -c^2/s^2\epsilon_1 + \frac{c^2 - s^2}{s^2}\epsilon_2 + 2\epsilon_3\end{aligned} \tag{41}$$

where we define $\delta k'$ and δr_W by:

$$\bar{s}_W^2 = (1 + \delta k')s_0^2 \tag{42}$$

$$(1 - \Delta r) = (1 - \Delta\alpha)(1 - \delta r_W) \tag{43}$$

with

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \tag{44}$$

and $\Delta\alpha$ given in Eq. (9).

Note that in the last equation $\alpha(m_Z)$ has replaced α in the tree-level relation between

s^2c^2 and m_Z^2. Also note that $\delta k'$ is different from δk defined by Eq. (25), because δk relates $\bar{s}_W^2$ to $s_W^2 = 1 - m_W^2/m_Z^2$ while s_0^2 appears in the definition of $\delta k'$. The expression of δk in terms of ϵ_1, ϵ_2, ϵ_3 is:

$$\delta k = \frac{c^2}{s^2}(\epsilon_1 - \epsilon_2) - \epsilon_3 \tag{45}$$

In the Standard Model the large $G_F m_t^2$ terms appear in ϵ_1 (the A terms), while ϵ_2 and ϵ_3 are of order $\alpha_W \sim G_F m_W^2$ (the F terms). When the F terms are neglected, $\delta\rho$, $\delta k'$, δk and δr_W are all proportional to each other, while $\delta\rho$, $\delta k'$ and δr_W become independent quantities if ϵ_1, ϵ_2 and ϵ_3 all receive sizeable contributions. One might imagine to neglect the F terms with respect to the A terms. By dimensions one would expect $\epsilon_1 \sim \Lambda^2$. But actually, $\epsilon_1 \sim \delta\Lambda^2$ where $\delta\Lambda^2$ is the scale that breaks the custodial $SU(2)$ symmetry. For Λ large, in all sensible models $\delta\Lambda^2 \ll \Lambda^2$. For example, $\delta\Lambda^2$ is the splitting of a $SU(2)$ multiplet (e.g., $m_{\tilde{t}}^2 - m_{\tilde{b}}^2$ for the s–top/s–bottom doublet in SUSY models) while Λ^2 is the average mass-squared. Also, the F terms are dimensionless and can have a finite limit for $\Lambda \to \infty$. In general the F terms are of order $G_F m_W^2$ and can well compete with the $G_F m_t^2$ term of the Standard Model for m_t not too large. In fact, the rather precocious dominance of the $G_F m_t^2$ terms in the Standard Model is largely accidental. In conclusion, there are examples of new physics[45–52)] where the contributions to ϵ_1, ϵ_2 and ϵ_3 are of the same order or larger than the $G_F m_t^2$ terms of the Standard Model, so that in general these terms cannot be ignored.

There is a difference between ϵ_1, ϵ_2 on the one side and ϵ_3 on the other side. ϵ_1 and ϵ_2 are only different from zero if an imbalance between $W^\pm$ and W_3 is created[54)]. For example only a split $SU(2)$ multiplet of heavy particles can contribute to ϵ_1, ϵ_2 while an unsplit multiplet cannot. On the contrary an unsplit multiplet can contribute to $\epsilon_3 \simeq \frac{c}{s} F_{30}$[55)] (e.g., from transitions between left isospin and right hypercharge). An unsplit fermion multiplet contributes to ϵ_3 because of the breaking of chiral invariance. Each member of an unsplit multiplet contributes to ϵ_3 the quantity[52,55)]:

$$\Delta\epsilon_3 = N_c \frac{G_F m_W^2}{8\pi^2\sqrt{2}} \, \frac{4}{3} \, (T_{3L} - T_{3R})^2 \tag{46}$$

For example, an unsplit quark or lepton doublet leads to

$$\Delta\epsilon_3 = N_c \frac{G_F m_W^2}{12\pi^2\sqrt{2}} \tag{47}$$

or $\Delta\epsilon_3 \simeq 0.0014$ for one quark doublet.

