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Time interval statistics of the Brillouin spectrum

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Abstract. Based on the *unique* analytic form of the one-fold generating-function very recently given by the present author in [3], we provide in this paper a complete and an exact analysis of the time-interval-statistics of the Brillouin spectrum with its *central* Rayleigh line. We also distinguish the time-interval-distributions for the present Brillouin triplet from the time-interval-distributions for the *pure* Brillouin spectrum (a doublet) given in Ref. [4]. As in Ref. [4], we establish here too the invalidity of the earlier numerical-results due to Blake and Barakat [5] for the present Brillouin triplet, as they violate even the well known equation (32) for the time-interval-distribution for the conditional photocount.

1. Introduction

A fairly sensitive account of the one-fold photon-counting analysis of even a complex multiple-peaked spectrum like the Brillouin spectrum with its *central* Rayleigh line (a triplet) can be accomplished by a simple and direct study of the arrival statistics of the photocounts offered by a combination of Time to Amplitude Converter (TAC) and a Pulse Height Analyser (PHA) with a resolution of the order of 10^{-6}–10^{-7} s [1, 2]. Exact theoretical analysis of the time-interval-distributions (TIDs), on the other hand can be achieved by a *unique* analytic form of the one-fold generating function (g.f) pertaining to this spectrum—a task very recently accomplished by the present author in Ref. [3]. The detailed analysis of the TIDs for the Brillouin spectrum with its central Rayleigh line provided in this paper shows a clear contrast with the TIDs for the pure Brillouin spectrum recently given in Ref. [4]. Also we establish (as in Ref. [4] for the pure Brillouin spectrum) the inadequacy of the numerical-scheme employed by Blake and Barakat [5] to study the TID of the conditional photocount for the present Brillouin triplet. Before proceeding with these formal features of the photon counting analysis, it is worthwhile to first ponder over the following general aspects pertaining to this spectrum.

Inelastic laser light scattering from a medium (a liquid or condensed matter) undergoing thermal motion produces a Brillouin triplet. The frequency shift 'Δ' is given by the following relation [6]:

$$\Delta = \omega_s - \omega_i = \pm 2v\omega_i(\epsilon/c)^{\frac{1}{2}} \sin \theta/2 \tag{1}$$

where ω_s and ω_i are respectively the frequency of the scattered and the incident wave, 'v' is the velocity of sound in the medium, 'ϵ' the directional constant of the

medium and 'c' is the velocity of light. In liquids the central Rayleigh component is due to the pressure fluctuations [6, 7] whereas in the solids, the central component is thought to be present due to the *elastic* scattering from the spatial imperfections or reflection from the surfaces and the side Brillouin components manifest due to the *inelastic* scattering from the lattice vibrations (phonons) [8]. A *macroscopic* theory generally used (see Ref. [9] for example) to explain certain details of this spectrum is the one given by Landau and Placzek [6]. In this theory, the ratio of the intensities of the Rayleigh and the Brillouin components is given by the following relation:

$$I_R/2I_B = (C_p - C_v)/C_v = \gamma - 1 \tag{2}$$

where C_p and C_v are the specific heats at constant pressure and volume respectively and γ is known as the Landau–Placzek ratio. Now, if we were to explain the presence of the central Rayleigh line in solids through equation (2) above, we immediately encounter a difficulty as $C_p \sim C_v$ in solids [7] but the presence of this line has been confirmed by Vacher et al in [8]. These facts suggest that there is a need for a new *microscopic* theory which would uniformly explain the existence of the Brillouin triplet and the relative intensities of its distinct components, when we scatter laser light from a liquid or a solid.

However, for a study of the relaxation times τ_c of the different components of this spectrum, we can use the following interesting definition (valid for any class of an optical field-quantum or classical) given by Prof. Mandel in [10]:

$$\tau_c = 2\int_0^\infty |g(\tau)|^2\, dt \tag{3}$$

where $g(\tau)$ is the field-correlation. Here for the Brillouin spectrum $g(\tau)$ is given by [5]:

$$\begin{aligned} g(\tau) &= (1-\alpha)e^{-|\tau|} + \alpha e^{-\delta|\tau|}\cos\Delta\tau \qquad (0 \leqslant \alpha \leqslant 1) \\ &= (1-\alpha)e^{-|\tau|} + \alpha/2(e^{-\beta|\tau|} + e^{-\beta^*|\tau|}) \qquad (\beta, \beta^* = \delta \mp i\,\Delta) \end{aligned} \tag{4}$$

where the parameters, 'δ'—the ratio of the half-widths of the Brillouin and Rayleigh lines and 'Δ'—the frequency shift (given by equation (1)) characterize the scattering medium—solid or liquid.

