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Autor(en): **Massa, Corrado**

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Equation of hydrostatic equilibrium and temperature dependent gravitational constant

By Corrado Massa

Via Fratelli Manfredi 55 42100–Reggio Emilia Italy

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Abstract. The Oppenheimer–Volkoff (shortly OV) equation of hydrostatic equilibrium describes the balance between gravitational force and pressure gradient in a self-gravitating perfect fluid. In the following, a generalized form of the OV equation is obtained by the assumption that the gravitational constant G is temperature-dependent according to the law $G = G_0(1 - bT^2)^{-1}$ where b is a positive constant. Such a law is required by a variety of gauge theories used in the unification program. A direct consequence of the generalized OV equation is the existence of an upper bound $T < (2/b)^{1/2}$ on the temperature of any self-gravitating radiation field.

A variety of gauge theories used in the unification program imply a temperature-dependent gravitational constant G given by:

$$G = G_0(1 - bT^2)^{-1} \tag{1}$$

where T is the temperature, G_0 is the zero temperature value of G, very close to the currently observed value 6.67 $10^{-8}\,\mathrm{cm}^{-3}\,g^{-1}\,\mathrm{s}^{-2}$, and b is a positive constant whose numerical value depends on the details of the theory concerned [1–3]. The simplest way to fit the law (1) on the framework of Einstein's gravitational theory is to write the gravitational field equations [1, 4]

$$R^{ik} - (\tfrac{1}{2})g^{ik}R = 8\pi G T^{ik} \tag{2}$$

where $G = G(T)$ according to equation (1). R^{ik} is the Ricci tensor, g_{ik} is the fundamental tensor, $R = g^{ik}R_{ik}$ is the spacetime curvature scalar, and T^{ik} is the energy tensor. In this paper, latin indices run from 0 to 3, greek indices run from 1 to 3; commas denote partial derivative and semicolons denote covariant derivative with respect to the spacetime coordinates x^i; dots denote differentiation with respect to x^0, and apices denote differentiation with respect to x^1.

By equation (2) and by the Bianchi identity $R^k_{i;k} = (\tfrac{1}{2})R_{,i}$ we have $(GT^{ik})_{;k} = 0$, namely

$$\dot{G}T^{00} + Gt^0 = 0 \tag{3}$$

$$G'T^{11} + Gt^1 = 0 \tag{3a}$$

where $t^i = T^{ik}_{;k}$.

Consider now a self-gravitating mass described by the energy tensor of a perfect fluid:

$$T^{ik} = (\rho + P)U^i U^k - Pg^{ik} \tag{4}$$

where ρ is the rest frame mass density and P the scalar pressure; U^i is the 4-velocity of the local rest frame of the fluid.

Consider a static, spherically symmetric field; thus:

$$U^\beta = 0, \qquad ds^2 = e^\nu (c\,dt)^2 - e^\lambda dr^2 - d\Omega^2 \tag{5}$$

where ds^2 is the spacetime line-element, ν and λ are functions of r only, and $d\Omega^2$ means $(r\,d\vartheta)^2 + (r \sin\vartheta\, d\varphi)^2$ where $(ct, r, \vartheta, \varphi) = x^i$ are the Schwarzschild coordinates (r is the radial coordinate).

The normalization of 4-velocity $U^i U_i = 1$ determines $U^0 = e^{-\nu/2}$, and, by (4)(5), $T^{00} = \rho e^{-\nu}$, $T^0_0 = \rho$, $T^{11} = Pe^{-\lambda}$. For $\dot{G} = 0$, equation (3) gives $t^0 = 0$ (neither creation nor destruction of matter) and we have to consider equation (3b) only. Equation (3b) with $T^{00} = \rho e^{-\nu}$ and $T^{11} = Pe^{-\lambda}$ gives:

$$(G'/G) + P' + (\tfrac{1}{2})(\rho + P)\nu' = 0 \tag{6}$$

The (00) component of the field equation (2) in the proper reference frames of fluid elements is;

$$(1/r^2)(d/dr)[r(1 - e^{-\lambda})] = 8\pi G\rho,$$

which integrated gives;

$$\left(\frac{1}{4\pi}\right) m(r) = \int_0^r G\rho r^2\, dr \tag{7}$$

where $2m(r) = r(1 - e^{-\lambda})$.

The (11) component is

$$-(1/r^2) + (1/r^2)e^{-\lambda} + (1/r)e^{-\lambda}\nu' = 8\pi GP,$$

that gives:

$$(\tfrac{1}{2})\nu' = (m + 4\pi PGr^3)(r^2 - 2mr)^{-1} \tag{8}$$

Equations (1) (8) lead to;

$$P' = -(\rho + P)(m + 4\pi PGr^3)(r^2 - 2mr)^{-1} - PG'/G \tag{9}$$

with $m = m(r)$ given by equation (7). Equation (9), with $G = G_0 = \text{constant}$, reduces to the usual OV equation for a perfect fluid [5–7]. Consider now equation (1), which gives $G'/G = (T'/T)2bT^2(1 - bT^2)^{-1}$. If the fluid is a radiation field with the equation of state $\rho = 3P = aT^4$ ($a =$ the radiation constant) then equation (9) reads:

$$AT'/T + [Bm + (4\pi/3)aT^4 r^3 G_0](BC)^{-1} = 0, \tag{10}$$

where;

$$A = 1 + (\tfrac{1}{2})bT^2 B^{-1}, \tag{10a}$$

$$B = 1 - bT^2, \tag{10b}$$

$$C = r^2 - 2mr. \tag{10c}$$

A direct consequence of equation (10) is the existence of an upper bound on the temperature T of the fluid. To see this fact, assume $T > 1/\sqrt{b}$ at some point r_0 inside the fluid, namely $B < 0$. For equation (1), the equation (7) gives $m(r_0) < 0$, viz. (for equation (10c)) $C > 0$, and equation (10) leads to

$$AT' > 0 \tag{11}$$

T' is certainly negative. Indeed, if $T' > 0$, then $A > 0$, and (for equations (10,a,b) and for $B < 0$) $T > (2/b)^{1/2}$; all values between $(1/b)^{1/2}$ and $(2/b)^{1/2}$ would be forbidden, and T could not be continuous.

Put $T' < 0$ into equation (11) and obtain $A < 0$, viz. $T < (2/b)^{1/2}$ that is an upper bound on T.

A more severe bound, namely $T < (1/b)^{1/2}$, can be obtained via the quite different procedure described in Reference (1). In the special case of the Hawking black hole temperature, the bound $T < (1/b)^{1/2}$ can be derived by the very simple procedure of Reference (2). My thanks to an anonymous referee of this Review which brought Reference (3) to my attention.

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