In many models, ϵ_2 is negligible in comparison to ϵ_1 and ϵ_3. For example in technicolour models this is the case. It has been shown[45–49] that the contributions to ϵ_3 of a technifermion doublet, or of a whole technifamily (with the same content of quarks and leptons of a standard fermion family) are given by:

$$\Delta\epsilon_3 = \frac{G_F m_W^2}{2\sqrt{2}\pi} \begin{array}{ll} 0.4 + 0.09\ (N_{TC} - 4) & \quad 1 \text{ doublet} \\ 2.1 + 0.4\ (N_{TC} - 4) & \quad 1 \text{ technifamily} \end{array} \tag{48}$$

Numerically, for $N_{TC} = 4$ and one complete technifamily, one finds $\Delta\epsilon_3 = +0.018$. Similarly in the "BESS" model of Ref. 56) (a non-linear non-renormalizable model of electroweak symmetry breaking, with a strongly interacting electroweak sector and new ρ-like vector states) one finds[57] $\Delta\epsilon_1 = \Delta\epsilon_2 = 0$ and $\Delta\epsilon_3 = \frac{g^2}{g''^2} > 0$. If axial vector mesons are also present, then $\Delta\epsilon_3 = (1 - Z^2)\frac{g^2}{g''^2}$ can be negative in the BSS model[57] (Z describes the effect of axial vector mesons). But there are models where ϵ_2 and ϵ_3 are non negligible and of the same order[50,52]. For example, this may be the case in some models[50] where all vacuum polarization amplitudes vanish at $q^2 = 0$, so that $\Delta\epsilon_1 = 0$. Thus a general analysis of the data should include all three of them.

Once the proportionality relations valid in the Standard Model among $\delta\rho$, δr_W and $\delta k'$ are released these quantities can be separately obtained from the existing data in the following way[50].

From Eq. (2) rewritten in the form

$$\left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2} \frac{1}{1 - \delta r_W} \tag{49}$$

With $\alpha(m_Z) = 1/128.8$, given m_Z and $\frac{m_W}{m_Z}$ from Tables 1 and 2, one obtains δr_W. This leads to[50]

$$\delta r_W = -0.015 \pm 0.018 \tag{50}$$

in agreement with Eqs. (9), (17) and (43).

Assuming lepton universality ($e = \mu = \tau$), the partial widths of the Z into charged leptons and the asymmetries provide information on $\delta k'$ and $\delta\rho$. One can define effective vector (g_V) and axial vector (g_A) couplings of the on-shell Z to charged leptons by the relations, taken as exact:

$$\Gamma(Z \to \ell\bar{\ell}) = \frac{G_F m_Z^3}{6\pi\sqrt{2}}(g_V^2 + g_A^2) \tag{51}$$

$$A_{FB}^{\mu}(q^2 = m_Z^2) = 3\frac{g_V^2 g_A^2}{(g_V^2 + g_A^2)^2} \tag{52}$$

$\delta\rho$ and $\delta k'$ are given in terms of g_A and g_V/g_A by the relations:

$$g_A = -\sqrt{\rho}/2 = -\frac{1}{2}(1 + \frac{1}{2}\delta\rho) \tag{53}$$

$$\frac{g_V}{g_A} = 1 - 4\bar{s}_W^2 = 1 - 4(1 + \delta k')s_0^2 \tag{54}$$

where s_0^2 is defined in Eq. (44). We see that given α, G_F, m_Z there is a diagonalization of the form $\frac{m_W}{m_Z} \leftrightarrow \delta r_W$, $g_A \leftrightarrow \delta\rho$ and $g_V/g_A \leftrightarrow \delta k'$. Note that in general one should introduce[53] a pair of $\delta\rho$ and $\delta k'$ for each flavour of fermions. In the Standard Model, $\delta\rho$ and $\delta k'$ are universal only if small terms from box diagrams, vertex corrections and imaginary parts are neglected. We work in this approximation and we are interested in oblique corrections that are larger than these terms. Alternatively one could subtract the Standard Model contributions. We prefer not to do that because the standard prediction depends on m_t and m_H so that it is not really fixed.