Thus a simple calculation gives the following relaxation time for the complete Brillouin spectrum (triplet):

$$\tau_c = [(1-\alpha)^2 + \alpha/2]\,1/\delta + \alpha^2\delta/2(\delta^2 + \Delta^2) + 8\delta\alpha(1-\alpha)/(4\delta^2 + \Delta^2). \tag{5}$$

From equation (5) above now it is easy to deduce the following relaxation times for the pure Brillouin (a symmetric two-peaked spectrum) and the pure Rayleigh (a

single Lorentzian profile) respectively:

(i) Pure Brillouin ($\alpha = 1$)

$$\tau_c = (2\delta^2 + \Delta^2)/2\delta(\delta^2 + \Delta^2). \tag{6}$$

(ii) Pure Rayleigh ($\alpha = 0$)

$$\tau_c = 1/\delta. \tag{7}$$

Thus, whereas equation (7) reporduces the well-known relation for a single Lorentzian line, equations (6) and (5) give us the new relationships of the relaxation times for the doublet and the triplet respectively. In Fig. 0 we give the behaviour of the relaxation times corresponding to different situations found in equations (5)–(7) as a function of the frequency shift Δ. For pure Rayleigh (equation (7)), we observe a straight line as in this case there is no frequency shift at all! For the complete Brillouin triplet (equation (5)) we see a steady fall of τ_c as Δ and α increase but after $\alpha = 0.6$ we find that τ_c values start moving towards the pure Brillouin case (the dotted line for $\alpha = 1$ in Fig. 0). Even though the relaxation times for the Brilouin triplet and the pure Brillouin are of the same order, yet it is possible to distinguish between these two cases even for small frequency shifts, as the

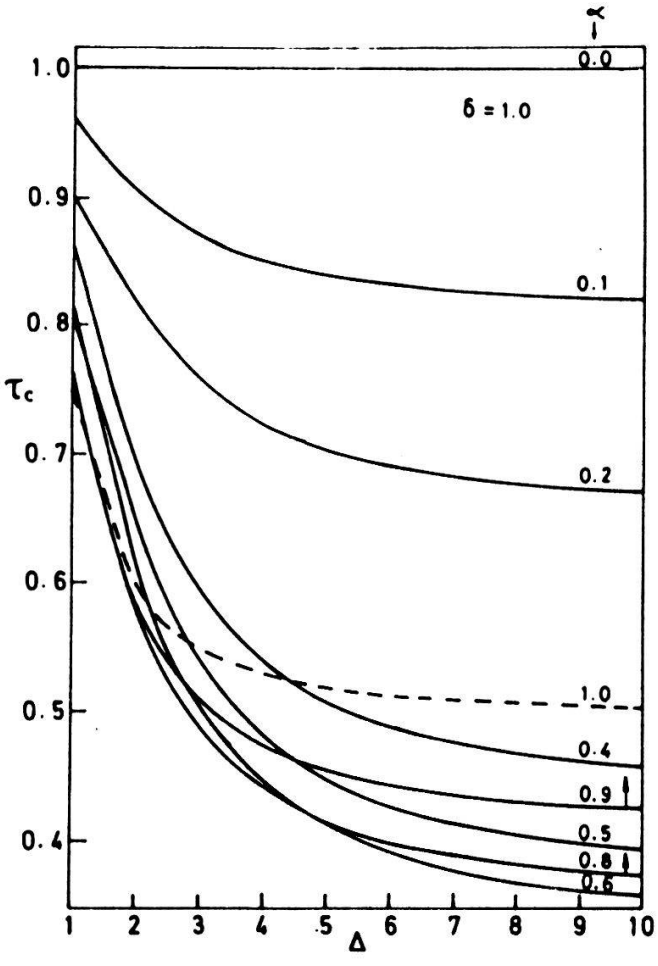

Figure 0
The relaxation times for the Rayleigh, pure Brillouin and the Brillouin triplet as defined by equation 3.

heterodyning of the Brillouin components amongst themselves and that of the Brillouin components with the central Rayleigh line ought to produce different effects! This indeed is the case as evident from Fig. 0.

2. The Generating Functions

For the optical fields of weak or moderate strength we find the following relation to be true,

$$\int_t^{t+\delta t} I(t')\,dt' \sim I(t)\delta t, \tag{8}$$

as only one photocount is registered in the time interval $[t, t+\delta t]$. In such a situation Glauber [11] has shown that we can study the Time Interval Statistics (TIS) by employing a one-fold g.f pertaining to a given spectrum. However for a rigorous analysis when the field strengths are arbitrary, Barakat and Blake [12] have shown that we would require a third-order g.f instead and we have successfully dealt with the case of Gaussian–Lorentzian (G–L) light in [13] by employing the exact third-order g.f derived in [14]. However, for the present, we confine ourselves to the results obtained from the one-field g.fs. The one-fold g.f is obtained on solving the following Fredholm integral equation of the second-kind:

$$s\int_0^T g|t-t'|\phi_k(t')\,dt' = \lambda_k\phi_k(t) \tag{9}$$

where 's' is a useful parameter, $g|t-t'|$ the auto-correlation of the spectrum under study and ϕ_k s and λ_k s are respectively the eigenfunctions and the eigenvalues. For a Gaussian light of any spectral shape, it is well-known that the one-fold g.f can be expressed as the following infinite-product involving the eigenvalues of eq. (9):

$$Q(s) = \prod_k (1+s\langle I\rangle\lambda_k)^{-1} \tag{10}$$

where $\langle I\rangle$ is the mean count rate of the scattered light.

The one-fold g.f given by Blake and Barakat [5], based on the numerical-procedure suggested by Glauber in [15], is given by the following relations:

$$Q(s, T) = \exp\sum_{r=1}^{\infty}[(-\langle I\rangle s)^r/r]I_r(T) \tag{11}$$

where

$$I_r(T) = \int_0^T g_r(t', t')\,dt' \tag{12}$$

and $g_r(t', t')$ is the rth iterated kernel of the integral equation (9) defined recursively as,

$$g_1(t', t'') = g(t', t'') \tag{13}$$

and

$$g_r(t', t'') = \int_0^T g(t', t) g_{r-1}(t, t'')\, dt \qquad (r \geqslant 2). \tag{14}$$

Clearly, no functional forms of the TIDs can be expected from this sort of a g.f stemming from a numerical-scheme, unlike in the case of our exact analysis of the TIDs here, based on a *unique* analytic g.f for the Brillouin spectrum, given by the present author in [3]. This one-fold g.f for the Brillouin triplet obtained on following certain boundary conditions on the g.f and the usage of the Hadamard's Theorem [16], has the following general form:

$$Q(s, T) = P(0)/P(-s\langle I\rangle) \tag{15}$$

where both $P(0)$ and $P(-s\langle I\rangle)$ are the specific forms of the entire function $P(\xi)$ given by the following determinantal forms:

$$P(\xi) = \begin{vmatrix} R_1(p_1) & R_1(-p_1)E_1(p, T) & R_2(p_1) & R_2(-p_1)E_2(p_1, T) & R_3(p_1) & R_3(-p_1)E_3(p_1, T) \\ R_1(-p_1) & R_1(p_1)E_1(-p_1, T) & R_2(-p_1) & R_2(p_1)E_2(-p_1, T) & R_3(-p_1) & R_3(p_1)E_3(-p_1, T) \\ R_1(p_2) & R_1(-p_2)E_1(p_2, T) & R_2(p_2) & R_2(-p_2)E_2(p_2, T) & R_3(p_2) & R_3(-p_2)E_3(p_2, T) \\ R_1(-p_2) & R_1(p_2)E_1(-p_2, T) & R_2(-p_2) & R_2(p_2)E_2(-p_2, T) & R_3(-p_2) & R_3(p_2)E_3(p_3, T) \\ R_1(p_3) & R_1(-p_3)E_1(p_3, T) & R_2(p_3) & R_2(-p_3)E_2(p_3, T) & R_3(p_3) & R_3(-p_3)E_3(p_3, T) \\ R_1(-p_3) & R_1(p_3)E_1(-p_3, T) & R_2(-p_3) & R_2(p_3)E_2(-p_3, T) & R_3(-p_3) & R_3(p_3)E_3(-p_3, T) \end{vmatrix} \times \{p_1p_2p_3(p_1^2 - p_2^2)(p_2^2 - p_3^2)^2(p_3^2 - p_1^2)\}^{-1} \tag{16}$$

where

$$R_i(p) = (\beta_i + p)\tilde{Z}_i(p), \qquad E_i(p, T) = \exp(-(\beta_i + p)T)$$

with

$$\beta_1 = 1, \quad \beta_2 = \beta, \quad \beta_3 = \beta^* \quad \text{and} \quad \tilde{Z}_i(p) = Z_j(p)Z_k(p) \tag{17}$$

where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$ and the variables p_is are obtained from the following sixth-order polynomial [3]:

$$p^6 + Ap^4 + Bp^2 + C = 0 \tag{18}$$

where

$$A = (s\xi a_1 - 1) - (\beta^{*2} + \beta^2),$$

$$B = (s\xi a_1 - 1) + (\beta^{*2} + \beta^2)(s\xi a_1 - 1) + \beta^{*2}\beta^2,$$

$$C = s\xi(a_3 - a_2) - s\xi(a_2 - a_1)(\beta^{*2} + \beta^2) + (s\xi a_1 - 1)\beta^{*2}\beta^2$$

with

$$a_1 = 2[(1 - \alpha) + \alpha/2(\beta^* + \beta)],$$

$$a_2 = 2[(1 - \alpha) + \alpha/2(\beta^{*3} + \beta^3)],$$

$$a_3 = 2[(1 - \alpha) + \alpha/2(\beta^{*5} + \beta^5)]. \tag{19}$$

An important measure of the g.f is given by the probability of zero-counts defined as,

$$P(0, T) = Q(s, T)|_{s=1} \tag{20}$$

which has the maximum value of 1 at $T = 0$, as there are no photocounts just on switching the photocounter [3]. However, such a simple aspect related to the g.f has been overlooked by Blake and Baraket [5] while studying the TIS of the Brillouin spectrum.

3. Time Interval Statistics

3.1. *Definitions and General Features*

The one-fold g.f can be alternatively written as,

$$Q(s, T) = \langle e^{-sE(t)} \rangle \tag{21}$$

where

$$E(t) = \int_t^{t+T} I(t')\, dt' \tag{22}$$

and $I(t)$ is the instantaneous intensity of the scattered field. The TIDs of registering the first photocount $V(T)$ and the conditional photocount $P(T)$, are respectively given by,

$$V(T) = \langle I(T) e^{-sE(T)} \rangle \tag{23}$$

and

$$P(T) = \langle I(0) I(T) e^{-sE(T)} \rangle \tag{24}$$

where $t = 0$ in equation (22). In terms of the one-fold g.f given by equation (21), the above equations (23) and (24) for the TIDs can be recast as follows:

$$V(T) = -\frac{\partial Q(s, T)}{\partial T}\bigg|_{s=1} \tag{25}$$

and

$$P(T) = \frac{1}{\langle I \rangle} \frac{\partial^2 Q(s, T)}{\partial T^2}\bigg|_{s=1}. \tag{26}$$