All four LEP experiments are now giving results both for the partial width and for the lepton asymmetries, so that the values of g_V and g_A can be separately extracted. An average of all LEP experiments gives[5,58]:

$$g_A = -0.5004 \pm 0.0021 \tag{55}$$

$$g_V/g_A = 0.085 \pm 0.010 \tag{56}$$

From these results and Eqs. (53) and (54), one obtains:

$$\delta\rho = 0.0016 \pm 0.0084 \tag{57}$$

$$\bar{s}_W^2 = 0.229 \pm 0.0025 \tag{58}$$

$$\delta k' = -0.011 \pm 0.011 \tag{59}$$

Another important input is obtained from neutrino deep inelastic scattering and from atomic parity violation experiments on Cesium atoms. From the experimental values[15,16] of $R_\nu = \frac{\sigma^{NC}}{\sigma^{CC}}$ for $\nu - N$ scattering, given by Eq. (10), and the analogous quantity $R_{\bar{\nu}}$ for $\bar{\nu} - N$, given by an identical equation with $r \to \bar{r} = 1/r$, one can separately extract ρ and $\bar{s}_W^2$. The experimental values of R_ν, $R_{\bar{\nu}}$ and r corrected for the non-isoscalarity of the target, QED effects, weak boxes and vertices, but no oblique corrections are given in Ref. 59). After also correcting for the small effect due to the change of scale from the typical q^2 of the neutrino scattering experiments up to m_Z^2 [45,59] one obtains[50] for the allowed range in the $\delta\rho$, $\delta k'$ plane, the ellipse which is plotted in Fig. 6. Altogether, from δr_W and the results in Fig. 6, we obtain

$$\epsilon_1 \equiv \delta\rho = 0.0025 \pm 0.0075 \tag{60}$$

$$\epsilon_2 = 0.001 \pm 0.019 \tag{61}$$

$$\epsilon_3 = -0.004 \pm 0.012 \tag{62}$$

These values contain the whole information from m_Z, $\frac{m_W}{m_Z}$, neutrino scattering and the LEP data on leptonic widths and asymmetries.

An interesting additional input is derived from atomic parity violation measured on Cesium[51]. For an atom with Z protons and N neutrons ($Z = 55$, $Z = 78$ for Cs) the relevant quantity which is measured is $Q(Z, N)$, proportional to $T_3 - 2Q\sin^2\theta_W$ evaluated for the atom:

$$Q(Z, N) \sim \rho(Z - N - 4Z\bar{s}_W^2) \tag{63}$$

In general $Q(Z, N)$ depends on ϵ_1 and ϵ_3 (but not ϵ_2) through $\delta\rho$ and $\delta k'$. The peculiarity of Cs is that for the corresponding values of Z, N there is an accidental, almost exact, cancellation of the dependence on ϵ_1[51]. Therefore Q_{Cs} is a direct measure of ϵ_3. From the present experimental value of Q_{Cs}[60], with some additional, small, radiative corrections taken into account[51], one obtains $Q_{\exp} = -71.04 \pm 1.8$, $Q_{TH} = -73.20 \pm 0.13 - 0.85 \cdot S$ with $\epsilon_3 = \alpha S/4s^2$ [46], so that

$$\epsilon_3 = -0.021 \pm 0.018 \tag{64}$$

An estimate of the theoretical error associated with the wave function calculations[61] is included in Eq. (64). We see that the resulting accuracy for ϵ_3 is remarkable given that the experimental result on Cs was obtained by a team of three people[60] by a table-top kind