For a gross-analysis of the TIS of the Gaussian light Barakat and Blake have given certain interesting details in their review article [12]. We recapitulate some of these details required here. For a completely polarized light the TIDs have the following simple relations [12]:

$$V(T) = \langle I \rangle (1 + \tfrac{1}{2} \langle I \rangle T)^{-2} \tag{27}$$

and

$$P(T) = 2\langle I\rangle(1 + \langle I\rangle T)^{-3} \tag{28}$$

where the length of the counting interval T is short and the mean count rate $\langle I\rangle$ can be anything. *Clearly these equations* (27) *and* (28) *are completely insensitive to the spectral shape of the chaotic light as the auto-correlation* $g(\tau)$ *does not figure in anywhere*! However, a fair idea of the TIDs under extreme conditions, can be had from the following relations derived from equations (27) and (28):

(i) where $T = 0$

$$V(0) = \langle I\rangle \tag{29}$$

and

$$P(0) = 2\langle I\rangle = 2V(0) \tag{30}$$

(ii) when $\langle I\rangle T \gg 1$, we have

$$P(T)/V(T) \approx (2\langle I\rangle T)^{-1} \tag{31}$$

which implies that $P(T)$ decays-off faster than $V(T)$ for large count rates and the crossing point is at about $T = 0.088$ for $\langle I\rangle = 5.0$ for example.

Whereas $P(T)$ given by eq. (28) is completely insensitive to the spectral shape of the chaotic light, a better and a well-known approximation for $\langle I\rangle T \ll 1$ is given by,

$$P(T) = \langle I\rangle(1 + g^2(T)) = \langle I\rangle C(T) \tag{32}$$

where $C(T)$ is the intensity-correlation function for the short counting times. Equation (32) has been recently used by Dhadwal et al. [17] for constructing a 'Time of Arrival Correlator' operative in the μs regime.

At $T = 0$ both equations (28) and (32) give $P(0) = 2\langle I\rangle$ as $g(0) = 1$ for the Brillouin triplet (equation (4)). Though equations (27) and (28) have been confirmed for the case of G–L light by Barakat aand Blake in [12], unfortunately the same authors report the $P(T)$ values for the Brillouin triplet which neither follow equation (28) (for any $\langle I\rangle$) nor the well-known equation (32) (for $\langle I\rangle T \ll 1$). This is indeed strange as a simple comparison of their numerical-results for the $P(T)$ [5] with the well-known equation (32) would have given a clear warning!

In our exact analysis of the TIDs of the Brillouin spectrum we allow for any arbitrary values of the parameters involved α, δ, Δ, T but $\langle I\rangle$ is confined to the moderate values as mentioned in the beginning of Section 2. The direct dependence of the TIDs on these parameters comes through the one-fold g.f determined by equations (15)–(19). In Section 3.2 below, we now give the explicit expressions for the various TIDs for the Brillouin spectrum, based on the determinantal form of the g.f given by equations (15)–(19).

3.2. *TIDs for the Brillouin Spectrum*

Whereas for the case of the pure Brillouin spectrum [4] where we could give the explicit *functional* forms for the various TIDs, we find it more convenient here to use the following elegant relation due to Bodewig [18]:

$$DA = A \operatorname{Tr}(\mathscr{A}^{-1} D\mathscr{A}) \tag{33}$$

where $\mathscr{A}$ is the matrix of the determinant A and Tr stands for the trace operation. Now rewriting equation (15) as,

$$Q(s, T) = A(0)/A(s, T) \tag{34}$$

and on using equation (33) in equations (25)–(26) we find,

$$V(T) = -\frac{\partial Q}{\partial T}(s, T)\bigg|_{s=1} = \frac{P(0, T)}{A(1, T)} \frac{\partial A}{\partial T}(1, T)$$

$$= P(0, T) \operatorname{Tr}\left[\mathscr{A}^{-1}(1, T) \frac{\partial \mathscr{A}}{\partial T}(1, T)\right] = P(0, T) \operatorname{Tr}(\mathscr{M}) \tag{35}$$

where

$$\mathscr{M} = \mathscr{A}^{-1}(\,.\,) \frac{\partial \mathscr{A}(\,.\,)}{\partial T}. \tag{36}$$

and $P(0, T)$ is the probability of zero-counts given by equation (20), and

$$P(T) = \frac{P(0, T)}{\langle I \rangle} \left\{2(A^{-1}(1, T) \frac{\partial A}{\partial T}(1, T))^2 - A^{-1}(1, T) \frac{\partial^2 A}{\partial T^2}(1, T)\right\}$$

$$= \frac{P(0, T)}{\langle I \rangle} \{\operatorname{Tr}^2(\mathscr{M}) + \operatorname{Tr}(\mathscr{M}^2) - \operatorname{Tr}(\mathscr{N})\} \tag{37}$$

where

$$\mathscr{N} = \mathscr{A}^{-1}(\,.\,) \frac{\partial^2 \mathscr{A}(\,.\,)}{\partial T^2}. \tag{38}$$

On using equations (35) and (36) we find that equation (37) can be recast as,

$$P(T) = \frac{1}{\langle I \rangle} \left\{\frac{V^2(T)}{P(0, T)} + P(0, T)[\operatorname{Tr}(\mathscr{M}^2) - \operatorname{Tr}(\mathscr{N})]\right\}. \tag{39}$$

From above it is interesting to observe that the higher TID is related to the lower-order ones as $V(T)$—the TID of registering the first photocount is related to the probability of zero-counts $P(0, T)$ and the TID of the conditional photocount $P(T)$ is related to both $V(T)$ and $P(0, T)$.