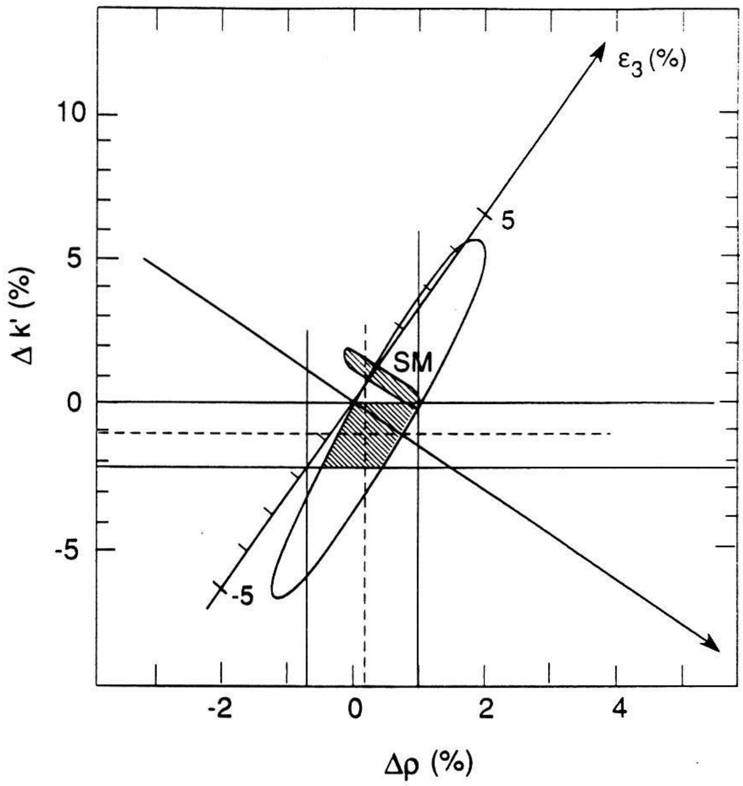


FIGURE 6

6. Constraints[50)] on $\Delta k'$ and $\Delta\rho$ (at $q^2 = m_Z^2$) imposed by the measurements of g_A (vertical band), g_V/g_A (horizontal band) and neutrino or antineutrino deep inelastic scattering (the area inside the ellipse[59)]). The shaded area denotes the $1-\sigma$ intersection. The region corresponding to the prediction of the Standard Model, including complete radiative corrections[45)], for $m_Z = 91.177$ GeV, $m_t = 90$-190 GeV and $m_H = 40$-1000 GeV is shown. The measured value of $\Delta k'$ is slightly smaller than required for a perfect agreement with the Standard Model. The ϵ_3 axis is also shown. Note that the Standard Model predicts a slightly positive value of ϵ_3.

of experiment. Also the fact that the central value is negative and relatively large leads to a powerful constraint on all models predicting positive values for ϵ_3 (as for technicolour, Eq. (48), or for unsplit multiplets, Eq. (47)). However, it is also evident that the LEP experiments are more precise (Eq. (62)) and of much simpler theoretical interpretation than the atomic physics measurements, so that already now, but especially in the near future, Cesium cannot compete with LEP. By combining the result in Eq. (62) with ϵ_3 from Cesium (Eq. (64)) we finally get:

$$\epsilon_3 = -0.009 \pm 0.010 \tag{65}$$

or $\epsilon_3 < +0.004$ at 90 %. Considering that the Standard Model predicts a small positive value for ϵ_3, we see that essentially no space is left for models predicting additional positive contributions to ϵ_3.