3.3. *Reduction in the Computation Time*

While introducing relation (33), what we had in mind was the reduction of the computation time by converting the rather involved algebra of a determinant A to that of a matrix $\mathscr{A}$. Explicitly speaking, evaluation of an *Nth* order determinant

implies $N!$ multiplications and one differentiation of the determinant implies summing up of N determinants whose one row (or column) has been differentiated at a time. Both of these operations can be phenomenally reduced by employing the usual Gauss-elimination [19] for inverting a matrix. Here, we need the following number of multiplications for inverting a matrix of $N \times N$ dimensions [19]:

$$\mu(N) = 4/3N^3 - 3/2N^2 + N/6 \tag{40}$$

and the evaluation of the Nth order determinant demands only N multiplications! Further, it is instructive to have the following ratios defining the number of times *more* we have to use the multiplications via the direct determinantal algebra as compared to the one given by the matrix inversion by Gauss-elimination [19] and the usage of equation (33):

(i) For $P(0, T)$ (equation (20))

$$v(N) = \frac{N + N!}{N + N + \mu(N)} = \frac{(1 + (N-1)!)}{2 + \mu(N)/N}. \tag{41}$$

(ii) For $V(T)$ (equation (35))

$$\eta(N) = \frac{N + N! + \frac{N}{2} \cdot N!}{N + N + 2N + \mu(N)} = \frac{1 + (N-1)!\left(\frac{N}{2} + 1\right)}{4 + \mu(N)/N}. \tag{42}$$

(iii) For $P(T)$ (equation (37))

$$\zeta(N) = \frac{N + N! + \frac{N}{2} \cdot N! + \frac{N}{2} \cdot N!}{N + N + 2N + 2N^2 + 2N + \mu(N)} = \frac{1 + N(N-1)!}{6 + 2N + \mu(N)/N}. \tag{43}$$

In our present calculations involving a sixth-order determinant (equation (15)), we have the following values for $v(6)$, $\eta(6)$ and $\zeta(6)$:

$$v(6) = 2.93927,$$

$$\eta(6) = 11.142857,$$

$$\zeta(6) = 14.71137. \tag{44}$$

For a comparison with the earlier case of the pure Brillouin spectrum [4, 20], we give below the values of $v(4)$, $\eta(4)$ and $\zeta(4)$ as the g.f in this case is represented by a fourth-order determinant (see equations (23–26) and (43) in Ref. [20]):

$$v(4) = 0.4000,$$

$$\eta(4) = 0.97436,$$

$$\zeta(4) = 1.0508. \tag{45}$$

Equation (45) clearly shows that there would not have been any distinct advantage in having employed equation (33) unlike in the present case where equation (44) shows a definite advantage in saving the computation time and it is maximum for $P(T)$,

which implies—'greater the number of differentiations involved, greater is the saving in the computation time'. Also it is easy to infer from equations (41)–(43) that 'higher the order of the determinant, greater is the saving in the computation time'. These deductions are really important as the algebra of the chaotic light with arbitrary number of peaks would otherwise obviously pose serious computational difficulties.

3.4. *Results and Discussion*

The probability of zero counts $P(0, T)$ is a direct measure of the g.f (equation (20)) and as the name implies, it ought to be maximum at the origin $T = 0$ and we find that $P(0, 0) = 1$ in Fig. 1. Even in this lowest order Time Interval measurement, there is a clear difference in the values of the Brillouin doublet ($\alpha = 1.0$) and the Brillouin triplet ($\alpha = 0.4$) as evident from Fig. 1. Also as the count rate goes higher, the probability of zero counts decays-off faster, thereby suggesting a faster registering of a photocount at high count rates.

Figure 2 gives the behaviour of the TID $V(T)$ for registering the first photocount. First we notice that $V(0) = \langle I \rangle$ at $T = 0$ as suggested by equation (27) but in our calculations we have arrived at this truth via the actual evaluation of $V(T)$ (equations (35–36)) through the one-fold g.f given by equations (15)–(19).

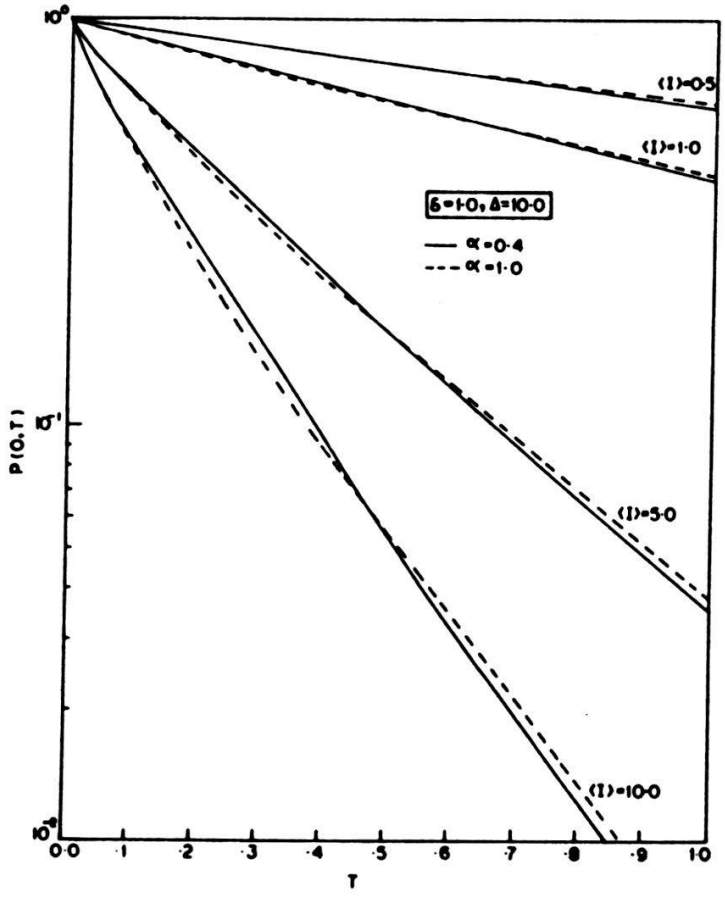

Figure 1
The probability of zero-counts.

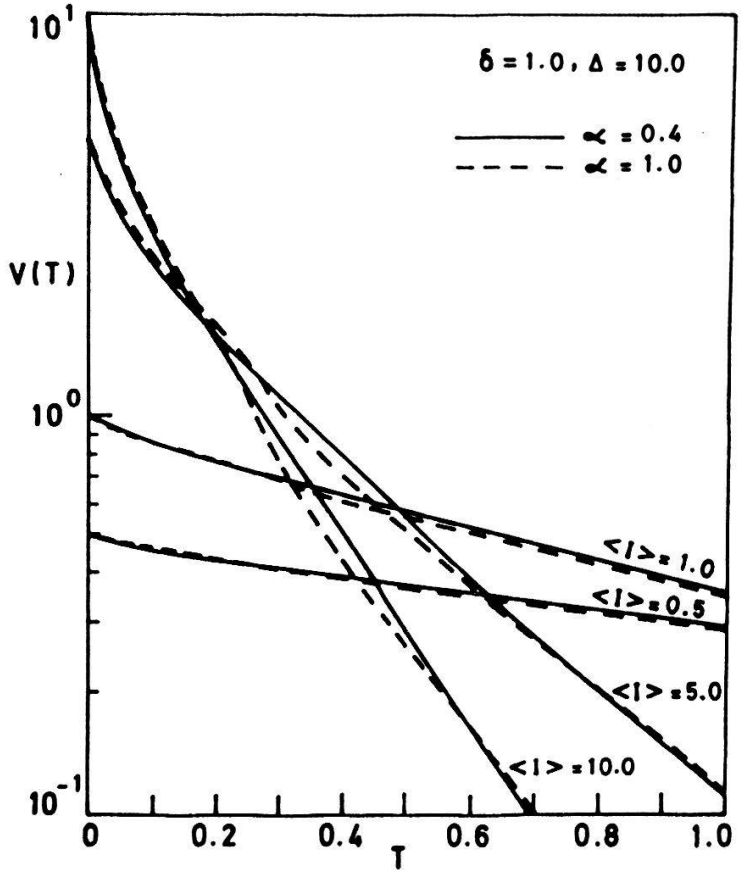

Figure 2
The time-interval-distribution for the first photocount.

As T grows, $V(T)$ represented by equation (27) becomes completely insensitive to the spectral shapes involved and in Fig. 2 we clearly see that the pure Brillouin case ($\alpha = 1.0$) is distinguishable from the Brillouin triplet ($\alpha = 0.4$), as shown by our exact calculations. At this point we need to mention that the TIS of the other multiple-peaked spectrum like the one encountered in the polydispersity case [21] is indeed distinguishable from the Brillouin doublet or triplet which do exhibit 'kinks' unlike the polydispersity case.

In Fig. 3 we give the details of the TID for the conditional photocount $P(T)$—the most important quantity in the Time Interval analysis. First we observe at $T = 0$ that $P(0) = 2\langle I \rangle$ as suggested by both equations (30) and (32), though we have arrived at this result by an actual evaluation of $P(T)$ via equations (37–38). Unlike the case of $V(T)$ (Fig. 2), we now see the emergence of pronounced oscillations in the behaviour of $P(T)$. This is to be expected only as indicated even by a simple equation (32) for $P(T)$ where we now have an explicit dependence on the auto-correlation unlike the case of $V(T)$. The general oscillatory behaviour of both $V(T)$ and $P(T)$ for the Brillouin spectrum is mainly due to the cos ΔT factor in the auto-correlation $g(\tau)$ in equation (4). However, the heterodyning of the pure Brillouin ($\alpha = 1.0$) components amongst themselves is different from the heterodyning of the Brillouin components with the central Rayleigh line of the Brillouin