2.2. Extended Gauge Models

Models with an enlarged gauge structure offer a conspicuous example of new physics that appears at tree level. The new LEP data impose important restrictions on extensions of the Standard Model with new heavy Z' [25–29]. We discuss here the simplest gauge extensions of the Standard Model, where only one extra $U(1)$ factor is added to the $SU(2) \otimes U(1)$ group of the standard electroweak theory. We follow the analysis of Refs. 27. The light and the heavy physical states Z_L and Z_H (Z_L is observed at LEP) are superpositions of the standard Z_S and of a new state $Z_N = Z'$

$$\begin{pmatrix} Z_L \\ Z_H \end{pmatrix} = \begin{pmatrix} \cos\xi_0 & \sin\xi_0 \\ -\sin\xi_0 & \cos\xi_0 \end{pmatrix} \begin{pmatrix} Z_S \\ Z_N \end{pmatrix} \tag{66}$$

At the tree level, even for doublet Higgses,

$$\cos^2\theta_W = \frac{m_W^2}{m_{Z_L}^2 \rho} \tag{67}$$

with $\rho = 1 + \delta\rho_M$, where $\delta\rho_M$ is due to the ξ mixing: the physical mass m_{Z_L} is pushed down with respect to m_{Z_S}. As a consequence, $\delta\rho_M \geq 0$. This important inequality holds in all extra $U(1)$ models. It could be violated if the $W^\pm$ were also mixed with some other heavy states[28,29]. In particular, in a large class of models we have, for m_{Z_H} large:

$$tg\xi_0 = a\frac{m^2_{Z_L}}{m^2_{Z_H}} \tag{68}$$

with a being a constant, and one finds

$$\delta\rho_M = a^2\frac{m^2_{Z_L}}{m^2_{Z_H}} \tag{69}$$

All effects of Z' at the Z_L peak arise because of mixing through $\delta\rho_M$ and ξ_0. For example the partial widths, in the improved Born approximation become

$$\begin{aligned}\Gamma_f =& N_c\frac{G_F m^3_{Z_L}\rho}{6\pi\sqrt{2}}\left[\cos^2\xi_0(v_f^2 + a_f^2)\right.\\ &+2\cos\xi_0\sin\xi_0(v_f v'_f + a_f a'_f)\\ &\left.+\sin^2\xi_0(v_f'^2 + a_f'^2)\right]\end{aligned} \tag{70}$$

where $\rho = 1 + \delta\rho_M + \delta\rho_{top} + \ldots$, $v_f = (T_{3f} - 2Q_f\bar{\bar{s}}^2_W)$, $a_f = -T_{3f}$, $\bar{\bar{s}}^2_W = \bar{s}^2_W - \frac{\bar{s}^2_W\bar{c}^2_W}{\bar{c}^2_W-\bar{s}^2_W}\delta\rho_M$ (for $f \neq b$). Note that the effective $\sin^2\theta_W$, $\bar{\bar{s}}^2_W$, differs from the standard value $\bar{s}^2_W$ (i.e., computed from α, $G_F, m_Z\ldots$ by using the Standard Model value of Δr) because of $\delta\rho_M$. More in general at the Z_L peak:

$$g^{\text{eff}}_{V,A} = \cos\xi_0 g_{V,A} + \sin\xi_0 g'_{V,A} \tag{71}$$

with

$$g_V = +\sqrt{\rho}v_f, \quad g_A = +\sqrt{\rho}a_f \tag{72}$$

and $g'_{V,A}$ are the Z_N couplings. As we see there would be no effect on Γ_f if $\delta\rho_M$ and ξ_0 would vanish (given the relation Eq. (69), it is enough that $\xi_0 = 0$). For $\xi_0 = 0$, Z_N would decouple from Z_L and LEP 1 could not constrain its mass at all. However, Z_N could of course be produced and observed at hadron colliders.

As discussed in previous sections, a limit on $\delta\rho_M$ can be derived from the measured values of m_{Z_L} and of m_W/m_{Z_L}. Clearly, because $\delta\rho_M \geq 0$, there is less space for a Z' if m_t is increased.

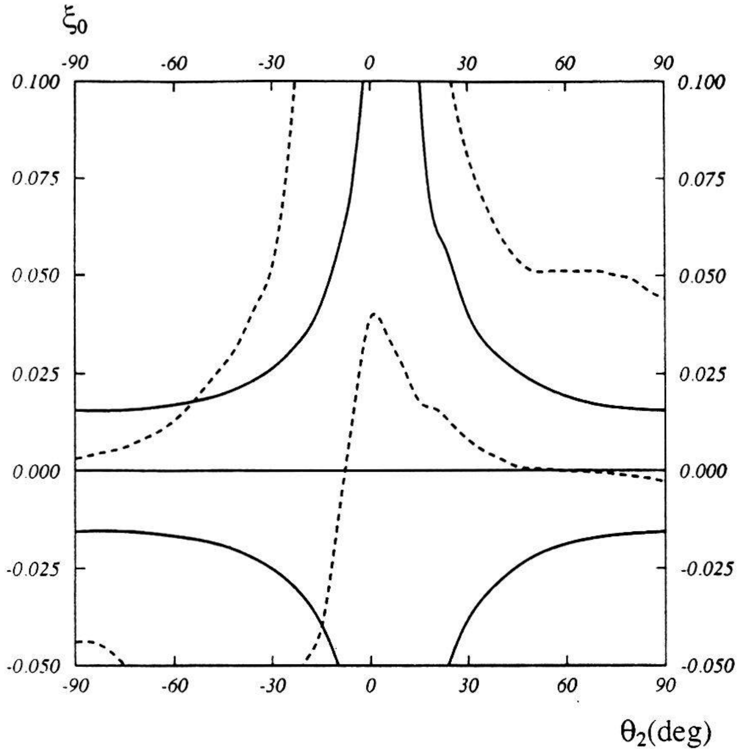
Bounds[27] from LEP data on $\delta\rho$ and ξ_0, treated as independent quantities, valid for extra $U(1)$ models of the $E(6)$ type[25], are shown in Figs. 7a and 7b. The angle θ_2 describes[25-27] the position of Z_N in $E(6)$ space. The allowed region in the $\xi_0 - \theta_2$ plane from neutral current experiments, taken from Ref. 18, is also shown for comparison. We see from Fig. 7 that LEP data have added much to the constraints on ξ_0 and $\delta\rho$. If we consider the effects of a Z' on the leptonic widths and asymmetries we see that $\delta\rho_M$ induces a positive shift to ϵ_1 and a negative shift to $\delta k'$, while the terms from the mixing angle ξ_0 can contribute with either sign to ϵ_1, ϵ_3.