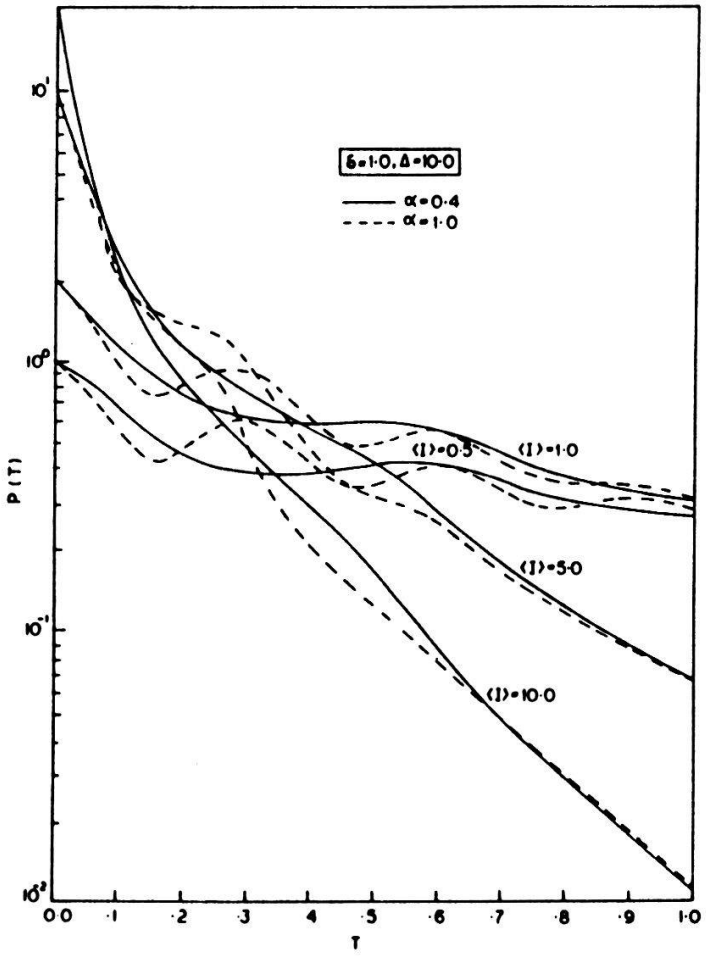

Figure 3
The time-interval-distribution of registering a conditional photocount at time T.

triplet ($\alpha = 0.4$) as shown in Fig. 3 where $P(T)$ curves for the Brillouin triplet ($\alpha = 0.4$) are flatter in comparison with the pure Brillouin ($\alpha = 1.0$) and this difference is essentially due to the central Rayleigh line which decays off exponentially unlike the side Brillouin components.

It is instructive to compare the TIDs $V(T)$ and $P(T)$ and this we do in Fig. 4. We notice that $P(0) = 2V(0)$ at the origin for both the count rates $\langle I \rangle = 5.0$ and $\langle I \rangle = 0.1$. For a high count rate $\langle I \rangle = 5.0$, $P(T)$ remains higher than $V(T)$ until about $T = 0.9$, the crossing point, after which $P(T)$ decays-off faster than $V(T)$. On the other hand for a low count rate $\langle I \rangle = 0.1$, $P(T)$ oscillates above the $V(T)$ values throughout.

Finally, we compare the approximate values of the TID $P(T)$ for the Brillouin triplet as given by Blake and Barakat in [5], with our exact ones provided here. At the outset, we notice in both Figs. 5(a) (for $\langle I \rangle = 0.1$) and 5(b) (for $\langle I \rangle = 1.0$) that the results of Blake and Barakat [5] violate even the well-known equation (32) valid for $\langle I \rangle T \ll 1$. At the origin $P(0) = 1.0375$ for $\alpha = 0.4$ in the reported values of Blake and Barakat [5] whereas equation (32) suggests $P(0) = 0.2$ and thus an error of $\sim 418.75\%$ exists at the origin itself! The same order of error is found to exist for the $\alpha = 0.6$ case when $\langle I \rangle = 0.1$ (Fig. 5(a)). For a relatively higher count rate

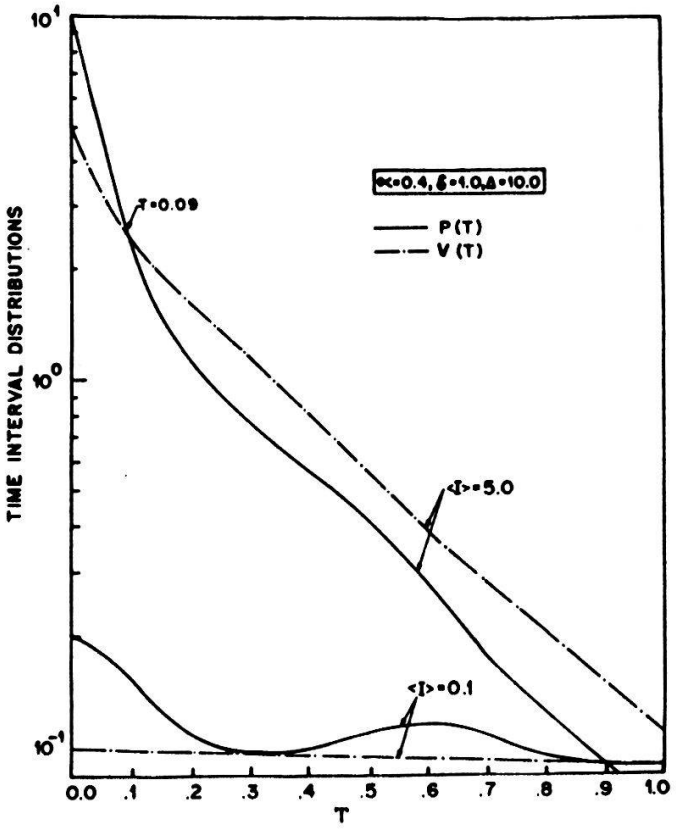

Figure 4
A comparative study of the time-interval-distribution $V(T)$ and $P(T)$, as a function of count rates. For $\langle I \rangle = 5.0$ (moderate count rate) $P(T)$ decays-off faster than $V(T)$ after $T = 0.09$ but $P(T)$ oscillates above $V(T)$ values throughout when $\langle I \rangle = 0.1$ (low count rate).