3. CONCLUSION AND OUTLOOK

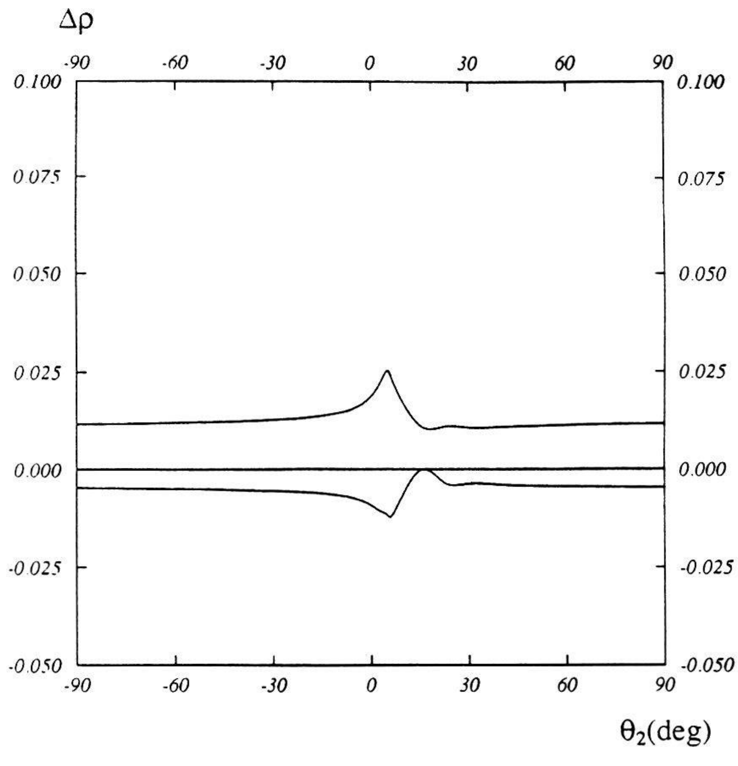
All the results of LEP are in perfect agreement with the standard electroweak theory. So much that there is almost a sense of deception in the LEP community. One could in fact hope for a sensational discovery, e.g., the production of some new particle. Instead the limit on the Higgs has been set at $m_H > 44$ GeV, the number of light neutrinos has been fixed at $N_\nu \simeq 3$, no new charged particles have been observed and so on. All these limits are indeed quite impressive but are not as fulfilling as a real discovery. For precision tests of the electroweak theory we knew from the start that the widths cannot compete with asymmetries, which need a large integrated luminosity. The absolute error on $\bar{s}_W^2$ from the leptonic width is given by:

$$\delta\bar{s}_W^2 = \pm 0.27 \quad \bullet \quad \frac{\delta\Gamma_e}{\Gamma_e} \simeq \pm 0.0025 \tag{73}$$

for $\delta\Gamma_e/\Gamma_e \simeq 1$ %. The ultimate precision on $\frac{\delta\Gamma_e}{\Gamma_e}$ cannot be brought down by very much in the future. The precision on $\bar{s}_W^2$ expected from the asymmetries is reported[62)] in Table 4.



(A)



(B)

FIGURE 7

7. (A) Bounds on ξ_0 obtained from LEP data in $E(6)$-type extended gauge models (the allowed region is internal to the solid lines) where θ_2 is the angle describing the orientation in $E(6)$ space of the additional $U(1)$ generator. For comparison the region allowed by existing neutral current data is also shown (dashed lines). The overall allowed region is the intersection of the above two domains.

7. (B) Allowed region for $\Delta\rho = \Delta\rho_{top} + \Delta\rho_M + \ldots$ from LEP data.

No Polarization	$\int \mathcal{L}dt = 200pb^{-1}$	
Asymmetry	$\delta\bar{s}^2_W$	$\delta(\delta k')$
A^{μ}_{FB}	$\pm$ 0.0017	
A^{c}_{FB}	$\pm$ 0.0015	
A^{b}_{FB}	$\pm$ 0.0009	$\pm$ 0.005
A^{τ}_{pol}	$\pm$ 0.0014	$\pm$ 0.007
Polarization $< P_L > = 0.5$	$\int \mathcal{L}dt = 40pb^{-1}$	
A_{LR}	$\pm$ 0.0004	$\pm$ 0.0016
A^{μ}_{FB}	$\pm$ 0.0006	

TABLE 4

Similarly m_W can be measured with $\delta_{m_W} = \pm$ 100 MeV at LEP 2 or at hadron colliders (given m_Z). $\delta m_W = \pm$ 100 MeV corresponds to $\delta s^2_W = \pm$ 0.002 in terms of $s^2_W = 1 - \frac{m^2_W}{m^2_Z}$. For $\delta\bar{s}^2_W$ one can use Eq. (28) to find $\delta\bar{s}^2_W = \pm$ 0.0006.

In spite of the fact that no striking discoveries have been found in the first few months of LEP there are all reasons to be satisfied. The big discoveries will presumably occur in the next few years. LEP 1 plus LEP 2 have in fact very good chances to discover new physics. Precision tests of the Standard Model, the search for the Higgs and for signals of new physics remains a very promising and exciting programme for the future of LEP.

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