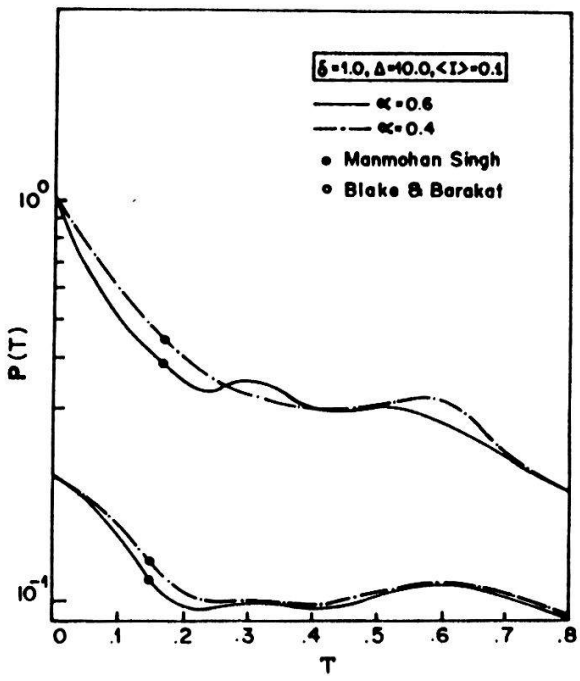

Figure 5(a)
The approximate values of the time-interval-distribution for the conditional photocount reported by Blake and Barakat [5] for $\langle I \rangle = 0.1$, compared against the exact ones obtained here.

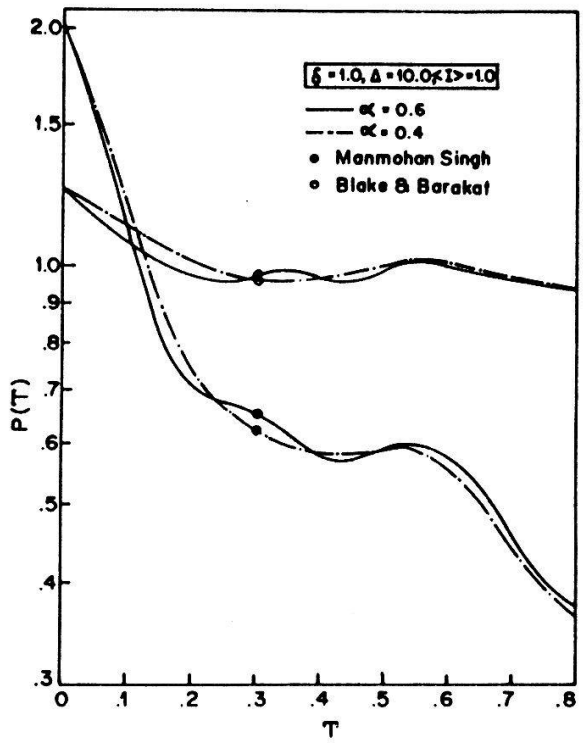

Figure 5(b)
The approximate values of the time-interval-distribution for the conditional photocount reported by Blake and Barakat [5] for $\langle I \rangle = 1.0$, compared against the exact ones obtained here.

$\langle I \rangle = 1.0$, we notice in Fig. 5(b) the value of $P(0) = 1.2444$ for both $\alpha = 0.4$ and $\alpha = 0.6$ as reported by Blake and Barakat [5] and the equation (32) on the other hand suggests $P(0) = 2.0$, which now suggests an error of the order of $\sim 38\%$ at the origin for this count rate $\langle I \rangle = 1.0$. However, as equation (32) itself is *strictly* valid under the restriction $\langle I \rangle T \ll 1$, the overall and the exact estimate of the error in the $P(T)$ values reported by Blake and Barakat [5] can be had only from our exact values here. On comparison we find that an average error of the order of 216% for $\alpha = 0.6$ and 234% for $\alpha = 0.4$ exists when the mean count rate $\langle I \rangle = 0.1$ (Fig. 5(a)) and for $\langle I \rangle = 1.0$ (Fig. 5(b)) the average error is of the order of 66.5% for $\alpha = 0.6$ and $\sim 70\%$ for $\alpha = 0.4$. However, there is some resemblance of truth (!) in the approximate results of Blake and Barakat [5] as the qualitative features (the nature of oscillations in Figs. 5(a) and 5(b)) do agree with our exact ones here. We would also like to add that as the numerical-scheme suggested by Glauber [15] (equations (11–14) here) is indeed correct, the error is thus only in the computations of Blake and Barakat [5] while using this numerical-scheme [15] for the Brillouin spectrum (doublet or triplet).

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